Examiners' Report
Principal Examiner Feedback

Summer 2019

Pearson Edexcel GCE AS Mathematics
In Further Pure Mathematics Paper 21
(8FM0/21)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2019
Publications Code 8FM0_21_1906_ER

General Introduction

This paper was accessible across the ability range, and most candidates found plenty of opportunities to demonstrate their knowledge. There were some testing questions involving the vector product and coordinate geometry that allowed the paper to discriminate well between the higher grades.

In a significant number of cases, candidates' solutions, particularly to Q3 and Q5, proved difficult to read. This was a result of either poor handwriting, incoherent working or disorganised presentation.

In summary, Q1(a), Q1(b)(i), Q1(c), Q2, Q3 and Q4(a) were a good source of marks for the average student, mainly testing standard ideas and techniques, whereas Q1(b)(ii), Q4(b) and Q5 were discriminating at the higher grades.

Question 1

Q1 was accessible with some candidates struggling to gain access to Q1(b)(ii).

In Q1(a), nearly all candidates recalled a correct $\sin x = \frac{2t}{1+t^2}$.

In Q1(b)(i), most candidates used $\tan\left(\frac{x}{2}\right) = \sqrt{2}$ to deduce and substitute $t = \sqrt{2}$ into their answer from Q1(a). Many achieved $\sin x$ as an exact $\frac{2}{3}\sqrt{2}$.

In Q1(b)(ii), most candidates either applied $\sin x = \frac{2t}{1+t^2}$ to $\sin^2 x + \cos^2 x \equiv 1$ or applied Pythagoras' Theorem to a right-angled triangle with angle x, opposite edge $2t$ and hypothenuse $1+t^2$. Some of these attempts contained incorrect algebra or were incomplete because of errors in forming and factorising a quartic expression or an error in using a numerator of $t^2 - 1$. A few candidates applied $\tan x = \frac{\sin x}{\cos x}$ with $\tan x = \frac{2t}{1-t^2}$, $\sin x = \frac{2t}{1+t^2}$ and were generally more successful in showing that $\cos x = \frac{1-t^2}{1+t^2}$. Those candidates who applied $\cos x \equiv \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$ to a right-angled triangle with angle $\frac{x}{2}$, edges 1, t and $\sqrt{1+t^2}$, did not receive any credit as they had not used their answer to part (a). Some candidates received no credit because they attempted to verify that $\cos x = \frac{1-t^2}{1+t^2}$ was true, usually with $t = \sqrt{2}$.

In Q1(c), most candidates provided a fully correct solution to achieve both angles $x = 143.1°,\ 292.6°$. Many substituted $\cos\theta = \dfrac{1-t^2}{1+t^2}$ and their $\sin\theta = \dfrac{2t}{1+t^2}$ into the given $7\sin\theta + 9\cos\theta + 3 = 0$. Nearly all candidates formed and attempted to solve a quadratic equation in *t*. At this stage some candidates manipulated a correct $\tan\left(\dfrac{\theta}{2}\right) = -\dfrac{2}{3},\ 3$ to give an incorrect $\theta = \arctan(-2)$ or $\theta = \arctan(6)$. Other candidates who achieved a correct $\dfrac{\theta}{2} = \{71.5650...,\ 146.3099...\}$ then halved (rather than doubled) their results to give an incorrect $\theta = \{35.8°,\ 73.1°, ...\}$. Other common errors included failing to round their final answers to one decimal place; premature rounding earlier on in their working which led a loss of accuracy in their value(s) for θ; additional solutions found in the range $0 < \theta \le 360°$; and leaving their final answer(s) in radians. Some candidates used the alternative method of substituting their values of *t* into the *t*-formulae for $\sin\theta$ or $\cos\theta$. Only a few of these candidates achieved both correct values for θ.

Question 2

Q2 was well-answered, although some candidates struggled to give clear explanations in Q2(a).

In Q2(a)(i), most candidates were aware of the sign error in line 3 and many explained that $x(x+11) - x - 24$ should have been written as $x(x+11) - (x-24)$ or $x(x+11) - x + 24$.

In Q2(a)(ii), most candidates realised that the student of the question had found the regions where the inequality is < 0, but this was inadequately explained in some candidates' work, e.g. 'solved for $x < 0$'. Some candidates said that the set notation was wrong, e.g. 'it should be $\cap$ and not $\cup$'.

