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Principal Examiner Feedback

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This paper offered plenty of opportunity for students to show what they had learnt. The early questions were intended to be accessible to candidates of all abilities and indeed the modal mark for the first 4 questions was full marks. The longer, later questions provided suitable challenge for stronger candidates but also gave opportunities for restarts for students who struggled with early parts of questions. Presentation was often good but some candidates did not take heed of the requirement in some questions to show all stages of their working or to not to rely on calculator technology. For example, in question 6(c) where an application of the Newton Raphson process was required, there was a clear demand for students to show their method and yet many candidates just wrote down an answer. The formal proof question, Question 11, saw more success than in previous series as students become more familiar with this type of question.

Question 1

The first question on the paper provided a good start for many candidates who were able to score full marks. A fairly small minority solved by squaring both sides of the equation. This method was generally very accurate when used. Most candidates however, attempted the alternative strategy of setting up two separate equations.

Of those who did not get full marks, most were able to identify that they should be solving $3 - 2x = 7 + x$ but a common error was in the incorrect manipulation of the equation to get $x = -3/4$. Others lost minus signs along the way or deliberately removed them, perhaps because this was a question involving modulus signs and they held the misconception that values should therefore be positive.

Many candidates were also able to identify that a second solution came from $2x - 3 = 7 + x$ but some incorrectly assumed no negatives were permitted and instead used $2x + 3 = 7 + x$. As with the first solution, the most common error, once identifying the right equation, was due to incorrect rearrangement to give $x = 4$. Some students however, only set up and solved one equation failing to recognise that two would be required.
Although not required by the question, some candidates annotated their graphs; clearly constructing the line $y = 7 + x$ to “show” the 2 solutions. Others unnecessarily calculated y coordinates, perhaps misunderstanding what was required by a ‘solution’. A small number of candidates obtained the correct answers, but then rejected $x = -4/3$. Other candidates incorrectly gave two positive solutions. Candidates should be advised to refer back to the graph provided in the question as a means of checking the validity of their solutions. In this case that one was clearly positive and one negative. A substantial minority of candidates demonstrated a lack of understanding of the modulus function and simply changed negative signs to positive signs or assumed that the modulus function is distributive i.e. $|a + b| = |a| + |b|$. A small number of candidates included inequalities in every step of their attempt and others maintained the presence of modulus signs throughout their entire attempt.

Question 2

This question on an exponential function saw full marks for most. Of the small number of errors that were seen in part (a), a common one was an incorrect intercept of (0, 4) from replacing 4^0 with 4 instead of 1. The intercept was occasionally seen as (1, 0) although this was condoned if it was positioned correctly on the sketch. A few graphs did not go into the second quadrant and others were not carefully drawn – some did not get close enough to the asymptote of $y = 0$, sometimes due to a poor choice of scale. Other graphs bent back on themselves in the first quadrant. Some students offered an incorrect or extra horizontal or even a vertical asymptote. Although almost all knew the correct shape, a few responses had a straight line graph.
Part (b) was similarly well-answered and most used logarithms correctly to obtain a correct numerical expression for x. Those who did not use $x = \log_4 100$ and arrived at $x = \frac{\log 100}{\log 4}$ sometimes processed this incorrectly, arriving at log 96, log 25 or even 25.
Some candidates who did not know how to solve the equation resorted to trial and improvement with varied success and is not a recommended strategy.

Question 3

This question proved to be successful for many candidates and many scored full marks here.
In part (a)(i), the vast majority were able to successfully write down the sequence of terms and hence demonstrate that the sequence was periodic. An algebraic proof was not required here, and one was only rarely seen.
Perhaps surprisingly having identified the pattern in the sequence in (a)(i), not all were able to identify the order of the sequence as '2' in (a)(ii). This perhaps highlights a lack of awareness of what is meant by "order" of a sequence. Common incorrect answers here were: 'first order'; 'order 3, 5'; '± 2'or just a re-writing of their sequence from part (a)(i).
In part b), candidates were most successful when they considered the nature of the sequence rather than trying to resort to a formula. A fairly common first step was to divide 85 by 2 to give 42 with remainder 1. Many students then correctly broke down the sum into either
$42 \times 3 + 43 \times 5$ or equivalently $(42 \times 8) + 3$. Errors occurred when candidates struggled with the odd number of terms and believed they should calculate
$42.5 \times 5 + 42.5 \times 3$ resulting in an answer of 340, or when candidates mistakenly thought that the 85th term was '5' rather than '3' resulting in an answer of 341. Other errors included summing only 84 terms ($42 \times 8 = 336$) or attempts to 'chunk' incorrectly into blocks of e.g. size 5: in such attempts the sum of the first five terms was obtained
$(3 + 5 + 3 + 5 + 3 = 19)$ followed by deduction that $\frac{85}{5} = 17$ leading to an incorrect answer of $19 \times 17 = 323$. A sizeable number of incorrect attempts involving use of the sum of arithmetic or geometric series were also commonly seen.
Perhaps most surprising was the significant minority who were able to set up a correct approach and write down a correct method but who then made arithmetical errors to write down an incorrect total which cost them the final A mark.

