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Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Further Pure Mathematics 2 (6668/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

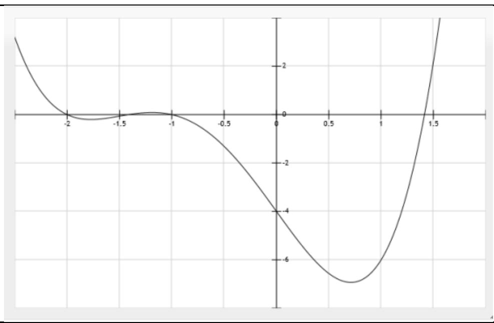
Normal marking procedure is as follows:

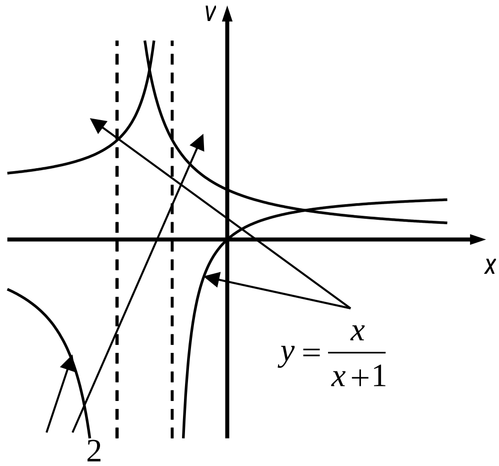
Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

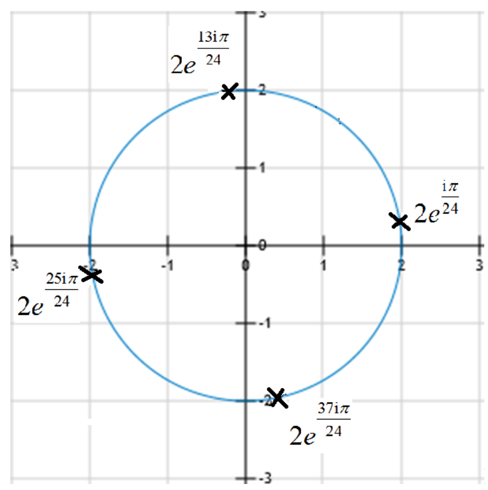
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
NB	Question states "Using algebra..." so purely graphical solutions (using calculator?) score 0/6. A sketch and some algebra to find CVs or intersection points can score according to the method used. (No B marks here so CVs - 1, -2 without working score 0)		
1	$\frac{x}{x+1} < \frac{2}{x+2}$		
	$x(x+1)(x+2)^2 < 2(x+1)^2(x+2)$	Multiply through by $(x+1)^2(x+2)^2$	M1
	$(x+1)(x+2)(x(x+2)-2(x+1)) < 0$	Collect terms and attempt to factorise	M1
	$(x+1)(x+2)(x^2-2) < 0$		
		Sketch need not be seen	
	$(\text{CVs:})\quad -2, -1, \pm\sqrt{2}$	Any 2 values correct(A1A0); all correct(A1A1) with no extras	A1,A1
	$-2 < x < -\sqrt{2} \quad -1 < x < \sqrt{2}$ $\left(-2, -\sqrt{2}\right) \cup \left(-1, \sqrt{2}\right)$	Any one correct interval (A1A0); all correct(A1A1) with no extras Set notation may be used $\cup$ or "or" but not "and" Penalise final A only if ,, used in final intervals.	A1,A1
NB	All marks available if above working is done with = instead of < in lines 1 and 2		(6)
			Total 6
Alts 1	$\frac{x}{x+1} - \frac{2}{x+2} < 0$		
	$\frac{x(x+2)-2(x+1)}{(x+1)(x+2)} < 0$	Attempt common denominator	M1
	$\frac{(x+\sqrt{2})(x-\sqrt{2})}{(x+1)(x+2)} < 0$ or $\frac{\left(x^2-2\right)}{(x+1)(x+2)}$	Simplify numerator (errors such as $\left(x^2-1\right), \left(x^2+2\right)$ qualify)	M1
NB	All marks available if above working is done with = instead of < in lines 1 and 2		
Alts 2	$x(x+1)(x+2)^2 = 2(x+1)^2(x+2)$	Multiply through by $(x+1)^2(x+2)^2$	M1
	$x^4 + 3x^3 - 6x - 4 = 0$	Obtain a 4 term quartic equation and attempt to solve to obtain at least 2 non-zero values for x	M1
	$x = \; -2, -1, \pm\sqrt{2}$	Solution by calculator requires all 4 values correct (A1A1 or A0A0)	A1A1

