# M2 JUNE 2010

**1.** A particle $P$ moves on the $x$-axis. The acceleration of $P$ at time $t$ seconds, $t \geqslant 0$, is $(3t + 5)$ m s$^{-2}$ in the positive $x$-direction. When $t = 0$, the velocity of $P$ is 2 m s$^{-1}$ in the positive $x$-direction. When $t = T$, the velocity of $P$ is 6 m s$^{-1}$ in the positive $x$-direction. Find the value of $T$.

**(6)**

$\text{acc} = 3t + 5$

$\Rightarrow \text{vel} = \int 3t + 5 \, dt = \frac{3t^2}{2} + 5t + C$

$v = 2, t = 0 \Rightarrow C = 2 \Rightarrow \text{vel} = \frac{3t^2}{2} + 5t + 2$

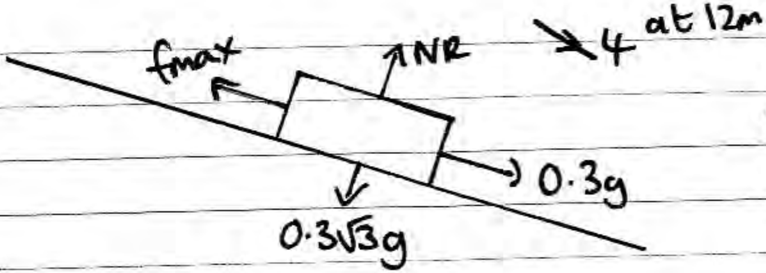
when $v = 6 \Rightarrow 6 = \frac{3}{2}t^2 + 5t + 2 \Rightarrow 3t^2 + 10t - 8 = 0$

$(3t-2)(t+4) = 0 \Rightarrow t = \frac{2}{3}$ sec

**2.** A particle $P$ of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When $P$ has moved 12 m, its speed is 4 m s$^{-1}$. Given that friction is the only non-gravitational resistive force acting on $P$, find

(a) the work done against friction as the speed of $P$ increases from 0 m s$^{-1}$ to 4 m s$^{-1}$, **(4)**

(b) the coefficient of friction between the particle and the plane. **(4)**




$u=0 \quad v=4 \quad s=12$

$v^2=u^2+2as$

$16 = 2a(12) \quad a=\frac{2}{3}$

RF↘ $0.3g - f_{max} = 0.6 \times \frac{2}{3} \Rightarrow f_{max} = 0.3g - 0.4$

wd against friction $= (0.3g - 0.4) \times 12 = \underline{30.5\text{J}}$ (3sf)

b) $f_{max} = \mu NR \qquad 0.3g - 0.4 = \mu(0.3\sqrt{3}g)$

$$\mu = \frac{0.3g - 0.4}{0.3\sqrt{3}g} = 0.499 \text{ (3sf)}$$

3.

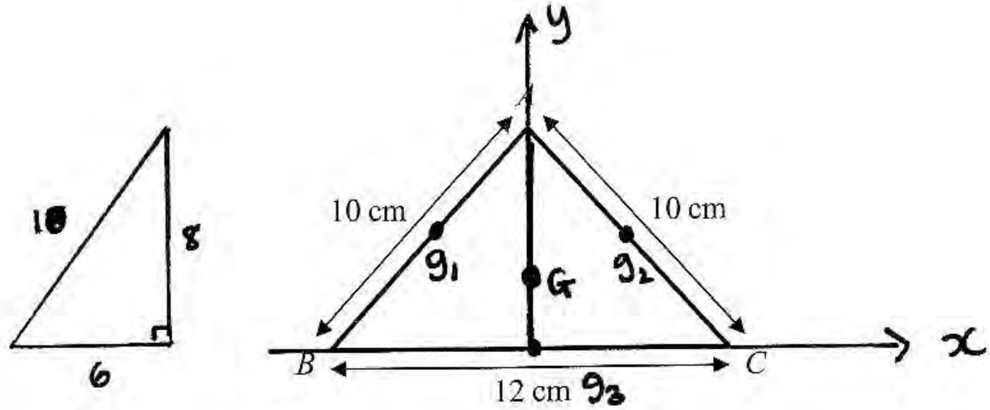



**Figure 1**

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle $ABC$, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

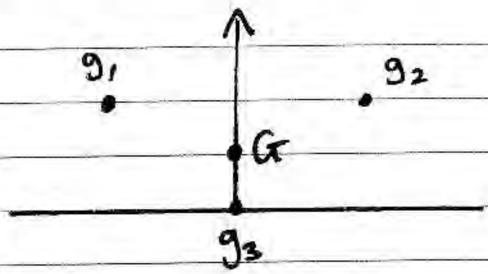
(a) Find the distance of the centre of mass of the frame from $BC$.