Question 4

This question on differentiation from first principles saw good scoring and the method was widely recalled. A small number of slips with the algebra were seen and some replaced
$f(x + h)$ with $2x^2 + h$ or $(2x + h)^2$ or $2(x^2 + h)$ instead of $2(x + h)^2$. There was the occasional poor squaring and mishandling of the "2"s e.g. expanding $2(x + h)^2$ to get $2x^2 + 2xh + h^2$ was surprisingly common. Some leeway was afforded to the students with notation here although use of $h = 0$ instead of $h \to 0$ was penalised if no indication of the limiting process was seen. A small number of responses did not have an "$f'(x) =$" or equivalent and so forfeited the final mark.

Question 5

Part (a) of this question required candidates to use a given table of function values in an application of the Trapezium Rule. This was generally done very well with candidates knowing how to apply the rule. Most achieved $h = 1.5$ although a common mistake was for students to think that the number of strips was the same as the number of x values instead of being one less. Thus dividing by 5 instead of 4 resulting in $h = 1.2$. Most knew how to apply the trapezium rule and had the bracketing correct and so achieved the correct answer. A few left out some of the brackets followed by an incorrect calculation.

$$\frac{\mathbf{1.5}}{\mathbf{2}}(\mathbf{1.63} + \mathbf{2.63}) + \mathbf{2}(\mathbf{2} + \mathbf{2.26} + \mathbf{2.46})$$

With or without an outer bracket either at the beginning or end – would lead them to then calculate the expression as it is written, often leading to an answer like: -

$$\frac{\mathbf{1.5}}{\mathbf{2}}(\mathbf{4.26}) + \mathbf{13.44} = \mathbf{3.195} + \mathbf{13.44} = \mathbf{16.635}$$

There were a few who worked out the areas of individual trapezia and then summed the areas achieving a correct answer. A minority didn't know how to apply the rule

Part (b) of the question clearly stated that the result from part (a) was to be used and that the method used was to be made clear. Thus, candidates who proceeded to use the Trapezium Rule again with revised values of y received no credit.
In (b)(i) many realised that the power law of logs had to be used and realised that their answer to part (a) had to be multiplied by 10, so achieving a correct answer. Many on the other hand unfortunately raised their answer to part (a) by a power of 10.

Most of those who attempted (b)(ii) didn't realise that they had to split the expression into two parts applying log laws. Part of the expression then had to be integrated and added to their answer to part (a). Often seen was the calculation $9\log_3(2x)$ or 9×13.275. Some seemed to think that $\log_3(18x)$ meant that the graph/function had been subjected to a stretch of either 9 or 1/9. Some didn't show sufficient working to be sure that they were applying the correct method whilst others showed the correct use of logs but then just added 2 to their part (a) answer, not integrating this between the limits. Even when a correct relationship of
$\log_3 18x = \log_3 2x + \log_3 9$ was established, the subsequent argument often did not have sufficient detail to show where their final answer came from. The question clearly stated "making your method clear". Very few achieved full marks on this part.

<u>Question 6</u>

Part (a) was generally well done with many candidates scoring 3 out of 4 but very few scoring 4 out of 4. Candidates generally understood that the question required an initial differentiation and this was done correctly on the whole. However, incorrect attempts at differentiation included:

- $16\cos\left(\frac{1}{2}x\right) - 3$
- $8\cos\left(\frac{1}{2}x\right) - 3$
- $^{-}4\cos\left(\frac{1}{2}x\right) - 3$

Candidates also understood that they had to set their derivative to zero which required a combination of manipulating the algebra, applying arccos, and doubling their answer. The difficulty encountered in this part was the recognition that it was the 3rd solution which was required and most gave either the 1st or 2nd turning point. As a diagram was given in the question, candidates should use this to help them understand what is required. Some students who gave the 1st turning point also checked by further differentiation that their point was a maximum. A few candidates unnecessarily found the corresponding y value for their x value. In establishing their value of x, it was disappointing to see all of the following:

- candidates halving instead of doubling
- using degrees
- arriving at a correct answer but leaving it as 14.01 instead of 14.0

In part (b), most candidates only referred to the change of sign and didn't mention continuity. Occasionally a candidate wanted to allude to continuity but was unable to articulate this correctly. For example incorrect wording including referring to the graph as "constant". Some candidates who referred to continuity used incorrect statements such as "because x is continuous" or "because the interval (or data) is continuous".

Part (c) was answered well with many candidates scoring full marks. Amongst those who didn't score full marks, many did not heed the demand in the question to "Show your method" and just quoted their calculator's solution without any explanation whilst others just wrote down a formula but did not show an application with values embedded. Others made rounding mistakes such as giving a final answer of 4.805 or 4.81. Premature rounding of
f (5) and f′(5) also led to a final answer of 4.81. A significant number of candidates erroneously used degrees but could score the method mark if appropriate working was shown. Some candidates were able to "recover" minimal working due to the accuracy of their answer 4.804...., and gained the two marks. Some candidates also evaluated f(5) incorrectly even though it was given in the question.

It was disappointing to see students misquoting The Newton-Raphson formula, typically with f(x) and f′(x) the wrong way round, even though the formula was given in the formula book.

Question 7

This question on binomial expansion was well attempted and generally well answered by most candidates. In part (a), almost all candidates achieved at least 1 mark, either by extracting the factor of $4^{1/2}$ correctly, or for having a correct binomial coefficient multiplied by the corresponding power of x on their 3rd or 4th term. A few candidates omitted the $-9/4$ and wrote $\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}x^2$ as their 3rd term, which was not awarded the method mark.

Some candidates used 9/4 erroneously, instead of $-9/4$ in their expansion, and this could still gain the method mark but meant the two accuracy marks were lost. Candidates bracketing was sometimes incorrect but often recovered by later work.

A common response seen was a correct expansion of $\left(1-\frac{9x}{4}\right)^{1/2}$ but failure to multiply this by the "2" they had factorised outside the bracket at the beginning.

A minority of candidates attempted a direct expansion method, and many of these achieved the first mark only for a correct first term, as they often did not find the correct binomial coefficients.

Unfortunately, several candidates made number of slips, or omitted an exponent, in their final line and did not write the correct answer following correct working and so lost the final accuracy mark.

In part (b) very few could articulate correctly why this was an overestimate. Many left this part out or just wrote overestimate or underestimate with no explanation. Some were very inventive in their reasons. Poor wording/clarity on subtracting terms, such as 'decreasing trend', or 'terms decreased' but not specifically stated to be subtracted from the leading 2. Many tried substituting in values to show it was smaller which was not sufficient. The majority did not relate the reasoning back to the binomial expansion directly. Some went on to evaluate the binomial using $x = 1/9$. Some said the terms were getting smaller rather than pointing out that they were negative. Some, however, gave a very clear reasoned argument.

Several students answered part (b) with an explanation about the validity of the expansion using $|x|<1$. Some students thought the overestimation was due to the fact that the expansion had only used 4 terms and that the approximation would be better if more terms were used, with no reference to the fact that they should be subtracted from 2. A common mistake was stating that $\boldsymbol{x} < \left[\frac{\mathbf{4}}{\mathbf{9}}\right]$ which in their opinion meant that the approximation was over/underestimation due to the fact that $\frac{\mathbf{4}}{\mathbf{9}} < \frac{\mathbf{1}}{\mathbf{9}}$ or the other way round.

Question 8

Most students knew that this question was to be solved by integration although a very small number thought that the question was about partial fractions, differentiation or a mixture of differentiation with integration. There were multiple approaches to solving this question including integrating a function with simplified indices, integrating by parts and integrating by substitution. The most common and the simplest method seen was multiplying out and simplifying indices. For the candidates who recognised that $4\sqrt{x}$ in the denominator should be rewritten as $\frac{1}{4} x^{-\frac{1}{2}}$ in the numerator there was a lot of success on this question. However, there were many candidates who brought the 4 to the top and left it as 4 rather than ¼ and earned a maximum of 3/6. Some wrote $4x^{\frac{1}{2}}$ in the numerator multiplying out to give incorrect powers and losing the first two method marks.

Finding the correct limits and substituting them in a changed function was done by most leading to quite a few correct answers. Quite a few gave part or all of the final answer as a decimal, this was not surprising as most were using their calculators to do the final evaluation and calculator use did not leave the result of e.g. $(\sqrt{2})^5$ as a surd. Some candidates recognised that the area needed to be positive, however the final answer was permissible even with a negative area.

There were often errors in solutions constructed using integration by parts. There were mistakes made using the formula correctly, some bracketing errors, and some attempted parts the other way round getting harder expressions in the integral which mostly resulted for them a maximum of 1 mark (3rd M mark) obtained.

There were a few incorrect substitutions selected for integration by substitution approaches, however candidates who chose $u = \sqrt{x}$ or $u = 4\sqrt{x}$ were generally successful. A few candidates who set e.g. $u = x - 2$ in a substitution method scored few marks, if any, as such attempts did not progress the candidate towards a function they could integrate. In substitution method a few candidates made arithmetic errors in finding the new limits or did not attempt to change them at all.

Question 9

This was a challenging question for the majority of the candidates with very few scoring all four of the marks in part (a). For those that attempted the question, most scored at most 1 or 2 marks. These were the initial B1 mark for realising that $A = 50$, however many wrote that $H = 50$ and did not relate that H meant A. The second B mark for finding a value for b was very poorly answered, with many not realising a simple calculation of $\frac{180}{720}$ was needed and attempted to use an '$R - \alpha$' method or failed to take into account transformations of the sine function, showing a lack of understanding of the properties of the sine wave. Some also tried to use $\frac{\mathrm{d}H}{\mathrm{d}t}$ with very limited success. A small minority of students achieved b = 1/4. The best answers tended to use transformation arguments, recognising that the full cycle depicted must represent a scale factor of 4 though many students attempting this approach did not appreciate the effect of the modulus in determining the value of b, obtaining instead a value of b = ½. A common incorrect method came from consideration of the wheel taking 720 seconds to complete 1 revolution leading candidates to solve the equation $720b + 1.15 = 180$ or from thinking that the maximum occurs at $t = 360$ leading to $360b + 1.15 = 90$. Of those that managed to access the question, few mixed radians with degrees and managed to give an answer as $\frac{\pi}{720}$ or $\frac{2\pi}{720}$, which scored no marks. 1010 was the most common mark profile for this question, as a significant number of students went on to find α correctly (albeit occasionally in radians) as they were able to spot that at $t = 0, H = 1$. Sadly, a few that did achieve all three elements did not conclude with an equation of the model and so lost the (easy) last mark, with this final mark being scored very rarely, at less than 5% of the cohort (the second least commonly scored mark in the paper).

In part (b), many candidates incorrectly referred to the fact that the passenger was 1 m above the ground and related the value of d to this fact, for the most part because they had not appreciated that the original model already took this into account. Some responses referred to the fact that the height needed to be a positive value or could not be negative, again missing that the modulus signs in the original model accounted for this. Some responses did not refer to the context of the model in their explanation. Overall, around a fifth of candidates understood that the passenger would touch the ground without adding the constant 'd.' There were also a few illogical answers referring to air resistance, etc. A few students also incorrectly though that '+ d' was in reference to the different heights of the passengers. A small number of candidates seemed to think that the graph showed the path of the position of the passenger, referencing that the passenger "bounced off the ground" and that the '+ d' had the effect of smoothing out the ride. When students were successful in this part the most common responses were that '+ d' meant that: the Ferris wheel would not touch/scrape the ground, the passenger/seat cannot touch the ground or there needed to be a minimum gap between the passenger/wheel and the ground.

There were many attempts at part (b) without any attempt at part (a). It was unfortunately quite common that responses to this part were very difficult to read and centres need to stress to candidates that work cannot be awarded marks if it is illegible.