Question Number	Scheme	Notes	Marks
ALT:		Draw sketch of graphs of $y=\dfrac{x}{x+1}$ and $y=\dfrac{2}{x+2}$ showing the area where they intersect. Graphs do not need to be labelled 2 vertical asymptotes and 2 intersection points needed. Only award if followed by some algebra to find the *x* coordinates of the points of intersection. Must obtain a quadratic eg $x(x+2)-2(x+1)=0$	M1
	$\dfrac{x}{x+1}=\dfrac{2}{x+2}$	Eliminate *y*	
	$x(x+2)-2(x+1)=0$		
	$x=\pm\sqrt{2}$	Solve to *x* = ...	M1
	(CVs) $\pm\sqrt{2},-1,-2$	need not be seen until the intervals are formed	A1,A1
	$-2<x<-\sqrt{2}\quad -1<x<\sqrt{2}$	Any one correct interval (A1A0), all correct(A1A1) Set notation may be used	A1,A1
NB	Finding CVs for $x(x+2)<2(x+1)$ w/o a sketch scores M0M1 and possibly A1		
	If all 4 CVs are given (ie $-1,-2$ included) score M1M1A1 and possibly A1A1A1		

Question Number	Scheme	Notes	Marks
2.(a)	$\dfrac{r(r+1)(r+2)-3(r+1)(r+2)+r+2-(r+1)}{(r+1)(r+2)}$	Attempt common denominator with at least two correct expressions in the numerator. Denominator must be seen now or later.	M1
	$=\dfrac{r^3-7r-5}{(r+1)(r+2)}$ **	No errors seen and at least one intermediate step shown.	A1 cso
			(2)
ALTs	1) Start with RHS and use partial fractions		
	2) Start with RHS, divide and then use partial fractions on the remainder.		
	For either: M1 complete method as described;	A1cso No errors seen and at least one intermediate step shown.	
(b)	$\sum_{r=1}^{n}(r-3)=\frac{1}{2}n(n+1)-3n$ or $\sum_{r=1}^{n}(r-3)=\frac{1}{2}n\left(-2+(n-3)\right)$	Use formula for sum of the natural numbers from 1 to n and '$-3n$' or either formula for the sum of an AP. If general formula not quoted the sub must be correct. (See general rules on "Use of a formula" page 7)	M1
	$\frac{1}{2}-\frac{1}{3}$ $\frac{1}{3}-\frac{1}{4}$ $\frac{1}{n}-\frac{1}{n+1}$ $\frac{1}{n+1}-\frac{1}{n+2}$	Method of differences with at least 3 lines shown, (2 at start and 1 at end or 1 at start and 2 at end). Last Line may be missing $=\frac{1}{n+1}$ Ignore extra terms at start due to including the "(r - 3)" term in each line (or some lines).	M1
	$=\frac{1}{2}-\frac{1}{n+2}$	Extract the 2 remaining terms. Second M mark only needed.	A1
	$\sum_{r=1}^{n}=\dfrac{n(n+1)(n+2)-6n(n+2)+n+2-2}{2(n+2)}$ Or $\sum_{r=1}^{n}=\dfrac{n(n+1)(n+2)-6n(n+2)+n}{2(n+2)}$	Attempt the correct common denominator using all their terms. dependent upon previous M (but not the first). The numerators must be changed. Denominator to be present now or later.	dM1
	$=\dfrac{n(n^2-3n-9)}{2(n+2)}$	$a=-3, b=-9$ Need not be shown explicitly.	A1cso
			(5)
			Total 7

Question Number	Scheme	Notes	Marks
3	$z^4 = 8(\sqrt{3}+\mathrm{i})$		
(a)	$\left(\lvert z^4\rvert = \sqrt{\left(8\sqrt{3}\right)^2 + 8^2} = \sqrt{256} =\right)\ 16$ or $\left(\lvert z\rvert =\right)\ 2$	Give B1 for either 16 or 2 seen anywhere	B1
	$\left(\arg z =\right)\arctan\dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$	$\dfrac{\pi}{6}$ Accept 0.524	B1
	$r^4 = 16 \Rightarrow r = 2$		
	$4\theta = -\dfrac{23\pi}{6}, -\dfrac{11\pi}{6}, \dfrac{\pi}{6}, \dfrac{13\pi}{6}$	Range not specified, you may see $4\theta = \dfrac{\pi}{6}, \dfrac{13\pi}{6}, \dfrac{25\pi}{6}, \dfrac{37\pi}{6}$	
	$\theta = -\dfrac{23\pi}{24}, -\dfrac{11\pi}{24}, \dfrac{\pi}{24}, \dfrac{13\pi}{24}$	Clear attempt at both r and θ with at least 2 different values for their $\arg z$, ie $r = \sqrt[4]{\text{their } 16},\ \theta = \dfrac{\text{principal arg} + 2n\pi}{4}$ all 4 correct distinct values of θ cao. $\theta = \dfrac{\pi}{24}, \dfrac{13\pi}{24}, \dfrac{25\pi}{24}, \dfrac{37\pi}{24}$ scores A1	M1, A1
	Roots are		
	$2\mathrm{e}^{\frac{-23\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{-11\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{13\mathrm{i}\pi}{24}}$	All in correct form cao $2\mathrm{e}^{\frac{\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{13\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{25\mathrm{i}\pi}{24}}, 2\mathrm{e}^{\frac{37\mathrm{i}\pi}{24}}$ scores A1	A1
			(5)
(b)		B1: All 4 radius vectors to be the same length (approx) and perpendicular to each other. Circle not needed. Radius vector lines need not be drawn. If lines drawn and marked as perpendicular, accept for B1 B1: All in correct position relative to axes. Points marked must be close to the relevant axes. At least one point to be labelled or indication of scale given.	B1B1
			(2)
			Total 7
ALT:	Obtain one value - usually $2\mathrm{e}^{\frac{\mathrm{i}\pi}{24}}$ - and place on the circle. Position the other 3 by spacing evenly around the circle.		

