Examiners' Report

Principal Examiner Feedback

Summer 2022

Pearson Edexcel GCE

AL Further Mathematics (9FM0)

Paper 01 Core Pure Mathematics

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Introduction

Overall, there was good access into all of the questions, with question 10 perhaps suffering a little from being at the end of a long paper with a few more incomplete answers, though most did manage to attempt it, though there was little evidence that candidates ran out of time with some question answered multiple times by some candidates. The only questions with mode less than full marks were 9 and 10, where the mode was one off full marks, indicating good access was available in all questions.

A few candidates made little progress throughout but these in the minority, with a more sporadic performance (some very good, some very poor answers) being common. This may reflect that content coverage has not been complete due to the impact of Covid, with different centres having focussed on different topics. Each individual question provided access to candidates, but the holistic performance across the paper determined the overall outcomes.

There were some weaknesses noted in algebraic skills, sometimes causing candidates to do a lot of extra work, while marks were sometimes lost for failing to write conclusions after things were proved, something candidates should try to ensure they remember to do.

Question 1

This straightforward complex number question proved to be very accessible with nearly two thirds of candidates scoring full marks on this question, and over 85% obtaining at least 5 of the 6 marks. This was well placed as an opening question, easing candidates into the paper.

Almost all candidates achieved the mark in part (a), with non-responses being the main reason for loss of mark, although $-2+3\mathrm{i}$ was seen on occasion.

Part (b) was well done by a majority of candidates. Most were able to find the third root by considering the product of the roots $\left(-\frac{52}{13}\right)$, or by first finding the quadratic factor and then using this to find the linear factor. Some successfully attempted the inefficient method of long division to find the linear factor, while a few used the sum of roots, which was an efficient way. Sign errors caused some problems where candidates used the product of the roots or the pair sum. It is noteworthy that a significant proportion of candidates omitted to list the three roots in part (b)(i), losing the A mark, even though they were asked to solve completely.

Most candidates went on to use either their expanded equation or the pair sum of roots to obtain a value for *a* (with errors depending on the value of their third root — common mistake was finding *a* = 29 (=16 + 13) from an error in the signs.). Some attempted to use the sum of the roots in order to find a value for *a*, gaining no marks.

The Argand diagram for part (c) was usually correct, following through their third root, and most candidates achieved the mark for it, though a few candidates made the mistake of plotting their value of *a* (so (-3, 0) instead of (-4, 0)) rather than the root found in (b). The scale was often not good, but was not stringently assessed although labelling was expected. Some sketches had no, or poor, labels, or had -4 much closer to O than 2 was to O or equidistant. A greater care of scale and labelling is needed.

Question 2

The majority (over 75%) of candidates were able to access this question with full marks or 3 out of 4 marks (losing the final accuracy mark) being the most common mark profiles seen in a bimodal distribution.

Most candidates were able to successfully find values for $\cosh^2 x$.This was done usually by factorising their quartic equation, although there was use of the quadratic formula demonstrated frequently. There were also several instances where candidates used their calculator to find both values for $\cosh^2 x$ with no evidence of workings which immediately lost the first two marks since the question specified reliance on a calculator was not allowed. In such cases candidates need to be aware that full methods without resorting to a calculator need to be shown.

Those who found both values for $\cosh^2 x$ usually dismissed or omitted the value of $-\frac{1}{8}$ correctly and proceeded with their $\frac{9}{8}$ (usually the correct value), though some did attempt to find values of $\cosh x$ from their $-\frac{1}{8}$ (which did not affect their overall solution).

The candidates who obtained the second method mark were more often those who took the positive square root approach and using the correct formula for $\text{arccosh}\, x$. There was then mixed success in using logs to find answers in the correct form, with many candidates giving only the positive solution of $\frac{1}{2}\ln 2$, or giving spurious extra solutions from the negative square root. Those using the exponential form for $\cosh x$ and solving a quadratic in e^x were fewer in number, but the proportion achieving 2 correct solutions was higher via this method.

There was small number of candidates who attempted to use the exponential definition for $\cosh x$ in the original equation to reach a quadratic in e^{4x}. Some did not use a valid method to expand their brackets, while those who did so successfully would have found this a time-consuming approach and not many successfully achieve answers via this route.

Question 3

This question was overall very well answered, particularly by those candidates who wrote their expression for y in terms of cos x. Nearly 60% gained full marks, though scores of 0 and 4 were also common.