Question 10

Parts (a) and (b) were generally answered well by most candidates, however parts (c) and (d) proved to be good discriminators with many failing to score any marks.
Part (a) required candidates to find $\mathrm{f}^{-1}(\frac{3}{2})$. The most common approach seen was finding the inverse function then substituting in $x = \frac{3}{2}$. This was usually carried out successfully with a few candidates losing a mark for arithmetic slips when rearranging. Very few attempted the more straightforward method of solving $\mathrm{f}(x) = \frac{3}{2}$. Of those gaining no marks, the common misconception was substituting $x = \frac{3}{2}$ into the reciprocal of the function or evaluating $\mathrm{f}(\frac{3}{2})$. Several substituted $x = \frac{3}{2}$ into $\mathrm{f}'(x)$.
In part (b) most students seemed confident about how to start and the majority of those who attempted it achieved both marks. Candidates either equated the numerator to achieve
$8x + 5 = A(2x + 3) + B$ or used the long division method. There were some errors in calculating A after finding the correct value for B. Those who used the long division method made fewer mistakes than those who compared coefficients. A few candidates attempted to manipulate the numerator and expressed it in terms of the denominator with a good rate of success. A few candidates lost the final accuracy mark due to substituting the correct A and B into the wrong place to give $-7 + 4/(2x + 3)$
In part (c) it was common to see the correct boundary values given but many lost the mark by using incorrect notation. Common errors seen were $0 \le x \le 4$, $0 \le \mathrm{g}(x) \le 4$, or even
$0 \le \mathrm{f}(x) \le 4$; using < instead of $\le$; or missing out the lower boundary value altogether. Many did not realise that the values of the range of $\mathrm{g}^{-1}(x)$ are the same as the domain of $\mathrm{g}(x)$ and found $\mathrm{g}^{-1}(x)$ first but then failed to give the correct range, gaining no marks. Giving the range as 0 to 16, instead of 0 to 4 was a common incorrect answer. A significant number obtained only one of the correct values.
Part (d) proved to be demanding for most candidates. Some candidates realised they could simply evaluate f(0) and f(4) to find the boundary values but most started by trying to find an expression for $\mathrm{fg}^{-1}(x)$. Many were able to find $\mathrm{g}^{-1}(x) = \sqrt{(16 - x)}$ and used this to obtain
$\mathrm{fg}^{-1}(x) = (8\sqrt{(16 - x)} + 5)/(2\sqrt{(16 - x)} + 3)$ or $\mathrm{fg}^{-1}(x) = 4 - 7/(2\sqrt{(16 - x)} + 3)$ (using their answer to (b)). Some made no further progress from here and thus scored no marks. Of those that did carry on, common errors from this point included using 4 as the upper value rather than 16 or changing $\sqrt{(16 - x)}$ to an incorrect expression such as $4 - \sqrt{x}$. Since one of the boundary values was 0 some candidates found 37/11 from incorrect work and did not score the method mark. Some candidates, who did find the correct values, lost the final mark by using < instead of $\le$ in their inequality. It was not uncommon to see part (d) left blank.

Question 11

Most students made some attempt at this proof question, and there was a mixture of responses that achieved the full range of possible marks. It was striking that a number of students attempted to answer the question without doing any algebra, despite the clear instruction within the question. Most candidates who used algebra recognised that a strategy of considering odd and even numbers could be used to prove the statement with most using
$n = 2k$ and $n = 2k + 1$, although a minority only attempted one of these. Within the algebra, a common misconception was that $(2k)^2 = 2k^2$. Occasionally there were a few candidates that tried to consider the cases of $3k - 1$, $3k$, and $3k + 1$ which caused those candidates difficulty in progressing the proof. Although students knew that they needed to show the expression was a multiple of 2 to conclude that it was even, few seemed to be looking for the earliest opportunity to do this, and therefore did more algebraic manipulation than was necessary. Some also presented expressions like $8k^3 + 12k^2 + 16k + 6$ and declared them to be even without explaining why this is the case. A handful used
$n = 2n + 2$ for the even case, again having to do more algebraic manipulation than was needed. Most candidates who attempted this approach gained the method marks but the accuracy marks were sometimes lost for algebraic slips or more commonly for failing to give a final statement that the result was even for both odd and even cases, or simply stating that the statement was a multiple of 2 without concluding that it was therefore even. It was surprising for the odd case how many candidates had a preference for fully expanding their expressions into a cubic before factorising, i.e. $8k^3 + 12k^2 + 16k + 6 = 2(4k^3 + 6k^2 + 8k + 3)$ rather than simply factoring $(2k + 1)(4k^2 + 4k + 6)$ into $2(2k + 1)(2k^2 + 2k + 3)$ for example which is algebraically easier to achieve.
Where algebra was not used, the most common approach was to use a logic argument involving the rules of combining odd and even numbers. Where the argument was convincing students were sometimes able to gain the method marks. A few candidates attempted a proof by contradiction or by induction, but usually produced an argument that was incomplete and made little progress.