Question Number	Scheme	Notes	Marks
4.(i)			
NB	If candidates appear to be considering any/all of *p*, *q*, *r* to be non-positive, send the attempt to review.		
	By use of integrating factor:		
(i) (a)	$\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{q}{p}x=\frac{r}{p}$		
	$\mathrm{e}^{\int\frac{q}{p}dt}=\mathrm{e}^{\frac{qt}{p}}$		
	$x\mathrm{e}^{\frac{qt}{p}}=\int\frac{r}{p}\mathrm{e}^{\frac{qt}{p}}\ dt$	Obtain IF $\mathrm{e}^{\pm\int\frac{q}{p}dt}=\mathrm{e}^{\pm\frac{qt}{p}}$, multiply through by it and integrate LHS. Accept $\int r\mathrm{e}^{\pm\frac{qt}{p}}\mathrm{d}t$ for RHS	M1
	$x\mathrm{e}^{\frac{qt}{p}}=\frac{r}{q}\mathrm{e}^{\frac{qt}{p}}\left(+c\right)$	Integrate RHS $\mathrm{e}^{\frac{qt}{p}}\to k\mathrm{e}^{\frac{qt}{p}}$. Constant of integration may be missing. Dependent on the first M mark.	dM1
	$t=0,\ x=0,\ c=-\frac{r}{q}$	Substitute x = 0 and t = 0 to obtain c. Dependent on both M marks above.	ddM1
	$xe^{\frac{qt}{p}}=\frac{r}{q}e^{\frac{qt}{p}}-\frac{r}{q}$		
	$x=\frac{r}{q}-\frac{r}{q}e^{-\frac{q}{p}t}$	oe Change to x = ...	A1
ALT:	By separating the variables:		
(i) (a)	$\int\frac{p\mathrm{d}x}{r-qx}=\int\mathrm{d}t$	Attempt to separate variables	M1
	$-\frac{p}{q}\ln(r-qx)=t\ \left(+c\right)$	Integrate to give ln Constant of integration may be missing.	dM1
	Use $t=0,x=0$	Substitute x = 0 and t = 0 to obtain their constant.	ddM1
	$x=\frac{r}{q}-\frac{r}{q}e^{-\frac{q}{p}t}$	Oe	A1
			(4)
(b)	$t\to\infty,\ e^{-\frac{q}{p}t}\to 0,$		
	$(x\to)\frac{r}{q}$	Cao	B1
			(1)