The sole method mark in part (a) had quite a long scope, and those who were not able to identify the integrating factor usually scored no marks at all. However, the majority of candidates managed to score the first three marks by reducing the differential equation to one of the format $\frac{\mathrm{d}y}{\mathrm{d}x} + Py = \mathrm{g}(x)$, finding the correct integrating factor, and continuing to an equation of the format $y = \mathrm{f}(x)$ as required. Some integrated incorrectly from $y \sec x = \int \mathrm{e}^{2x} \mathrm{d}x$ to give $y = 2\mathrm{e}^{2x} + c$, but most were able to achieve the correct result and remembered to include the constant. A few candidates made sign errors or slips in the integrating factor leading to attempts at integration by parts on an expression involving $\mathrm{e}^{2x} \cos x$ or $\mathrm{e}^{2x} \cos^2 x$ or similar, gaining only the method mark.

For part (b), the main key factor seemed to be whether the candidates had written their particular integral in terms of cos x or in terms of sec x. While many candidates achieved the first mark and found a (correct) value of c, those candidates who had written the equation in terms of sec x often failed to see that $\frac{1}{\sec x} = 0$ when cos x = 0, and failed to find a correct method to solve their equation and hence could not reach a correct solution. Other less common errors included letting c be a function of x, leading to incorrect solutions. In contract, the vast majority of students who achieved an expression for y in terms of cos x achieved the correct answer of $\frac{\pi}{2}$.

Question 4

With the modal mark being full marks, scored by nearly 60% of candidates, this question was another that was very well answered. Candidates seemed to be very familiar on using the method of differences on problems involving logs, although a few problems were caused by the lower limit of 2, and some of the work in part (b) was sketchy.

In part (a), the majority of candidates correctly used log laws to convert the quotient into a difference, following the lead of the question, and went on to substitute values of typically at least r = 2,3,4, n-1 and n. The majority of candidates who used these values of r, continued to a fully correct answer. Occasionally, r = 0 or r = 1 was used, which lead to inaccurate answers due to logarithms having arguments outside their domain. A few did not write out sufficient terms to make the non-cancelling terms clear, and lost the last two A marks as a result. Also, some candidates gave answers with ln1 included before removing later, whereas others did not show ln1 in their intermediate answer. They then either used −ln2 as ln1/2 and multiplied all 3 terms or demonstrated laws of logs separately. Proofs should be convincing, and require a higher bar to be cleared than simply finding a result.
It is noteworthy that there was a significant proportion of candidates who combine logs to a single term and used a cancelling of ratios instead of applying differences. Such were able to score marks, as the methods are consummate, but candidates should aim to use approaches specified in papers.
In part (b), many candidates used part (a) and log laws correctly to lead to the correct answers. The most common incorrect methods involved failing to use the power law of logarithms or using it incorrectly. Also, accuracy was occasionally lost where candidates used $r = 49$ or $r = 51$ when subtracting. Some did not applying the result of part (a) and instead simply substituted values into the given printed summand.
There was some confusion with the power, with a few candidates incorrectly bracketing their log power while others completely missed the power of 35 altogether. A number of responses were given full marks on a technicality that no step was incorrect, even though it seemed the power was being mistreated by being left as a power until the last step instead of being taken down as a multiple before applying the result from (a). Those who got the answer fully correct usually simplified their answers fully.

Question 5

Another very well approached question with 96% scoring 4 or more marks out of the 6 available. Almost all candidates attempted to find the determinant, with most doing so correctly. Some candidates lost the A mark for either making a slip when simplifying their expression for the determinant, usually when dealing with the $-3(2a-12)$ incorrectly. Common incorrect values were 44 or 24, while some lost the A mark for not referencing that det $\mathbf{M} \neq 0$ within their conclusion, or failing to state the conclusion that the matrix was non-singular.

In part (b) most candidates worked through all the correct stages in the process to find the inverse matrix, with only minor slips being made. A good attempt was generally made at finding the matrix of minors. Where candidates took one step at a time in the process of finding the inverse matrix they were more likely to be successful and more unlikely to make basic errors. Minor errors when finding the matrix of cofactors and/or transposing the matrix still enabled them to gain the method mark as long as the process was clearly correct. Most completed the process by dividing by their det **M**.

Question 6

Still at this stage of the paper well over 50% of candidates were able to score full marks for the question, with the processes required being well demonstrated. For responses that were not fully correct the performance was much more mixed, with a fairly uniform distribution across all the other possible scores.

In part (a) a surprising number of students did not set up partial fractions in the correct form, which left them unable to make any real progress. The incorrect form was usually $\frac{A}{x+1}+\frac{B}{x^2+4}$ but $\frac{A}{x+1}+\frac{Bx}{x^2+4}$ was also seen. The majority however did have the correct form for the partial fractions and made a valid attempt to find values for their constants. Most multiplied through to form a suitable identity and substituted for x to find the values, with only relatively few equating coefficients, mostly successfully.