**(5)**

The frame has total mass $M$. A particle of mass $M$ is attached to the frame at the mid-point of $BC$. The frame is then freely suspended from $B$ and hangs in equilibrium.

(b) Find the size of the angle between $BC$ and the vertical.

**(4)**




Mass ratio $10:10:12$

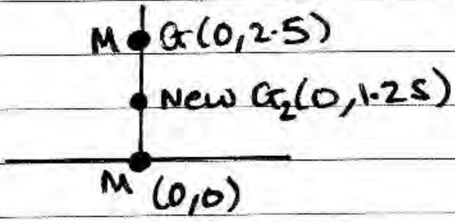
$\Rightarrow 5:5:6$

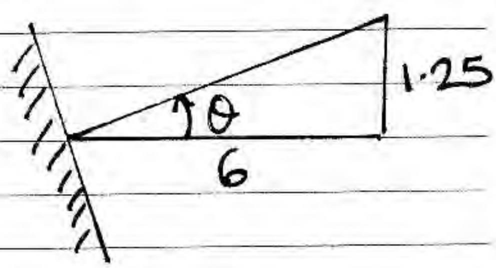
$g_1(-3,4)$ $g_2(3,4)$ $g_3(0,0)$

$G(0,\bar{y})$

$\curvearrowright x$ $\quad 5\times4+5\times4+6\times0 = 16\times\bar{y} \Rightarrow \bar{y} = 2.5\text{cm}$

b)




$\theta = \tan^{-1}\left(\frac{1.25}{6}\right) = 11.8^\circ$ (3sf)

**4.** A car of mass 750 kg is moving up a straight road inclined at an angle $\theta$ to the horizontal, where $\sin\theta = \frac{1}{15}$. The resistance to motion of the car from non-gravitational forces has constant magnitude $R$ newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of 20 m s$^{-1}$.

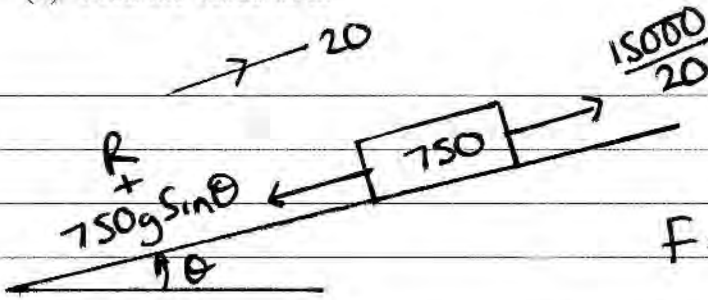
(a) Show that $R = 260$.

**(4)**

The power developed by the car's engine is now increased to 18 kW. The magnitude of the resistance to motion from non-gravitational forces remains at 260 N. At the instant when the car is moving up the road at 20 m s$^{-1}$ the car's acceleration is $a$ m s$^{-2}$.

(b) Find the value of $a$.

**(4)**




$\sin\theta = \frac{1}{15}$

$F = \frac{P}{V}$

$R_f \nearrow = 0 \Rightarrow 750 = 50g + R \Rightarrow R = 260\,N$

b) $P = 18kW$ $\qquad \frac{18000}{20} - 260 - 50g = 750a$

$R_f = ma$

$a = \frac{1}{5}\,ms^{-2}$

**5.** [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A ball of mass 0.5 kg is moving with velocity $(10\mathbf{i} + 24\mathbf{j})$ m s$^{-1}$ when it is struck by a bat. Immediately after the impact the ball is moving with velocity $20\mathbf{i}$ m s$^{-1}$.

Find

(a) the magnitude of the impulse of the bat on the ball,

**(4)**

(b) the size of the angle between the vector **i** and the impulse exerted by the bat on the ball,

**(2)**

(c) the kinetic energy lost by the ball in the impact.

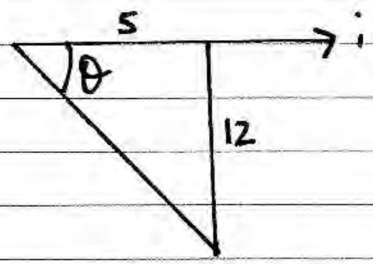
**(3)**

a) Mom before $= \frac{1}{2}(10i+24j) = 5i+12j$
Mom after $= \frac{1}{2}(20i+0j) = 10i$

Impulse = change in momentum $= 5i - 12j$

$|\text{Impulse}| = \sqrt{5^2+12^2} = \underline{13\,Ns}$

b)