4(ii)	$ye^{2\theta}=\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$	Multiply through by IF of the form $e^{\pm 2\theta}$ and integrate LHS (RHS to have integral sign or be integrated later). IF = $e^{2\theta}$ and all correct so far.	M1 A1
	$ye^{2\theta}=\left[-e^{2\theta}\cos\theta\right]+2\int e^{2\theta}\cos\theta\,\mathrm{d}\theta$ Or $\left[\frac{1}{2}e^{2\theta}\sin\theta\right]-\frac{1}{2}\int e^{2\theta}\cos\theta\,\mathrm{d}\theta$	Use integration by parts once (signs may be wrong)	M1
$\left(ye^{2\theta}=\right)$	$\left[-e^{2\theta}\cos\theta\right]+2\left\{\left[e^{2\theta}\sin\theta\right]-2\int e^{2\theta}\sin\theta\,\mathrm{d}\theta\right\}$ Or $\frac{1}{2}e^{2\theta}\sin\theta-\frac{1}{2}\left[\frac{1}{2}e^{2\theta}\cos\theta+\frac{1}{2}\int e^{2\theta}\sin\theta\mathrm{d}\theta\right]$	Use parts a second time (Sim conditions to previous use) Must progress the problem - not just undo the first application	M1
	$\left(ye^{2\theta}=\right)-e^{2\theta}\cos\theta+2e^{2\theta}\sin\theta-4\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$ Or $\frac{1}{2}e^{2\theta}\sin\theta-\frac{1}{4}e^{2\theta}\cos\theta-\frac{1}{4}\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$	RHS correct	A1
	$ye^{2\theta}=-e^{2\theta}\cos\theta+2e^{2\theta}\sin\theta-4ye^{2\theta}+c$ Or $ye^{2\theta}=\frac{1}{2}e^{2\theta}\sin\theta-\frac{1}{4}e^{2\theta}\cos\theta-\frac{1}{4}ye^{2\theta}+c$ $ye^{2\theta}=\int e^{2\theta}\sin\theta\,\mathrm{d}\theta=\frac{1}{5}e^{2\theta}(2\sin\theta-\cos\theta)(+c)$ $\theta=0,y=0\Rightarrow C=\frac{1}{5}$	Replaces integral on RHS with integral on LHS (can be $ye^{2\theta}$ or $\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$) **and** uses $\theta=0,y=0$ to obtain a value for the constant. Depends on the second M mark	dM1
	$y=\frac{1}{5}(2\sin\theta-\cos\theta)+\frac{1}{5}e^{-2\theta}$	oe	A1cso (7)
ALT:	By aux equation method:		
	$m+2=0\Rightarrow m=-2$	Attempt to solve aux eqn	M1
	CF $\left(y=\right)Ce^{-2\theta}$	oe	A1
	PI $\left(y=\right)\ \alpha\sin\theta+\beta\cos\theta$	PI of form shown oe	M1
	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=\alpha\cos\theta-\beta\sin\theta$		
	$\alpha\cos\theta-\beta\sin\theta+2\alpha\sin\theta+2\beta\cos\theta=\sin\theta$	Diff and subst into equation	M1
	$2\alpha-\beta=1,\alpha+2\beta=0\Rightarrow\alpha=\frac{2}{5},\beta=-\frac{1}{5}$	Both $\alpha=\frac{2}{5},\beta=-\frac{1}{5}$	A1
	$\theta=0,y=0\Rightarrow C=\frac{1}{5}$	Use $\theta=0,y=0$ to obtain a value for the constant	dM1
	$y=\frac{1}{5}(2\sin\theta-\cos\theta)+\frac{1}{5}e^{-2\theta}$	Must start $y=$ ….	A1cso(7) **Total 12**
NB	If the equation is differentiated to give a second order equation and an attempted solution seen – send to review.		

Question Number	Scheme	Notes	Marks
5.	$\sin^5\theta = a\sin 5\theta + b\sin 3\theta + c\sin\theta$		
(a)	$2\mathrm{i}\sin\theta = z - \dfrac{1}{z}$ or $2\mathrm{i}\sin n\theta = z^n - \dfrac{1}{z^n}$ oe	Seen anywhere "z" can be $\cos\theta + \mathrm{i}\sin\theta$ or $\mathrm{e}^{\mathrm{i}\theta}$ or z See below for use of $\mathrm{e}^{\mathrm{i}\theta}$	B1
	$\left(z - \dfrac{1}{z}\right)^5 = \left(z^5 - \dfrac{1}{z^5}\right) - 5\left(z^3 - \dfrac{1}{z^3}\right) + 10\left(z - \dfrac{1}{z}\right)$	M1: Attempt to expand powers of $z \pm \dfrac{1}{z}$ A1: Correct expression oe. A single power of *z* in each term. No need to pair. Must be numerical values; nCr s eg 5C2 score A0	M1A1
	$32\sin^5\theta = 2\sin 5\theta - 10\sin 3\theta + 20\sin\theta$	At least one term on RHS correct – no need to simplify.	M1
	$= \dfrac{1}{16}\sin 5\theta - \dfrac{5}{16}\sin 3\theta + \dfrac{5}{8}\sin\theta$	All terms correct oe Decimals must be exact equivalents. *a*, *b*, *c* need not be shown explicitly. Must be in this form.	A1cso (5)
Use of $\mathrm{e}^{\mathrm{i}\theta}$	$2\mathrm{i}\sin\theta = \left(e^{i\theta} - e^{-i\theta}\right)$ oe		B1
	$(2i\sin\theta)^5 = \left(\left(\mathrm{e}^{5i\theta} - \mathrm{e}^{-5i\theta}\right) - 5\left(\mathrm{e}^{3i\theta} - \mathrm{e}^{-3i\theta}\right) + 10\left(\mathrm{e}^{i\theta} - \mathrm{e}^{-i\theta}\right)\right)$		M1A1
	$\left(32i\sin^5\theta =\right)$ $\left(2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)\right)$ $\left(32\sin^5\theta =\right)$ $\left(2\sin 5\theta - 10\sin 3\theta + 20\sin\theta\right)$		M1
	$= \dfrac{1}{16}\sin 5\theta - \dfrac{5}{16}\sin 3\theta + \dfrac{5}{8}\sin\theta$		A1cso
ALTs:			
Way 1	De Moivre on $\sin 5\theta$		
	$\sin 5\theta =$ $\mathrm{Im}(\cos 5\theta + \mathrm{i}\sin 5\theta) = \mathrm{Im}(\cos\theta + \mathrm{i}\sin\theta)^5$	B1: $\sin 5\theta = \mathrm{Im}(\cos\theta + \mathrm{i}\sin\theta)^5$	B1
	$= 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$		
	$= 5\left(1 - \sin^2\theta\right)^2\sin\theta - 10\left(1 - \sin^2\theta\right)\sin^3\theta + \sin^5\theta$	M1 Eliminate $\cos\theta$ from the expression using $\cos^2\theta = 1 - \sin^2\theta$ on at least one of the cos terms.	M1
	$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	A1: Correct 3 term expression	A1
	Also: $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$		
	Thus: $16\sin^5\theta = \sin 5\theta + 20\sin^3\theta - 5\sin\theta$		
	$= \sin 5\theta + 5(3\sin\theta - \sin 3\theta) - 5\sin\theta$	M1: Use their expression for $\sin 3\theta$ to eliminate $\sin^3\theta$	M1

