Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

Summer 2019

Pearson Edexcel GCE AS Mathematics
In Further Pure Mathematics 2 (9FM0/4A)

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Introduction

This was the first sitting for this paper under the new specification, with small entry due to being an option 2 paper. It provided a good mix of some familiar topics (such as complex loci (Q1 and 17), eigenvalues and eigenvectors (Q2), reduction formula (Q5) and polar coordinates (Q8) as well as some new topics to the specification (recurrence relations (Q3), number theory and combinatorics (Q4) and group theory (Q6). Q8 proved the most challenging question, but candidates showed a good aptitude when it came to the new topics.

Question 1

This question was generally answered well, and provided a suitable opening to the paper, though there was a discriminating mark in part (b).

In part (a) the majority of candidates knew the procedure for this part and proceeded to apply the magnitude formula correctly to both sides. However some errors expanding and manipulating to the required form were seen leading to the loss of the accuracy mark. Only a small number of candidates forgot to square the 4, or ended up with negative y^2 terms.

The first mark of part (b) was scored by the majority of candidates via completing the square, usually correctly. Knowing what to do with this form proved more challenging, with many giving 16/15 as the answer at this stage. Some did state the centre and radius to gain the second mark, but went no further. Those who drew a sketch were more successful in spotting the correct method required. Only a minority went on to score the A by using their centre and radius appropriately. A few candidates tried to differentiate as a “maximum” was required, and such cases usually only scored the first mark.

Question 2

This question was well answered by candidates, although a lack of attention to details lost some marks. The methods required were shown to be well understood with accuracy in part (b) being the main cause for lost marks.

For part (a) most candidates understood how to find eigenvalues and successfully found the characteristic polynomial before attempting to factorise. There were only a small number of cases of incorrect factorisation, some of the algebraic manipulation being inaccurate and resulting in 3 distinct eigenvalues, such as 2, 4 and 6.

As part (a) was a “show” question, evidence was needed that there were repeated roots, not simply an assertion that such was the case, so some lost marks because they didn't specify either that 2 was a repeated eigenvalue or the other eigenvalue being 8.

In part (b) errors mainly arose from bad algebra, particularly for the eigenvector related to eigenvalue 8, but the correct method for finding eigenvectors was shown by most. There were many correct alternatives found for the eigenvectors for $\lambda = 2$ and attaining three correct eigenvectors **A** for was the most common outcome for this part.

Again, part (c) was answered well by candidates, who used their column vectors to produce a suitable matrix **P**. Due to the follow through mark here, that meant most candidates were able to score both marks in this part. A few candidates normalised their eigenvectors first, the process of diagonalising a symmetric matrix using **P** and its transpose, which still gains the marks here. Such a procedure was not necessary and indicates that some candidates are unaware of difference in the two cases.

Of those who did not achieve full marks in part (c), this was mainly due to only having found one eigenvector for the eigenvalue 2. In such cases either use of the zero vector as one column, or a repeated column or simply leaving part (c) unanswered resulted. There were a few cases where the two eigenvectors for $\lambda = 2$ were multiples, which could score only one mark in part (c).

Question 3

Recurrence relations are a new topic on the specification but the methods required for solving were shown by most. The main hindrance for this question was a lack of showing method to achieve a given solution in part (b).

Part (a) was accessible to most student who were able to write down the correct relation.

In part (b) the process of forming and solving the auxiliary equation before setting up and solving equations to find the coefficients was demonstrated well by most candidates. However, the question was a "show" and with the given answer on the page, candidates need to ensure that sufficient working is shown. An over reliance on calculator technology meant that many candidates simply wrote down the answers 1.2 and −0.2 to the auxiliary equation, and the values of a and b from the simultaneous equations – but these were given in the question. Particularly for the latter, evidence was required as to where these values came from and so many student lost marks for a failure to show method. While calculator technology is encouraged to be used appropriately in the new specification, candidates need to be more aware of where is and is not appropriate. Where answers are given, method should be evidenced.