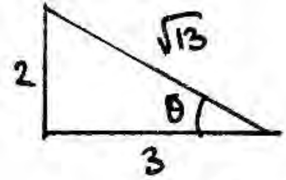



$\theta = \tan^{-1}\left(\frac{12}{5}\right) = 67.4°$ below i

c) Vel before $= \sqrt{10^2+24^2} = 26\,ms^{-1}$
Vel after $= 20\,ms^{-1}$

loss in K.E. $= \frac{1}{2}m(V_2{}^2 - V_1{}^2) = \frac{1}{2}\left(\frac{1}{2}\right)(26^2 - 20^2) = 69\,J$

**6.**




$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\theta = \frac{3}{\sqrt{13}}$

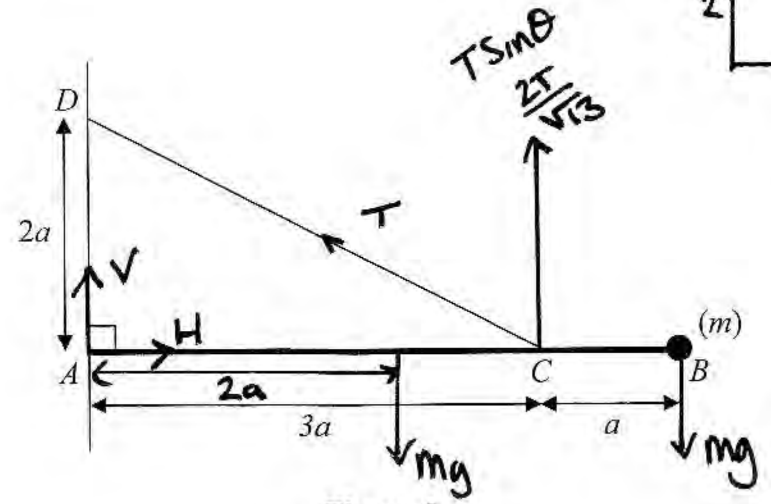



**Figure 2**

Figure 2 shows a uniform rod $AB$ of mass $m$ and length $4a$. The end $A$ of the rod is freely hinged to a point on a vertical wall. A particle of mass $m$ is attached to the rod at $B$. One end of a light inextensible string is attached to the rod at $C$, where $AC = 3a$. The other end of the string is attached to the wall at $D$, where $AD = 2a$ and $D$ is vertically above $A$. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is $T$.

(a) Show that $T = mg\sqrt{13}$.

**(5)**

The particle of mass $m$ at $B$ is removed from the rod and replaced by a particle of mass $M$ which is attached to the rod at $B$. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leqslant \frac{5}{2}m$.

**(3)**

a) $A\circlearrowleft \quad mg\times 2\cancel{a} + mg\times 4\cancel{a} = \frac{2T}{\sqrt{13}}\times 3\cancel{a}$

$\cancel{6}mg = \frac{\cancel{6}T}{\sqrt{13}} \Rightarrow T = \sqrt{13}\,mg$

b) $T \leq 2mg\sqrt{13}$

$A\circlearrowleft \quad mg\times 2\cancel{a} + Mg\times 4\cancel{a} \leq 2mg\cancel{\sqrt{13}} \times \frac{2}{\cancel{\sqrt{13}}} \times 3\cancel{a}$

$2mg + 4Mg \leq 12mg$

$\Rightarrow 4M\cancel{g} \leq 10m\cancel{g} \qquad M \leq \frac{5}{2}m$

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**7.**

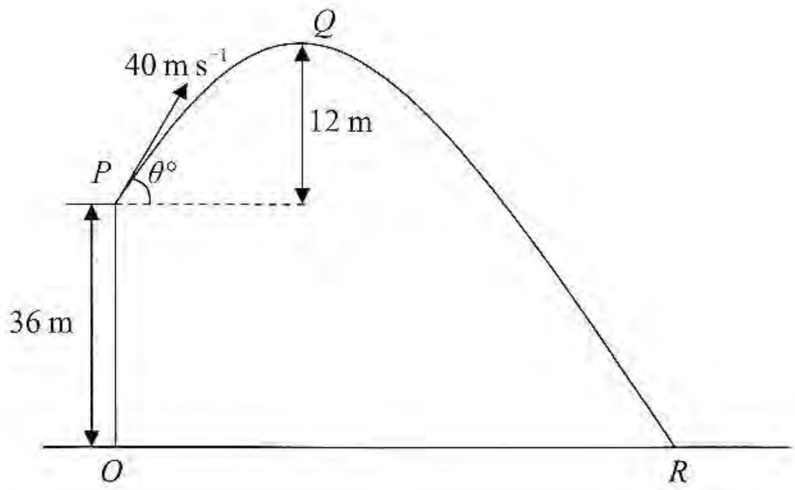



**Figure 3**

A ball is projected with speed 40 m s$^{-1}$ from a point $P$ on a cliff above horizontal ground. The point $O$ on the ground is vertically below $P$ and $OP$ is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point $Q$ is the highest point of the path of the ball and is 12 m above the level of $P$. The ball moves freely under gravity and hits the ground at the point $R$, as shown in Figure 3. Find

(a) the value of $\theta$,

**(3)**

(b) the distance $OR$,

**(6)**

(c) the speed of the ball as it hits the ground at $R$.

**(3)**

a) $u\uparrow = 40\sin\theta$ $\quad v^2 = u^2 + 2as \Rightarrow 0 = (40\sin\theta)^2 - 19.6 \times 12$

$a = -9.8$

$s = 12$ $\quad 40\sin\theta = \sqrt{19.6 \times 12}$ $\quad \theta = 22.544805$

$v = 0$ $\quad \theta = 22.5°$ (3sf)

$u = 15.336231$ $\quad s = ut + \frac{1}{2}at^2 \rightarrow -36 = 15.3..t - 4.9t^2$

$a = -9.8$

$s = -36$ $\quad 4.9t^2 - 15.33...t - 36 = 0$ $\quad t = 1.5649...$

$\vec{H}$ Vel $= 40\cos\theta = 36.943...$ dist = Vel × time $\quad OR = 57.8$ m

c) $v^2 = u^2 + 2as$ $\quad v^2 = (40\sin 22.54...)^2 - 19.6 \times -36 \Rightarrow \downarrow v = 30.672..$

$\vec{v} = 36.943$ $\quad$ Vel $= 36.943i - 30.672j$ $\Rightarrow$ speed $= 48$ ms$^{-1}$

pythag ⇒

8. A small ball $A$ of mass $3m$ is moving with speed $u$ in a straight line on a smooth horizontal table. The ball collides directly with another small ball $B$ of mass $m$ moving with speed $u$ towards $A$ along the same straight line. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find

(i) the speed of $A$ immediately after the collision,

(ii) the speed of $B$ immediately after the collision.

**(7)**

After the collision $B$ hits a smooth vertical wall which is perpendicular to the direction of motion of $B$. The coefficient of restitution between $B$ and the wall is $\frac{2}{5}$.

(b) Find the speed of $B$ immediately after hitting the wall.

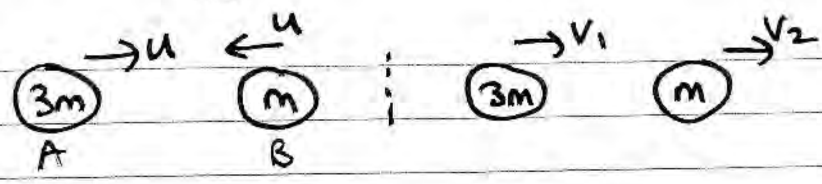
**(2)**

The first collision between $A$ and $B$ occurred at a distance $4a$ from the wall. The balls collide again $T$ seconds after the first collision.

(c) Show that $T = \dfrac{112a}{15u}$.

**(6)**

a)




$V_2 > V_1$

$e = \dfrac{V_2 - V_1}{2u} = \dfrac{1}{2} \Rightarrow V_2 = u + V_1$

$3mu - mu = 3mV_1 + m(u + V_1) \Rightarrow 2mu = 3mV_1 + mu + mV_1$

$\Rightarrow \cancel{m}u = 4\cancel{m}V_1 \Rightarrow V_1 = \frac{1}{4}u \quad V_2 = u + \frac{1}{4}u = \frac{5}{4}u$

b) $e = \dfrac{\overleftarrow{V}}{\frac{5}{4}\overrightarrow{u}} = \dfrac{2}{5} \Rightarrow \overleftarrow{V} = \frac{2}{5} \times \frac{5}{4}u \quad V = -\frac{1}{2}u$

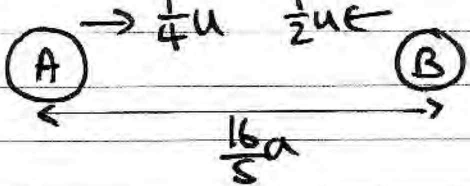
Speed $= \frac{1}{2}u$

c) B $\text{Vel} = \frac{5}{4}u \quad S = 4a \quad 4a = \frac{5}{4}u \times t_1 \Rightarrow t_1 = \dfrac{16a}{5u}$

A $\text{Vel} = \frac{1}{4}u \quad t = \dfrac{16a}{5u} \quad S = \frac{1}{4}u \times \dfrac{16a}{5u} = \dfrac{4a}{5}$

$4a - \frac{4}{5}a = \frac{16}{5}a$

so when B hits the wall A and B are $\frac{16}{5}a$ apart $t = \frac{16a}{5u}$.




speed of approach $= \frac{3}{4}u$

$\frac{16}{5}a = \frac{3}{4}u \times t_2 \quad t_2 = \frac{64a}{15u}$

total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{48a + 64a}{15u} = \frac{112a}{15u}$

qed