Many good solutions were seen in Q2(b) and only a few weak attempts were seen, such as invalid cross-multiplication or expansion to a quartic. Most candidates achieved the correct critical values and then went on to choose the appropriate regions. A small number failed to use set notation. A few candidates made the same sign error as the student in Q2(a) or thought that all they had to do in Q2(b) was to correct the student's line 7 error.

Question 3

Q3 was well-answered with many candidates scoring full marks.

Most candidates applied two iterations of $R_{n+1} = R_n + h\left(\dfrac{\mathrm{d}R}{\mathrm{d}t}\right)_n$ for $n = 0, 1$ with $R_0 = 20$ and found an estimate for the number of rabbits at 4 months. Common errors included using an incorrect value for *h* such as 0.2, 2 or $\dfrac{1}{12}$; using $t_0 = \dfrac{1}{6},\ t_1 = \dfrac{1}{3}$

$\left(\text{instead of } t_0 = 0,\ t_1 = \frac{1}{6}\right)$ in their first and second iteration, respectively; or incorrectly finding $\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)_0$ as $2(20) + 4\sin\left(\frac{1}{6}\right)$. A few candidates applied 4 iterations with $h = \frac{1}{12}$ and some candidates were unaware that they should be working in radians. Nearly all candidates gave a correct conclusion in context for their estimated value of R_2.

Question 4

Q4 was accessible to most candidates, but it was clear that there were a few candidates who had not revised the vector cross product formula.

In Q4(a), most candidates found the area of triangle *ABC* by applying the method $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$. Some candidates found the correct answer by applying the vector cross product between two other edges of triangle *ABC*. Some candidates applied the formula $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin(B\hat{A}C)$, where angle $B\hat{A}C$ had been found by using the scalar product formula. A few candidates applied $\frac{1}{2}(\text{base})(\text{height})$ after correctly deducing that *ABC* was isosceles. Common errors included incorrect cross product calculations; finding $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ even after previously stating a correct formula $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$; or misconstruing the area formula as $\frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$.

Q4(b) proved more challenging since no solution was possible without the application of the scalar triple product. Some candidates used a complete method of applying $\frac{1}{6}\left|\overrightarrow{AD}\bullet\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right| = 21$ to find the value of *d*, although some used $\frac{1}{3}, \frac{1}{2}$ or 1 instead of $\frac{1}{6}$. There were several manipulation and calculation errors seen in the evaluation of the scalar triple product in terms of *d*. Those that used $\overrightarrow{AB} \times \overrightarrow{AC} = 18\mathbf{i} + 0\mathbf{j} + 36\mathbf{k}$ from Q4(a) usually found a correct $d = 6$ after solving $\frac{1}{6}\left|\begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix}\right| = 21$. A common error was the confusion between the modulus of a scalar and the magnitude of a vector, leading to some candidates writing $\sqrt{(54)^2 + (3d - 144)^2} = 126$. Another common error was to simplify $|36d - 90|$ to give $3d + 90$ or $\sqrt{(36d)^2 + (90)^2}$.

Question 5

Q5 required an extended response and its unstructured nature proved too demanding for those who did not formulate a clear strategy. This meant that Q5 discriminated well across higher ability candidates and there was a significant minority who made no attempt or no creditable attempt at this question. A huge range of methods were viable, most of which were seen, although Way 1, as described in the mark scheme, was the most popular and successful route to a correct answer.

A significant number of candidates progressed as far as finding an equation for *l* (the tangent to *H* at *P*) and finding the points where *l* cut through the *x*-axis and the *y*-axis. Errors seen for those who made further progress included using $OA = 2OB$ rather than $OB = 2OA$; using $OA : OB = 1 : 3$; applying $(\text{base})(\text{height})$ when finding the area of triangle *OAB*; believing $t^2 = \frac{1}{2} \Rightarrow t = \frac{1}{4}$; or simplifying $\frac{1}{2}(x)(2x)$ to give either $\frac{1}{2}x^2$, $2x^2$ or $2x$. On the whole, it was pleasing to see a significant number of well-presented correct solutions.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom