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Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level in Mechanics 3
(WME03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

‘M’ marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

‘A’ marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

‘B’ marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\lfloor$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

6. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1(a)	$\omega = \dfrac{2\pi}{\frac{\pi}{4}} = 8$	M1
	$\ddot{x} = -\omega^2 x$	
	$20 = \lvert -8^2 a \rvert = 64a$	M1
	$a = \dfrac{20}{64} = \dfrac{5}{16}$ (m) oe (Accept 0.31, 0.313, 0.3125)	A1 (3)
(b)	$v_{\max} = a\omega = 2.5$ (m s^{-1})	B1ft (1)
(c)	Dist from A $= 1.5 - \dfrac{20}{16} = 0.25$	B1
	$x = a\cos\omega t = \dfrac{5}{16}\cos 8t$	
	$\dfrac{1}{16} = \dfrac{5}{16}\cos 8t$	M1
	$t = \dfrac{1}{8}\cos^{-1} 0.2$	A1
	Total time = $\dfrac{\pi}{4} + \dfrac{1}{8}\cos^{-1} 0.2 = 0.95657...$ 0.96 or better	A1ft (4) [8]
	Notes for Question 1	
	(a)M1 for attempting to obtain ω no need to simplify M1 using max mag of $\ddot{x} = \lvert -\omega^2 x \rvert$ with max mag accel = 20 and their ω A1 $a = \dfrac{5}{16}$ m oe fraction or 0.3125 m (b)B1ft $v_{\max} = a\omega = 2.5$ follow through their values for ω and a (c)B1 finding the distance from A when P has travelled 1.5 m = 0.25 m OR the distance from the centre = $\dfrac{1}{16}$ M1 using $x = a\cos\omega t$ with $x = \dfrac{1}{16}$ (their value), their ω and their a OR for using $x = a\sin\omega t$ with $x = \dfrac{1}{16}$ (their value), their ω and their a A1 for $t = \dfrac{1}{8}\cos^{-1} 0.2$ OR $t = \dfrac{1}{8}\sin^{-1} 0.2$ A1ft for $\dfrac{\pi}{4} + \dfrac{1}{8}\cos^{-1} 0.2 = 0.95657...$ 0.96 or better ft their time or equivalent using sine	

Question Number	Scheme	Marks
2.	Mass/area of half of lamina = $(\rho)\times\frac{1}{2}\times a\times\sqrt{3}a=(\rho)\frac{\sqrt{3}a^2}{2}$	B1
	$\int_0^{a\sqrt{3}} yx\mathrm{d}x=\int_0^{a\sqrt{3}}\frac{x^2}{\sqrt{3}}\mathrm{d}x$	M1
	$=\left[\frac{x^3}{3\sqrt{3}}\right]_0^{a\sqrt{3}}$	A1
	$=a^3$	A1
	For the half lamina in the first quadrant $\bar{x}=\frac{\int yx\,\mathrm{d}x}{\text{area}}=a^3\div\frac{a^2\sqrt{3}}{2}$	M1
	By symmetry, c of m of complete triangle is $\frac{2a}{\sqrt{3}}$ oe eg 1.15*a*, 1.2*a*	A1 [6]
	Alternative Work with the whole lamina by multiplying by 2 in lines 1 - 4. No mention of symmetry needed for final answer.	
	Notes for Question 2	
	B1 for the mass or area of half of the lamina M1 for attempting to integrate $\int_0^{a\sqrt{3}}\frac{x^2}{\sqrt{3}}\mathrm{d}x$ limits not needed here A1 for $\left[\frac{x^3}{3\sqrt{3}}\right]_0^{a\sqrt{3}}$ limits must be shown and correct but can be implied if result of sub is correct. A1 for sub limits to get a^3 M1 for using $\bar{x}=\frac{\int yx\,\mathrm{d}x}{\text{area}}$ with their previous answers A1cso for $\frac{2a}{\sqrt{3}}$ oe eg 1.15*a*, 1.2*a* "Symmetry" or "2 x " <u>must</u> be seen for all marks to be awarded. If missing, deduct final A mark. If no *a* in the integrals deduct final A mark unless similar triangles are mentioned. Use of a solid scores 0/6	

Question Number	Scheme	Marks
3	$T_a \cos 30 + T_b \cos 60 = 3g$	M1A1A1
	$T_a \sin 30 + T_b \sin 60 = 3r\omega^2$	M1A1
	$= 3 \times 0.4 \cos 30 \omega^2$	A1
	Solve: $T_a \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} T_b = 3g$	
	$\dfrac{1}{2} T_a + T_b \dfrac{\sqrt{3}}{2} = 3 \times 0.4 \times \dfrac{\sqrt{3}}{2} \times 36$	
	$T_b = 1.2 \times 36 \times \dfrac{3}{2} - 3g$	
	$T_b = 35.4$ (N)	DM1A1
	$T_a = 13.5$ (N) must be 2 or 3 sf	A1 [9]
	Notes for Question 3	
	M1 for resolving vertically. Two tensions (resolved) and a weight must be seen. A1 for two correct terms A1 for all terms (inc signs) correct M1 for NL2 horizontally. Two tensions (resolved) and mass x acceleration needed. The acceleration can be in either form A1 for the two tensions, correctly resolved and added A1 for $3 \times 0.4 \cos 30 \omega^2$ M1 dep for solving the equations to obtain either tension. Dependent on both previous M marks A1 for either tension correct A1 for the second tension correct. Both tensions must be given to 2 or 3 sf to gain the marks. (Penalise once for more than 3 sf)	

Question Number	Scheme	Marks
4(a)	$0.4\dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{4}{(t+5)^2}$	B1
	$v=-\dfrac{10}{(t+5)}+c$	M1A1
	$t=0,\ v=4\ \Rightarrow 4=-\dfrac{10}{5}+c,\ \ c=6$	DM1
	$v=6-\dfrac{10}{(t+5)}\qquad t\geqslant 0\quad \dfrac{10}{t+5}\geqslant 0\Rightarrow v\leqslant 6$	A1 (5)
(b)	$s=\int_2^7\left(6-\dfrac{10}{(t+5)}\right)\mathrm{d}t$	
	$=\left[6t-10\ln(t+5)\right]_2^7$	M1A1ft
	$=42-10\ln 12-(12-10\ln 7)$	M1
	$=30+10\ln\left(\dfrac{7}{12}\right)$ oe eg 24.6100.... 25 or better	A1 (4)
(c)	KE = $\dfrac{1}{2}\times 0.4\times\left(6-\dfrac{10}{12}\right)^2-\dfrac{1}{2}\times 0.4\times\left(6-\dfrac{10}{7}\right)^2$	M1A1ft
	$=1.1592....\text{J}$ Accept 1.2 or better	A1 (3) [12]
	Notes for Question 4	
	(a)B1 for a correct equation of motion with acceleration = $\dfrac{\mathrm{d}v}{\mathrm{d}t}$. Can be awarded by implication if work correct at next stage M1 for attempting the integration wrt t to obtain an expression for v A1 for correct result, constant not needed M1dep for using $t=0,\ v=4$ to obtain a value for c Dependent on the previous M mark A1cso for a correct concluding statement. Can have $\geqslant$ or > **(b)** M1 for attempting the integration of *their* expression for v Limits need not be seen for this mark A1ft for correct integration M1 for substituting the limits 2 and 7 A1cao a correct result, exact or decimal (min 2 sf) (c)M1 for attempting the difference of KE between the points A and B (either way round). Velocities to be calculated using *their* expression for v. Award for a gain or a loss. A1ft for KE at B - KE at A, with *their* expression for v. Need not be simplified, may be reversed. A1cso for $=1.1592....\text{J}$ Accept 1.2 or better Must be positive.	

Question Number	Scheme	Marks
5(a)	Energy A to B $\frac{1}{2}\times 2mv^2 - \frac{1}{2}\times 2mu^2 = 2mga\left(1-\cos 60°\right)$	M1A1 A1
	$v^2 = u^2 + ga$	
	C of M: $2mv = 3mV$	B1
	$V = \frac{2}{3}\sqrt{u^2 + ag}$ *	DM1A1 (6)
(b)	NL2 at bottom: $3m\frac{V^2}{a} = T - 3mg$	M1A1
	$T = 3m\left(\frac{V^2}{a} + g\right) = m\left(\frac{4u^2}{3a} + \frac{13g}{3}\right)$ (N) oe	A1 (3)
(c)	Energy from B to top: $\frac{1}{2}\times 3m\times\frac{4}{9}\left(u^2 + ag\right) - \frac{1}{2}\times 3mX^2 = 3mg\times 2a$	M1A1
	At top $T + 3mg = 3m\frac{X^2}{a}$	M1A1
	$T \geqslant 0 \quad \Rightarrow X^2 \geqslant ag$	DM1
	$\frac{4}{18}\left(u^2 + ag\right) - 2ag \geqslant \frac{ag}{2}$	
	$u^2 \geqslant \frac{41ag}{4}$ *	A1 (6) [15]

	Notes for Question 5	
	(a) M1 for an energy equation from A to B. Two KE terms and 2 PE terms (or a loss of PE) needed. A1 for correct KE terms (difference either way round) A1 for a correct loss of PE and all signs correct throughout the equation mass can be m or $2m$ for these two A marks, provided consistent B1 for a correct conservation of momentum equation M1dep for using the two equations to obtain the speed of the combined particle. Dep on the first M mark and using the C of M equation even if B0 has been given for it. A1cso for $V=\frac{2}{3}\sqrt{u^2+ag}$ * (b) M1 for using NL2 at the bottom, tension, weight and mass x accel terms required. Accel can be in either form. A1 for a fully correct equation, no need to substitute for the speed. A1 for substituting the speed (as given in (a)) to obtain a correct expression for the tension in terms of a, g, m and u. Must be simplified. Any equivalent expression scores A1 eg $\frac{m}{3a}\left(12u^2+13ag\right)$ (c)M1 An energy equation from the bottom to the top. Must have a difference of KE terms and a gain of PE. A1 for a fully correct equation M1 for NL2 along the radius at the top. Must have a tension, weight and mass x acceleration (in either form). A1 for a fully correct equation acceleration in either form. M1dep for using $T\geqslant 0$ at the top to obtain an inequality for the speed at the top and completing to an inequality for u^2. Dependent on both previous M marks in (c). OR: Eliminate X^2 between the two equations and then use the inequality $T\geqslant 0$ A1cso for $u^2\geqslant\frac{41ag}{4}$ *	

Question Number	Scheme	Marks
6(a)	$T=\dfrac{9mgpa}{6a}=mg$	M1
	$p=\dfrac{2}{3}$ *	A1 (2)
(b)	$T=\dfrac{9mg\left(\frac{2}{3}a+x\right)}{6a}$	
	$mg-\dfrac{9mg\left(\frac{2}{3}a+x\right)}{6a}=m\ddot{x}$	M1A1
	$-\dfrac{9gx}{6a}=-\dfrac{3gx}{2a}=\ddot{x}$	DM1
	Of form $\ddot{x}=-\omega^2 x$ $\therefore$ SHM	A1 (4)
(c)	Period $=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{\sqrt{\frac{3g}{2a}}}$, $=2\pi\sqrt{\dfrac{2a}{3g}}$	M1,A1ft (2)
(d)	The string never becomes slack or the SHM is complete	B1 (1)
(e)	Loss of EPE = $\dfrac{9mg\times(2a)^2}{2\times 6a}=3mga$	B1
	$mgh=3mga,\quad h=3a$	M1,A1
	$AE=AD-h=8a-3a=5a$	A1ft (4) [13]
	Notes for Question 6	
	(a)M1 for using Hooke's Law resolving vertically. A1cso for $p=\dfrac{2}{3}$ * (b)M1 for an equation of motion vertically. Must have a tension, a weight and a mass x acceleration. Allow with a for acceleration. Must be dimensionally correct, but allow for misuse of brackets. A1 for a correct equation, can still have a M1dep for rearranging to the form $\ddot{x}=-\omega^2 x$ Acceleration a scores M0 A1 for a correct equation and a conclusion eg $\therefore$ SHM Accept "shown" (c)M1 for using period $=\dfrac{2\pi}{\omega}$ with *their* ω to obtain the period. A1ft for $2\pi\sqrt{\dfrac{2a}{3g}}$ (d)B1 for any statement equivalent to those shown (e)B1 for the EPE lost or initial EPE. Need not be simplified. M1 for an energy equation equating their EPE to the PE gained A1 for a correct vertical distance risen A1ft for $AE=8a-$ *their* distance risen	

Question Number	Scheme	Marks
7(a)	Cylinder cone S	
	Mass $18\pi a^3$ $\frac{1}{3}\pi\times 4\times\frac{3}{2}a^3 = 2\pi a^3$ $16\pi a^3$	B1
	Dist from O a $\frac{3a}{8}$ $\bar{x}$	B1
	$18a - 2\times\frac{3}{8}a = 16\bar{x}$	M1A1ft
	$\bar{x} = \frac{69}{64}a$ *	A1 (5)
(b)	$\tan\theta = \frac{3a}{\bar{x}}$	M1
	$\theta = \tan^{-1}\left(\frac{64}{23}\right)$, $\theta = 70.23...° = 70°$ or 1.2257...rad	A1,A1 (3)
(c)	$R = mg\cos\phi$ $F = mg\sin\phi$ (or $F\cos\phi = R\sin\phi$ M1A1)	B1 B1
	$\frac{F}{R} = \tan\phi = \mu = 0.6$	M1
	$\phi_{\max} = 30.96... = 31$ *	A1 (4)
		[12]
	Notes for Question 7	
	(a) B1 for a correct mass ratio B1 for correct distances, probably from O but can be from another point, eg vertex of cone or centre of base. M1 for a moments equation with 3 terms A1ft for a correct equation, follow through *their* mass ratio and distances. A1cso for $\bar{x} = \frac{69}{64}a$ * **(b)** M1 for $\tan\theta = \frac{3a}{\bar{x}}$ with the given $\bar{x}$. Can be either way up. A1 for $\theta = \tan^{-1}\left(\frac{64}{23}\right)$ A1cao for $\theta = 70.23...°$ Accept 70^{o} or better. Or $1.2257...^{c}$ Accept 1.2^{c} or better. (c) B1 for $R = mg\cos\phi$ B1 for $F = mg\sin\phi$ M1 for using $F = \mu R$ with $\mu = 0.6$ to obtain an equation for $\tan\varphi$ A1cso for $\phi_{\max} = 31°$ *	

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