	$=\sin 5\theta-5\sin 3\theta+10\sin\theta$		
	$\sin^5\theta=\frac{1}{16}\sin 5\theta-\frac{5}{16}\sin 3\theta+\frac{5}{8}\sin\theta$	A1:cso Correct result with no errors seen.	A1cso (5)
Way 2	De Moivre on $\sin 5\theta$ and use of compound angle formulae		
	$\sin 5\theta=$ $\mathrm{Im}\left(\cos 5\theta+\mathrm{i}\sin 5\theta\right)=\mathrm{Im}\left(\cos\theta+\mathrm{i}\sin\theta\right)^5$	B1: $\sin 5\theta=\mathrm{Im}\left(\cos\theta+\mathrm{i}\sin\theta\right)^5$	B1
	$=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$		
	$=\frac{5}{2}\cos^3\theta\sin 2\theta-\frac{10}{4}\sin^2 2\theta\sin\theta+\sin^5\theta$	M1: Use $\sin 2\theta=2\sin\theta\cos\theta$	M1
	$\sin^5\theta=\sin 5\theta-\frac{5}{4}\left(\sin 3\theta+\sin\theta\right)\cos^2\theta+\frac{10}{4}\left(1-\cos^2 2\theta\right)\sin\theta$		A1
	$=\sin 5\theta-\frac{5}{8}\cos\theta\left(\sin 4\theta+2\sin 2\theta\right)+\frac{10}{4}\sin\theta-\frac{10}{8}\left(\sin 3\theta-\sin\theta\right)\cos^2\theta$		
	$=\sin 5\theta-\frac{5}{16}\left(\sin 5\theta+\sin 3\theta+2\left(\sin 3\theta+\sin\theta\right)\right)$ $+\frac{10}{4}\sin\theta-\frac{10}{16}\left(\sin 5\theta+\sin\theta-\sin 3\theta+\sin\theta\right)$		M1
	$=\frac{1}{16}\sin 5\theta-\frac{5}{16}\sin 3\theta+\frac{5}{8}\sin\theta$	A1cso	A1cso
Way 3	Working from right to left:		
	$\sin 5\theta=$ $\mathrm{Im}\left(\cos 5\theta+\mathrm{i}\sin 5\theta\right)=\mathrm{Im}\left(\cos\theta+\mathrm{i}\sin\theta\right)^5$		B1
	$\sin 3\theta=$ $\mathrm{Im}\left(\cos 3\theta+\mathrm{i}\sin 3\theta\right)=\mathrm{Im}\left(\cos\theta+\mathrm{i}\sin\theta\right)^3$		
	$5a\left(1-2\sin^2\theta+\sin^4\theta\right)\sin\theta-10a\left(1-\sin^2\theta\right)\sin^3\theta+a\sin^5\theta$ $+3b\left(1-\sin^2\theta\right)\sin\theta-b\sin^3\theta+c\sin\theta$ M1: Find the imaginary parts in terms of $\sin\theta$ and sub for $\sin 5\theta$, $\sin 3\theta$ in RHS A1: Correct (unsimplified) expression		M1A1
	$5a+10a+a=1$ $-10a-10a-3b-b=0$ $5a+3b+c=0$	M1: Compare coefficients to obtain at least one of the equations shown	M1
	$a=\frac{1}{16},\ b=-\frac{5}{16},\ c=\frac{5}{8}$	A1cso	A1cso