Most students who had the correct form for the partial fractions from part (a) realised that for part (b) they needed to split their integral into three parts and achieved the M mark for at least two integrated terms. Generally, good integration skills were shown in this part of the question. The integration using logs was usually successful but there were more errors in the integration of the arctan term. Many candidates continued to use the correct limits and the correct log rules to achieve a single ln term to achieve the final method.

A common error when integrating $\frac{x+2}{x^2+4}$ was not to split the terms and instead have some unpleasant muddle involving logs or arctan. Amongst those that integrated sensibly common slips were missing the half from $\ln\left(x^2+4\right)$ or integrating to $\frac{1}{2}\arctan\frac{x}{2}$.

Question 7

The first question on the paper where less than 50% scored full marks, however it was bimodal with 6 or 7 out of 7 both achieved by 35% of candidates, so did still provide good access. The final mark of the question was often the one lost, for including extra solutions.

In part (a) the majority of candidates found a correct expression for zz^*, although a few either missed a square on one of the terms or got a minus instead of a plus. Some candidates lost the A mark for not stating a conclusion, something candidates should make sure is routine part of their answers.

In part (b) most candidates started by writing $a^2 + b^2 = 18$, and then multiplied top and bottom by $a + b\mathrm{i}$ to form an equation with real denominators, and many were successful in extracting correct equations by equating real or imaginary parts. The approach of multiplying both sides by $a - b\mathrm{i}$ to leave $a + b\mathrm{i} = \ldots$ was also usually successful although not as clear as the first approach. A minority of candidates attempted the alternative approach of finding the modulus and argument for z^2 and then z. In general this approach was less successful as candidates tended to work in decimals and were often unable to find values for a and b.

A few candidates lost the final A mark for making slips in the algebra when solving their equations, but many more lost the final A mark for failing to 'choose' the right solutions at the end, with lots leaving all 4 solutions ±4 and $\pm\sqrt{2}\mathrm{i}$. Those using $ab = 4\sqrt{2}$ or equivalent, rather than two equations in a^2 and b^2, were more likely to find just the 2 correct solutions

Question 8

The difficult of the paper did increase over the last few questions with less than a quarter of candidates able to score full marks for this question, though 85% were able to score at least 7 out of 12 in a tapering distribution of scores from the mode of full marks all the way down to zero marks.

Part (a) tested the candidates ability to write out a proof, something that many are not good at, though the general approach was known by most. Success depended on candidates showing a complete logical sequence of steps leading to the required answer. While the majority of the candidates managed to find a correct expansion and grouped terms correctly, a significant minority did not state $\left(z+\frac{1}{z}\right)^6 = 64\cos^6\theta$, leading to loss of accuracy. Some confused $z^6+\frac{1}{z^6}$ with $\left(z+\frac{1}{z}\right)^6$ and did not make much progress. Other less common errors involved failing to show how the terms in the expansion could be grouped to achieve an expression where cosine had multiples of θ as arguments. Some candidates omitted $64\cos^6\theta$ and some the $32\cos^6\theta$. Losing the first or final mark.

Almost all candidates stated the correct value $H = 2$ for part (b).

In part (c), the majority of candidates succeeded in forming a suitable expression leading to a volume of revolution and went on to attempt to integrate and apply the correct upper and lower limits. However, obtaining the correct expression for the volume, and carrying out the integration, were not always done well. The most common mistakes were not spotting the argument had changed to $\frac{x}{4}$ leading to an incorrect integral, using the full volume of revolution through 360º, or incorrect use of limits. Some used θ and appropriate substitution which scored the M mark but changing to x caused some problems. There were some very good solutions and fully correct answers but also many incorrect answers. Common incorrect answers included 12.28 (half) and 49.12 (double).

Most candidates made a successful comment in part (d), usually either relating to the smoothness of the paperweight or the equation of the curve. It was noticeable that some candidates did not understand what a paperweight was and assumed it was made with paper! Some candidates incorrectly argued that the thickness of the material had not been had into account, but this paperweight was not hollow, so thickness was not relevant here.

Question 9

The first question for which the modal mark was not full marks, but was in this case 5 out of 6, with 6 and 4 marks being the next most common scores (about equally likely). Over 80% scored at least 3 marks, but there were many who made little progress in either part.

Part (i) (a) proved difficult for candidates to explain, despite being a deceptively simple question expecting the answer that one of the limits is infinite. The intention was often clear but incorrect phrasing or vocabulary made it difficult for marks to be awarded. While most candidates had some idea the interval being integrated over is unbounded they found it difficult to express this clearly, with the most common comment being that one of the limits is infinity. It was not always clear whether they were referring to one of the limits or the integral as a whole. A huge variety of wording used by candidates demonstrated a lack of depth in their understanding of the definition of an improper integral.