Where candidates had given an incorrect answer to (a), they either made little progress in (b) when their solutions to the auxiliary equation were not what was expected, or else managed to find the correct solutions for their incorrect equation – as the answers were on the page.

There were only a very small minority of candidates who tried to prove the formula using proof by induction, and this was rarely fully successful.

For part (c) very few candidates used the envisaged method as they did not realise that $(-0.2)^N$ is negligible for large N. However, this was not a given answer and so in this part use of calculator method were acceptable and many gave $N = 55$ as the answer from little or no working, or as a restart after failed attempts to solve a log equation with $(-0.2)^N$ included.

Question 4

Another of the new topics to the specification and again candidates found this question accessible, with most able to make a start on both parts.

For part (i) the candidates who used Fermat's Little Theorem in the form $6^{12} \equiv 1 \pmod{13}$ usually obtained full marks had more success than those who used the form $6^{13} \equiv 6 \pmod{13}$. Confusing the two versions and forming $6^{13} \equiv 1 \pmod{13}$, or other incorrect indices, were seen in a small number of cases, but it was pleasing to see that most were able to recall the correct formula.

The process of reducing the the power using its remainder modulo 12 or 13 (as appropriate) was usually carried out well, and most were able to reach a least positive residue, though some left the answer as 36. Attempts via $6^{13} \equiv 6 \pmod{13}$ tended to be cumbersome, with many more steps required, and thus were more prone to error.

For part (ii) the vast majority of candidates answers (a) correctly, and part (c) was also correct more often than not, with at least the method mark gained by most. Parts (b) and (d) proved the most challenging parts of this question. For (b) most candidates recognised 4! had some part to play, but many left it at that, treating the friends as a block but not considering the order in the block. Only a minority successfully completed this part. Part (d) was also found to be difficult by many candidates, though many did gain the method for attempting to subtract something from 5040, but the attempts at what to subtract were usually incorrect.

It was good to see some candidates attempting to explain their reasoning through diagrams - candidates who did this generally obtained good marks in this question and were able to think through some interesting other approaches to the question yielding the correct answer.

Question 5

This was another familiar topic to candidates, having been on the previous specification, but the proof of a reduction formula such as this one is still a challenge for many. Those who had practised similar questions to know the split required did well, while others found it difficult to make much progress.

Many candidates were able to answer part (a) using the main method shown on the mark scheme, splitting $\text{cosec}^n x$ into $\text{cosec}^{n-2} x$ and $\text{cosec}^2 x$, and correctly integrating with the appropriate choice of u and v. Those who attempted the alternative ways were fewer in number and mainly unsuccessful. Most of the algebra seen was correct, but some weaker candidates could not start this part or made errors at an early stage in their integration.

Good candidates made their final method in part (b) very clear, using a complete series of reduction formulae and substituting explicitly at the end to obtain 3 terms which summed to the given answer. A few candidates made arithmetic slips or misused the reduction formula, failing to substitute values for n correctly.

Both parts had given answers ("prove that" and "show that") and once again candidates need to remember to be clear in all steps of working to produce a convincing solution in such case. Many lost marks in part (b) in particular for arriving at the given answer without sufficient evidence of the substitution into their formula having been made.

Question 6

Another fresh topic to the new specification, but once again it was answer well by most, certainly part (i). It is clear that the new topics have been covered well in preparing candidates for the exam.

In part (i) the most common score profile was M1 M1 A0 A1. The general outline of the proof was attempted well, but many lacked the rigour required as they did not achieve suitable expressions for the expansions to verify both yielded the same. The definition of associativity was understood by nearly all of the candidates sitting the paper, but a small number of them gave a numerical example of it working for once case, rather than a proof. A single example will be required in order to show an operation is not associative, but is not sufficient to prove an operation is.

Part (ii) is where the most of the difficulty in this question lay. For part (a) many candidates realised the group was cyclic since 3 (only a small number used 5) was a generator, but some merely stated this was so because its order is 6 and did not show how it generated the rest of the group. Those who listed the powers of 3 (or 5) would usually gain both marks for completing the argument that the group was cyclic. Others assumed that writing out the group tables was proof of being cyclic.

Identifying the subgroups in part (b) produced a variety of response, but usually scoring two of the available marks. The subgroup {1, 17} was usually identified correctly, but {1,7,13} was sometimes missed, as were one or other of {1} and H itself. many also identified spurious subgroups such as {1,5}, {1,7}, {1,13} and so on, assuming 1 and any element formed a subgroup. Nevertheless, many were able to identify the three correct subgroups and no others.

Part (c) proved provided the most challenge for candidates in this question. Most commonly either 3 or 0 marks were scored on this part as candidates either knew what to do and got it correct, or did not even know where to start. Many would just match up elements in ascending order, which did not gain access to marks. Others attempted to explain what an isomorphism is in general terms rather than identify a specific isomorphism for the group.

There were some good attempts at defining mappings algebraically by identifying that powers of a generator needed to map to powers of a generator. Candidates attempting such an approach usually scored well.

Partially correct attempts at this approach tended to stem from attempting to match elements by order, which would give correct pairings $1 \rightarrow 1$ and $6 \rightarrow 17$ and that $\{2,4\} \rightarrow \{7, 13\}$ and $\{3,5\} \rightarrow \{5,11\}$, but would then choose incorrect pairings for the latter two elements for their pairings for $\{2,4\}$. Some did not make a selection at all, but left the set-wise mappings, which was not sufficient information to identify an isomorphism.

Question 7

The main challenge of this question came in part (b), with many good attempts at part (a) seen.

Candidates who used the main method exemplified in the scheme in part (a) were mainly successful in reaching the correct circle equation, although some did not then express it in Cartesian form. Attempts using the other methods were less common and far less successful, as the complications of algebra led almost inevitably to errors.

For part (b) a number of candidates were unable to place their circles correctly on the Argand diagram, placing their centres on the real axis not the imaginary axis, or above the real axis when it should have been below. Some omitted this part entirely - usually if they had made little progress in part (a), although circle C did not depend on any work in (a) and should have been an easily accessible mark. The sketching of simple loci such as these ones should be providing candidates some accessible marks at this stage of the paper, so it was surprising to see such difficulty with them.

Question 8

Candidates found this question very demanding, particularly part (b), but being the final question it was possible that timing was an issue for some candidates at this point. There were some good, complete answers seen, so well prepared candidates were able to complete the entire paper, but others may have spent more time than necessary on earlier questions leaving them short on time for this one. It was often difficult to follow the reasoning of candidates in their attempts at this question and generally those candidates who included a diagram of the correct curve obtained more marks than those responses with no explanation.

This question did test the problem solving skills of candidates, with the orientation of the diagram not directly relating to the curve, and this did give difficulty to many, with a number of candidates assuming rotation around the y-axis was required, rather than about the x-axis.

For part (a) most were able to correctly find $\mathrm{d}r/\mathrm{d}\theta$ and then substitute this and r^2 into a correct surface area formula (about either axis) and score the first three marks. However, achieving a correct simplified form proved much more of a challenge, with errors in algebra meaning attempts often ground to a halt. But there were also some very well executed solutions simplifying this form to a simple form to integrate. In such cases incorrect selection of limits prevented access to the final two marks. Those rotating about the y-axis in particular had trouble identifying correct limits, since the curve is not defined between the expected ones (though it is doubtful any candidates appreciated this).

Successful attempts at part (b) were rare, though some excellent solutions were seen, showing the understanding of what was required and carrying the processes correctly. Few who identified the correct method made errors in algebra. Some candidates used a maximisation process on the length of *CD* via completing the square on the trigonometric function rather than differentiation. It was also important that the candidates demonstrated that they were finding the length *CD*, and not the value of r, since these had the same value, and some candidates proceed only as far as r.

However, the majority of candidates did not realise what was required and either did not attempt this part or set off in a wrong direction. Many assumed the arc length was required and consequently attempted another integration. Many again mixed up directions and attempted to find $\mathrm{d}x/\mathrm{d}\theta$, for which partial credit could be scored, but in most cases they did not proceed far enough to do so.

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