Question 12

Part (a) was generally well answered. The quotient rule was usually applied correctly for the first two marks and, given the quadratic factor was given, most candidates were able to arrive at the correct expression for $g(x)$. Some used the product rule instead and usually gained the first two marks although it was a little more difficult to get the final answer and some algebraic errors were made. Although most candidates gained the M mark in (a) their brackets were very often incorrect or missing. This was usually recovered in later steps allowing full marks to be obtained. When differentiating $4x^2 + k$, some candidates left k in the answer, or differentiated to $8x + 1$, gaining no marks. Occasionally the terms on the numerator were reversed and these attempts gained no marks in (a) as in incorrect method was implied. Some missed the 3 when differentiating e^{3x} using the chain rule. Most candidates understood there was no need to expand the denominator. Where they did so, they generally did it correctly and no marks were lost. This was a "Show that" question so "$f'(x) =$" or "$\frac{dy}{dx} =$" needed to be seen and was sometimes missing, losing the final A mark.

Candidates found part (b) more challenging. Although most knew to set $f'(x)$ to zero for stationary points, there were many marks lost due to insufficient rigour in the presentation of method when looking for one or more roots. Generally candidates understood that it was the quadratic expression that needed to be used and not any part of $g(x)$. Overall, most realised that the number of solutions for a quadratic was determined by the discriminant although a significant number did not. The majority attempted $b^2 - 4ac$ with the appropriate values but candidates often set $b^2 - 4ac = 0$ rather than $b^2 - 4ac \geq 0$. The B mark was usually gained by the use of the discriminant > or ≥ 0, it was rarely gained by making an appropriate statement. Some stated $g(x) \neq 0$ and a few $g(x) > 0$ which may have been required if full justification had been required. Most candidates gained the M mark for calculating the value of the discriminant using the correct values for a, b and c. Other methods included completing the square to find the least value of the quadratic and then considering the y coordinate. Some differentiated the quadratic to find the minimum turning point and again considered the y value. These alternative methods were often incomplete and unsuccessful in most cases seen. Few realized the range had a lower limit, but this was condoned for the A mark as the k was defined as positive in the question. A common error was to give the final inequality as $k < \frac{4}{9}$.

<u>Question 13</u>

This question was a good differentiator, with the majority of candidates gaining some credit, but only the strongest achieving full marks. Overall, candidates often demonstrated good reasoning skills and were rewarded with good marks on this question but a significant number of candidates struggled with this topic. Throughout, methods of solution were many and varied, and often involved non vector approaches that were sometimes difficult to follow.
In part (a) candidates using a vector method often attempted to subtract the given position vectors to form at least two of the vectors $\pm\overrightarrow{AB}$, $\pm\overrightarrow{AC}$ or $\pm\overrightarrow{BC}$ although errors in their components were common. Candidates unfortunately often misread $\overrightarrow{OB}$ as $\begin{pmatrix}4\\0\\6\end{pmatrix}$ or $\begin{pmatrix}4\\6\\0\end{pmatrix}$. This was as far as many candidates reached, but some were able to successfully complete this approach by establishing $\overrightarrow{BC} = 4\overrightarrow{AB}$, $\overrightarrow{AC} = 5\overrightarrow{AB}$ or $5\overrightarrow{BC} = 4\overrightarrow{AC}$ and reached $p = 32$. Some candidates incorrectly used methods like $\overrightarrow{AC} = \overrightarrow{AB}$ or assumed that the original position vectors were parallel. Candidates attempting to work out the magnitudes of vectors usually gained no marks. Others who made progress with vector methods, established the vector equation of the straight line through A, B and C and used this to find p.

Candidates often used non vector methods as well. A common approach was to ignore one component and compare 'gradients' using the other two e.g. $\frac{4-(-3)}{0-4} = \frac{p-4}{-16-0}$. This was judged to be an acceptable approach, although errors with the substitution of values were common. Some even established an 'equation' such as $y = -\frac{7}{4}x + 4$ and then substituted in $x = -16$ to find p, which, again, was acceptable, but it was a shame that so many candidates resorted to such methods and seemed to lack confidence in their ability to construct a solution using vectors. In some cases where the first method mark was awarded, candidates went on to an incorrect equation such as $p = 4 \times 7$ and were much less likely to be awarded the second method mark, as it was often unclear which vector they were working with for this step. Many fully correct answers for (a) were seen, sometimes with minimal working. It should be noted here that candidates should try to make their intention clear, as there were many occasions where examiners struggled to follow the structure of candidates' work and were unable to award marks as a result.