(b)	$\int_0^{\frac{\pi}{3}} \sin^5\theta\,\mathrm{d}\theta$ $= \frac{1}{32}\left[-\frac{2}{5}\cos 5\theta + \frac{10}{3}\cos 3\theta - 20\cos\theta\right]_0^{\frac{\pi}{3}}$ **NB: Penultimate A mark has been moved up to here.**	M1: $\sin n\theta \to \pm\frac{1}{n}\cos n\theta$ for $n = 3$ or 5 A1ft: 2 terms correctly integrated A1ft: Third term integrated correctly.	M1A1ft A1ft
	$= \left(-\frac{1}{160} - \frac{5}{48} - \frac{5}{16}\right) - \left(-\frac{1}{80} + \frac{5}{48} - \frac{5}{8}\right)$ $= -\frac{203}{480} - \left(-\frac{256}{480}\right)$	M1:Substitute both limits in a changed function to give numerical values. Incorrect integration such as $\pm n\cos n\theta$ could get M0A0A0M1A0	M1
	$\int_0^{\frac{\pi}{3}} \sin^5\theta = \frac{53}{480}$ **	cso, no errors seen.	A1cso (5) **Total 10**
OR:(b)	$\sin^5\theta = a\sin 5\theta + b\sin 3\theta + c\sin\theta$	Or their *a,b.c* letters used or random numbers chosen	
	$\int_0^{\frac{\pi}{3}} \sin^5\theta\,\mathrm{d}\theta = \left[-\frac{a}{5}\cos 5\theta - \frac{b}{3}\cos 3\theta - c\cos\theta\right]_0^{\frac{\pi}{3}}$	M1: $\sin n\theta \to \pm\frac{1}{n}\cos n\theta$ for $n = 3$ or 5 A1ft: Correct integration of their expression oe	
		M1:Substitute both limits, no trig functions	
		A0 A0 (A1s impossible here)	

Question Number	Scheme	Notes	Marks
6(a)	$\mathrm{f}(x)=\tan x$		
	$\mathrm{f}'(x)=\sec^2 x$ or $\dfrac{1}{\cos^2 x}$		B1
	$\mathrm{f}''(x)=2\sec x(\sec x\tan x)$ $=2\sec^2 x\tan x$	Use of Chain Rule (may use product rule)	M1
	$\mathrm{f}'''(x)=2\sec^2 x(\sec^2 x)$ $+2\tan x(2\sec x(\sec x\tan x))$ $=2\sec^4 x+4\sec^2 x\tan^2 x$	M1: Attempt the third derivative A1: Correct third derivative, any equivalent form.	M1 A1
	$\mathrm{f}\left(\dfrac{\pi}{4}\right)=1,\quad \mathrm{f}'\left(\dfrac{\pi}{4}\right)=2,$ $\mathrm{f}''\left(\dfrac{\pi}{4}\right)=4,\quad \mathrm{f}'''\left(\dfrac{\pi}{4}\right)=16$	Use $\dfrac{\pi}{4}$ in $\mathrm{f}(x)$ and in their 3 derivatives.	M1
	$\tan x=1+2\left(x-\dfrac{\pi}{4}\right)+2\left(x-\dfrac{\pi}{4}\right)^2$ $+\dfrac{8}{3}\left(x-\dfrac{\pi}{4}\right)^3$	M1: Attempt a Taylor series up to $\left(x\pm\dfrac{\pi}{4}\right)^3$ using their derivatives. $\tan x$ not needed and coeffs need not be simplified but must be numerical. A1cso: A correct Taylor series. Equivalent fractions and factorials allowed. Must start $\tan x=\ldots$ or $y=\ldots$ or f(x) = provided y or $\mathrm{f}(x)$ have been defined to be $\tan x$.	M1 A1cso
	Some alternative derivatives:		(7)
	Using sin and cos $f(x)=\dfrac{\sin x}{\cos x}$ $f'(x)=\dfrac{\cos x(\cos x)-\cos x(-\sin x)}{\cos^2 x}=\dfrac{1}{\cos^2 x}$ $f''(x)=\dfrac{(\cos^2 x)(0)-1(-2\cos x\sin x)}{\cos^4 x}=\dfrac{2\sin x}{\cos^3 x}$ $f'''(x)=\dfrac{\cos^3 x(2\cos x)-2\sin x(-3\cos^2 x\sin x)}{\cos^6 x}=\dfrac{2\cos^2 x+6\sin^2 x}{\cos^4 x}$		
	alternative third derivatives (replacing $\sec^2 x$ with $1+\tan^2 x$ in second derivative then chain rule) $f''(x)=2\sec^2 x\tan x$ $f''(x)=2(1+\tan^2 x)\tan x=2\tan x+2\tan^3 x$ $f'''(x)=2\sec^2 x+6\tan^2 x\sec^2 x$		