Part (i)(b) was much more successful but again, students did not always appreciate the importance of the correct language. Most candidates attempted to write down the integral, but a significant number did not replace ∞ with a parameter. They then went on to integrate, but some substituted ∞ instead of using a parameter. Candidates either integrated $\cosh x$ directly to give $\sinh x$, or expressed $\cosh x$ in terms of exponentials and then integrated, with incorrect processing of limits being the most common reason for dropped marks. Those who did use a parameter and correctly applied the limits 0 and t (or a) usually attempted to find the limit as their t tended to infinity. If they got this far they often made the correct statement and drew the conclusion that it is divergent, but some omitted this. Incomplete statements were more common in those who used exponentials, so the last A mark was often not awarded. Those who opted to integrate to $\frac{1}{2}\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)$ sometimes did not consider what happens as t tends to infinity for **both** e^{t} and e^{-t}.

Part (ii) was mostly answered well with both approached shown in the scheme being seen frequently. Those who went straight to $\tanh x = \frac{p}{4}$ were able to achieve and answer swiftly, however some made lots of work for themselves by working in exponentials and this increased the chances of making an error somewhere along the way. The most efficient responses used the range of $\tanh x$ to state the range for $\tanh\left(\frac{x}{4}\right)$. A few of those who used the exponential form struggled to reach an expression in e^{2x}. There were some algebraic errors in their working but the most common error was to use the inclusive inequality, the inequality the wrong way round, or a single inequality, usually $p < 4$.

Among those that took the exponential approach some tried to form an equation in terms of e^{x} and use the discriminant to form an inequality in p – it rarely led to the correct final answer, being often abandoned. For those who continued to reach an inequality for p from an exponential definition, most managed to identify 4 as a boundary value but then left this as a single inequality, often $p > 4$.

Question 10

Again for this question, the modal mark was one less than full marks, with one mark either side of this providing the next most common score, followed by zero marks. Perhaps being the last question had an effect, with some not reaching this stage, there were signs that they were pressurised for time. The distribution across the remaining marks was uniform.

Part (a) did not work quite as expected, with many candidates not realising that to show a function is a particular solution requires only to show it satisfies the differential equation, and instead setting out to find the complementary function first, so the form of the particular integral could be decided. Even among those who realise the complementary function was not needed, many used a general form and found values for constants, rather than using the specific function given, making the differentiation harder as often a cosine term was included as well. Fortunately, this did not seem to affect their overall marks for part (a), but it did result in many inefficient methods being seen when attempting the particular integral. No more than 30% of candidates used the most efficient method of starting with the given particular integral, differentiating twice and substituting into the given differential equation. This proved very straight forward to do, although a notable number then omitted any kind of conclusion and gained only 3 of the 4 marks available. Many candidates used the laborious method of using the form $\theta = \lambda t \sin 3t + \mu t \cos 3t$ for their particular integral.

Most students were able to construct the general solution successfully for part (ii), though many repeated work finding the auxiliary equation and hence complementary function having already found it in part (i) beforehand to decide on a form for the particular integral. The most common auxiliary equation was the correct equation of $m^2 + 9 = 0$, however $m^2 + 9m = 0$ was also commonly seen. The question was generally answered well here although $\theta =$... was sometimes missed or the letter x used instead of t.

Those who successfully navigated part (a) usually went on to do well in part (b).Candidates who had the correct form for their General Solution found it easy enough to find their A with mixed results for B. Those who recognised how to differentiate successfully here, were usually able to find $B = 0$, though some guessed this or obtained in fortuitously. Assuming an answer was reached with the qualifying methods gained, candidates were then able to find an estimate for the angle at $t = 10$ seconds, and many correct answers were seen, although 0.66 was sometimes given, losing accuracy.

Assessing the model in part (c) was done with varying degrees of success. Those offering a percentage error and a conclusion were most successful in obtaining the B1 mark. The difference in signs between their answer to part (b) and the given value caused some confusion with a significant number of candidates thinking that the model was not good because it was predicting that the pendulum was on the other side of the vertical. Those who suggested it was a good model were more likely to get the mark here but many failed to back this up with a quantitative comparison of their answer and the given 0.62.

A fair number of candidates did give the correct response to part (d), but there was a, perhaps surprisingly, also a large proportion that did not. Many did not attempt it at all or were simply unclear about the correct answer, despite simple harmonic motion begin expected on the specification. Occasionally incorrect variables were seen, perhaps as they were recalling the correct form from a text book.