Part (b) was found to be challenging by most candidates, with few gaining any marks and with many not attempting it. Many candidates again worked with magnitudes of vectors and made no progress.
Those candidates who did gain credit usually used the fact that $k\overrightarrow{OB} - \overrightarrow{OC}= \begin{pmatrix}16\\4k-p\\6k-10\end{pmatrix}$ was parallel to $\overrightarrow{OA}$ to find $k = 5$ giving $\overrightarrow{OD}= \begin{pmatrix}0\\20\\30\end{pmatrix}$. Many candidates failed to realise that $\overrightarrow{OD} = k\begin{pmatrix}0\\4\\6\end{pmatrix}$, and made no progress, whilst others incorrectly assumed that

$\begin{pmatrix} 16 \\ 4k-p \\ 6k-10 \end{pmatrix} = \overrightarrow{OA}$. Candidates often drew diagrams to visualise the problem but full marks in this part were rarely awarded. A significant number of candidates who had found the correct vector, failed to find its magnitude.

Question 14

In part (a) a large proportion of candidates demonstrated a good understanding of how to find the partial fractions and the methods seen were often very clear. Substitution of $x = -1$ and
$x = 1/2$ was much more common than comparing coefficients. Generally the only mistakes made on this part of the question were arithmetic. A small number lost the final mark for failing to write out the complete partial fractions. A minority of candidates used
$A + B(x + 1) + C(2x-1)$ thinking it was an improper fraction but many proceeded correctly to found $A = 0$.
The most challenging aspect of part (b) for the candidates was to separate the variables correctly. A large proportion of candidates did not make the link to part (a) and often divided both sides by $3V$ as a starting point, i.e. $\int \frac{1}{3V}.dV = \int \frac{1}{(2t-1)(t+1)}.dt$

Where candidates did this they often would incorrectly replace the right hand side with the partial fraction they had found in part (a). For the candidates that got as far as this, the integration of the partial fractions they put in place was generally completed well with the key …ln($2t$ – 1) - …ln(t + 1) being found more often than not although a common error was leaving a multiplier of 2 when integrating the first fraction to give $2\ln(2t-1)$. Regardless of the integration a candidate had achieved, there were a large number of candidates that did not consider the constant of integration and tried to jump straight to the answer at this stage. Generally, if a candidate remembered their integration constant, they were more often than not successful in using the given limits to obtain a value which they could use to find an equation for V. A few attempts at solving the first order linear differential equation using an Integrating Factor were seen. Those employing this method were usually reasonably successful.

In part (c) the solution of 30mins was only obtained by the strongest candidates although many did not attempt this part of the question or possibly failed to see it. Of those that did attempt this part, a common answer was to obtain ½ but not recognise the units of the question were in hours, some giving it incorrectly as 0.5 minutes

As the equation for the volume was a given in this question most candidates were able to attempt this in spite of their attempt at part (b). Candidates gave the correct 6m^3 although a substantial number thought that 3m^3 was the answer. Although candidates were not required to provide units, many did so correctly and marks were rarely lost for stating incorrect units although a few instances of mm^3 were seen.

Question 15

The majority of candidates found this question accessible and were able to score consistently throughout. Where marks were lost they were usually in parts (b) and (c). Part (a) was attempted successfully by most candidates, starting usually with $\frac{5+2sin\theta}{12cos\theta} = \frac{6tan\theta}{5+2sin\theta}$. Candidates then successfully cross multiplied and used the appropriate identity to reach the required result. Another starting point regularly seen was $6tan\theta = 12cos\theta\left(\frac{5+2sin\theta}{12cos\theta}\right)^2$. Errors in the proof were rare if candidates started from a correct statement. Occasionally the " = 0" was carelessly missing.
In part (b) most candidates correctly solved the equation either by factorising or using calculator technology, but many reached the acute angle $\frac{\pi}{6}$ rather than the required obtuse angle of $\frac{5\pi}{6}$. Others found a number of solutions but failed to identify $\frac{5\pi}{6}$ as the required answer. A few candidates worked in degrees and again lost the accuracy mark. Those who had obtained an angle of $\frac{\pi}{6}$ in (b) then usually carried this error into (c) and were in a position only to earn the method marks. Candidates using $\frac{\pi}{6}$ in (c) often would reach $9(1+\sqrt{3})$, but did not seem to worry that this did not match the required answer format given in the question. There were many attempts, however, to use the correct formula for the sum to infinity with appropriate values for the first term and the common ratio. Some candidates unfortunately put effort into finding and rearranging (often incorrectly) algebraic expressions for the sum to infinity involving θ, without substituting an angle in, gaining no credit, and others used $\frac{a}{1+r}$ incorrectly as the formula for the sum to infinity. The most common mistake in (c) was failure to show the steps where the denominator was rationalised. Many candidates clearly typed an expression like $\frac{-6\sqrt{3}}{1-\left(-\frac{\sqrt{3}}{3}\right)}$ straight into their calculators to obtain $9-9\sqrt{3}$ losing the last two marks in the question.