(b)	$\tan\frac{5\pi}{12}\approx 1+2\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)+2\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)^2$ $+\frac{8}{3}\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)^3$	Sub $x=\frac{5\pi}{12}$ in their part (a)	
	$\tan\frac{5\pi}{12}\approx 1+2\left(\frac{\pi}{6}\right)+2\left(\frac{\pi}{6}\right)^2+\frac{8}{3}\left(\frac{\pi}{6}\right)^3$	change $\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)$ to $\left(\frac{\pi}{6}\right)$ in at least one term of their expansion or just use $\left(\frac{\pi}{6}\right)$ $\left(\frac{\pi}{6}\right)$ to be seen explicitly ie second term $\left(\frac{\pi}{3}\right)$ does not qualify	M1
	$\tan\frac{5\pi}{12}\approx 1+\frac{\pi}{3}+\frac{\pi^2}{18}+\frac{\pi^3}{81}$ **	Must start $\tan\frac{5\pi}{12}$ and justify use of $\left(\frac{\pi}{6}\right)$ - ie $\frac{5\pi}{12}-\frac{\pi}{4}=\frac{\pi}{6}$ either seen separately or a term of the expansion changed from $k\left(\frac{5\pi}{12}-\frac{\pi}{4}\right)^n$ to $k\left(\frac{\pi}{6}\right)^n$	A1cso
			(2)
			Total 9

Question Number	Scheme	Marks
7 (a)	$x=\mathrm{e}^u \quad \dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}=x$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{x}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}u}\times\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u}$	M1A1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-u}\dfrac{\mathrm{d}u}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}u}+\mathrm{e}^{-u}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u}+\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y=-x^{-2}$	
	$\mathrm{e}^{2u}\times\mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u}+\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)-2\mathrm{e}^{u}\times\mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u}+2y=-\mathrm{e}^{-2u}$	dM1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}-3\dfrac{\mathrm{d}y}{\mathrm{d}u}+2y=-\mathrm{e}^{-2u}$ *	A1cso (6)
(a)		
M1	obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later (may not be shown explicitly but appear in the substitution)	
A1	correct expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ any equivalent form (again, may not be seen until substitution)	
M1	obtaining $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	a correct expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form	
dM1	substituting in the equation to eliminate x **Only** u and y now Depends on both previous M marks. Substitution must have come from their work	
A1cso	obtaining the given result from completely correct work.	

	ALTERNATIVE 1	
	$x = \mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u = x$	
	$\frac{\mathrm{d}y}{\mathrm{d}u} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}u} = x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2} = 1\frac{\mathrm{d}x}{\mathrm{d}u} \times \frac{\mathrm{d}y}{\mathrm{d}x} + x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \times \frac{\mathrm{d}x}{\mathrm{d}u} = x\frac{\mathrm{d}y}{\mathrm{d}x} + x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2y}{\mathrm{d}u^2} - \frac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}u^2} - \frac{\mathrm{d}y}{\mathrm{d}u}\right) - 2x \times \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u} + 2y = -x^{-2}$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2} - 3\frac{\mathrm{d}y}{\mathrm{d}u} + 2y = -\mathrm{e}^{-2u}$ *	dM1A1cso (6)
M1	obtaining $\frac{\mathrm{d}y}{\mathrm{d}u}$ using chain rule here or seen later	
A1	correct expression for $\frac{\mathrm{d}y}{\mathrm{d}u}$ here or seen later	
M1	obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ any equivalent form	
dM1A1cso	As main scheme	
	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u}$	M1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x}\frac{\mathrm{d}^2y}{\mathrm{d}u^2} \times \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right) - 2x \times \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u} + 2y = -x^{-2}$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2} - 3\frac{\mathrm{d}y}{\mathrm{d}u} + 2y = -\mathrm{e}^{-2u}$ * Depends on both previous M marks	dM1A1cso

	There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.	
M1	obtaining a first derivative with chain rule	
A1	correct first derivative	
M1	obtaining a second derivative with product rule (Chain rule errors are penalised through A marks)	
A1	correct second derivative with 2 or 3 variables present	
dM1	Either substitute in equation I or substitute in equation II according to method chosen **AND** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)	
A1cso	Obtaining the required result from completely correct work	

Question Number	Scheme	Notes	Marks
(b)	$m^2 - 3m + 2 = 0 \Rightarrow m = 1, 2$	M1: Forms AE and attempts to solve to m = ...or values seen in CF A1: Both values correct. May only be seen in the CF	M1A1
	$(\text{CF} =)\, A\text{e}^{u} + B\text{e}^{2u}$	CF correct oe can use any (single) variable	A1
	$y = \lambda \text{e}^{-2u}$		
	$\dfrac{\text{d}y}{\text{d}u} = -2\lambda \text{e}^{-2u}$ $\dfrac{\text{d}^2 y}{\text{d}u^2} = 4\lambda \text{e}^{-2u}$	PI of form $y = \lambda \text{e}^{-2u}$ (or $y = \lambda u \text{e}^{-2u}$ if $m = -2$ is a solution of the aux equation) **and** differentiate PI twice wrt u. Allow with x instead of u	M1
	$4\lambda \text{e}^{-2u} + 6\lambda \text{e}^{-2u} + 2\lambda \text{e}^{-2u} = -\text{e}^{-2u}$ $\Rightarrow \lambda = -\dfrac{1}{12}$	dM1 substitute in the equation to obtain value for λ Dependent on the second M1 A1 $\lambda = -\dfrac{1}{12}$	dM1A1
	$y = A\text{e}^{u} + B\text{e}^{2u} - \dfrac{1}{12}\text{e}^{-2u}$	A complete solution, follow through their CF and PI. Must have y = a function of u Allow recovery of incorrect variables.	B1ft
			(7)
(c)	$y = Ax + Bx^2 - \dfrac{1}{12x^2}$ Or $y = A\text{e}^{\ln x} + B\text{e}^{2\ln x} - \dfrac{1}{12\text{e}^{2\ln x}}$	Reverse the substitution to obtain a correct expression for y in terms of x No ft here $\dfrac{1}{12x^2}$ or $\dfrac{1}{12}x^{-2}$ Must start $y = \ldots$	B1
			(1)
			Total 14

Question Number	Scheme	Notes	Marks
8(a)	$7\cos\theta = 3+3\cos\theta \Rightarrow \cos\theta = \frac{3}{4} \quad \theta = ...$	Solve to $\theta = ...$	M1
	$P(\frac{21}{4},\alpha)$ and $Q(\frac{21}{4},-\alpha)$ or $\left(\frac{21}{4},2\pi-\alpha\right)$ where $\alpha = \arccos\frac{3}{4}$ or 0.7227...	A1: Angles correct, Decimal for α to be 3 sf minimum A1: r to be $\frac{21}{4}, 5\frac{1}{4}$ or 5.25 Need not be in coordinate brackets	A1 A1cao (3)
(b)	$\left(2\times\frac{1}{2}\right)\int (7\cos\theta)^2 \mathrm{d}\theta$ $=\frac{49}{2}\int (\cos 2\theta+1)\mathrm{d}\theta$ $=\frac{49}{2}\left[\frac{1}{2}\sin 2\theta+\theta\right]$ OR Area of sector $=\frac{1}{2}r^2(\beta-\sin\beta)$ $=\frac{1}{2}\left(\frac{7}{2}\right)^2\left(\pi-2\alpha-\sin(\pi-2\alpha)\right)$	M1: Use of $\frac{1}{2}\int r^2\mathrm{d}\theta$ for C_1 leading to $k\int(\cos 2\theta\pm 1)\mathrm{d}\theta$ OR Area of a segment A1: Correct integration of $49\cos^2\theta$ Ignore any limits shown. OR correct expression for the area of the sector	M1 A1
	$\left(2\times\frac{1}{2}\right)\int (3+3\cos\theta)^2\mathrm{d}\theta$ $=9\int\left(1+2\cos\theta+\cos^2\theta\right)\mathrm{d}\theta$ $=9\int\left(1+2\cos\theta+\frac{1}{2}(\cos 2\theta+1)\right)\mathrm{d}\theta$	M1: Set up the integral for C_2 and reach $k\int\left(1+2\cos\theta+\frac{1}{2}(\cos 2\theta\pm 1)\right)\mathrm{d}\theta$	M1
	$=\left[9\theta+18\sin\theta+\frac{9}{2}(\frac{1}{2}\sin 2\theta+\theta)\right]_0^{\alpha}$	dM1: Correct integration of the trig functions $\cos\theta\to\pm\sin\theta$, $\cos 2\theta\to\pm\frac{1}{2}\sin 2\theta$ A1 Fully correct integration with limits, $0\to\alpha$ or $-\alpha\to\alpha$	dM1 A1
	Area $=9\alpha+18\sin\alpha+\frac{9}{2}(\frac{1}{2}\sin 2\alpha+\alpha)$ $+\frac{49\pi}{4}-\frac{49}{2}(\frac{1}{2}\sin 2\alpha+\alpha)$	dM1: **Depends on all 3 M marks above** Combine areas correctly to find the required area. Use of correct limits required.	dM1
	$\frac{49\pi}{4}+\frac{3\sqrt{7}}{4}-11\alpha$ or 32.5	A1cso Correct answer, exact or awrt 32.5	A1cso (7) **Total 10**
NB	The area can be found by "area of circle – area of crescent":		
	Send to review.		