Question 16

This question proved to be a good differentiator between candidates. It provided a challenge for many candidates but equally there were a number of accessible marks here. Most candidates attempted part (a), many attempted part (b) and at least partially attempted part (c), usually attempting to find the lower limit for k. There were, however, very few completely correct answers to this question and a substantial minority made no attempt at any part, perhaps having run out of time by the time they reached the last question on the paper.
Part (a) was mostly approached correctly with a good number scoring full marks. Candidates generally noticed the instruction to use parametric differentiation. Whilst $\frac{\mathrm{d}x}{\mathrm{d}t}$ was most often correct, $\frac{\mathrm{d}y}{\mathrm{d}t}$ proved a challenge for many who struggled to differentiate $\sec 2t$. Many did not seem to know that these functions were standard derivatives presented in the formula booklet and instead there were some long-winded and overly complex attempts to convert to sine or cosine functions and such approaches often led to errors. However, even some of the most inefficient approaches managed to persevere through lengthy algebra to obtain a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ eventually. Almost all who attempted $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ knew and attempted $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$.
Those who made errors when determining $\frac{\mathrm{d}y}{\mathrm{d}x}$ often exhibited an over-reliance on trying to match the printed result and thus 'adjusted' their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in an attempt to obtain the required value for the normal gradient. Often this meant the loss of a method mark. Candidates would be better advised to utilise a correct method rather than incorrectly manipulating their answers to match the result they are attempting to achieve and caution should always be advised when working backwards from a given answer. Nonetheless, application of negative reciprocals and equations of lines was generally successfully attempted, and candidates were clearly confident in these areas.
There were a few attempts at a Cartesian approach in part (a) which gained zero marks as the question had specified the use of parametric differentiation.
In part (b), there were a multitude of approaches seen and candidates were often at least partially successful. Most candidates recognised the need to begin with a trigonometric identity, usually $\sec^2 t = 1 + \tan^2 t$ and rearranged the expressions for x and y to obtain expression for $\sec^2 t$ and $\tan^2 t$ before substituting into the identity to obtain an equation in x and y which could in turn be rearranged to the required form. However, sometimes candidates took a far lengthier approach, beginning with e.g. $\sin^2 t + \cos^2 t = 1$ and using x and y to find expressions for $\sin t$ and $\cos t$. An alternative and popular, though inelegant, method was to start from the target Cartesian equation and substitute in the trigonometric forms for x and y before rearranging to obtain an identity. Provided a conclusion was given, this approach was credited with both marks. Occasionally candidates used the equation for x to obtain an expression for t in terms of arctangent before substituting into the expression for y which, if correctly done, was an acceptable approach. Several students just substituted the coordinates of points such as P into the given Cartesian equation, stating that they satisfied the equation. Such attempts did not constitute a proof and so gained no marks.
Part (c) provided a particular challenge and usually only the lower limit was attempted. Most candidates appeared to completely miss the need to find an upper limit. Of those who did manage to find both limits, only a small minority of candidates got to the final solution with correct inequality interpretation.

It is worth noting that candidates achieving full marks for this part very often had clear construction lines on their graph that enabled them to see the correct approach.
Most often though, candidates gained the method mark for the lower limit, $\frac{43}{8}$, for identifying that two intersections were required and set up a correct inequality for the discriminant for the quadratic equation obtained by equating the equations of the line and the curve. Common errors in manipulation, particularly of the coefficient of ½ from the quadratic meant that a number of candidates did not earn the accuracy mark for 43/8.
Many candidates stopped after obtaining a lower limit for *k*, but others explored the parameter limits at $t = -\frac{\pi}{4}$ and $t = \frac{\pi}{3}$ to identify k values of $\frac{13}{2}$ and $\frac{23}{2} - \sqrt{3}$ for the line passing through the associated points. Very few, however, recognised $\frac{13}{2}$ as an upper limit and a number of candidates assumed incorrectly that $\frac{13}{2} < k < \frac{23}{2} - \sqrt{3}$. A minority of candidates incorrectly thought that setting $\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = -\frac{1}{2}$ would provide limiting criterion for *k* values.
In summary this was an interesting question which gave rise to a wide variety of responses in all three parts. There were instances of both succinct and ingenious approaches seen, particularly in part (c) and it was pleasing to see tenacity in some response where candidates had inadvertently opted for less efficient approaches but had persevered nonetheless to obtain the required results.

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