C1 JUNE 2010

**1.** Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where $k$ and $x$ are integers. **(2)**

$= \sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3} = 5\sqrt{3} - 3\sqrt{3} = \underline{2\sqrt{3}}$

**2.** Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx$$

giving each term in its simplest form. **(4)**

$= \dfrac{8x^4}{4} + \dfrac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 5x + C = \underline{2x^4 + 4x^{\frac{3}{2}} - 5x + C}$

**3.** Find the set of values of $x$ for which

(a) $3(x-2) < 8-2x$ **(2)**

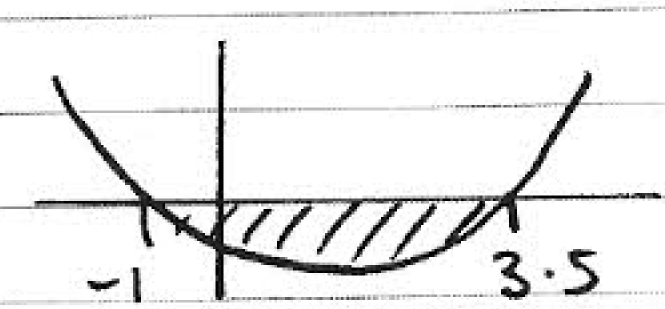
(b) $(2x-7)(1+x) < 0$ **(3)**

(c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$ **(1)**

a) $3x - 6 < 8 - 2x \Rightarrow 5x < 14 \Rightarrow \underline{x < 2.8}$

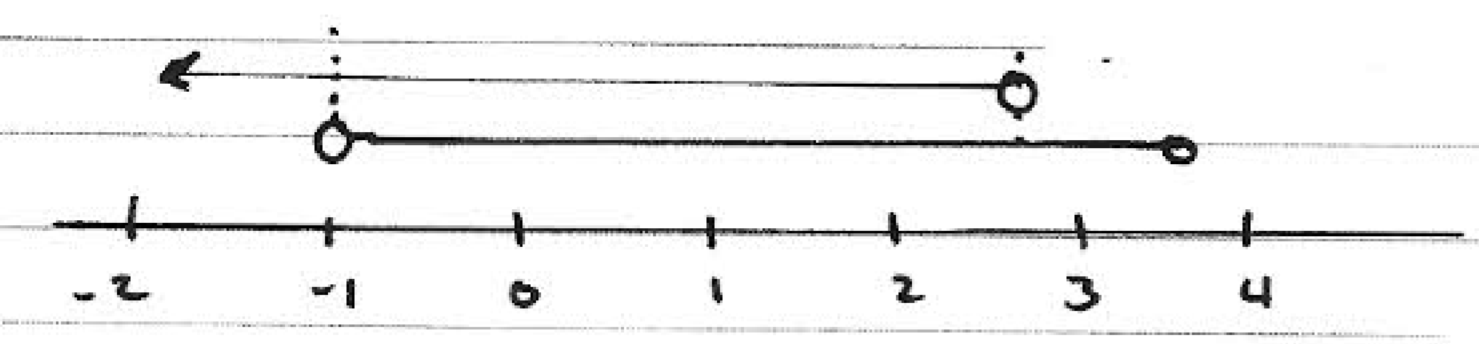
b) $(2x-7)(1+x) < 0$

$3.5 \quad -1$




$\underline{-1 < x < 3.5}$

c)




$\underline{-1 < x < 2.8}$

**4.** (a) Show that $x^2 + 6x + 11$ can be written as

$$(x+p)^2 + q$$

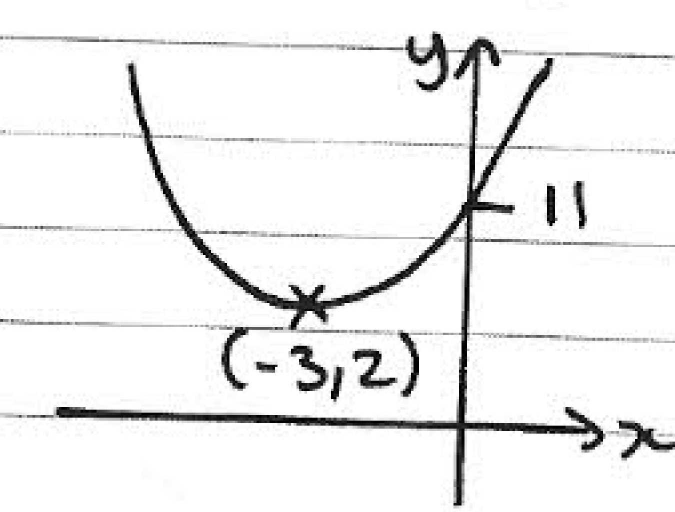
where $p$ and $q$ are integers to be found. **(2)**

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes. **(2)**

(c) Find the value of the discriminant of $x^2 + 6x + 11$ **(2)**

a) $(x+3)^2 - 9 + 11 \Rightarrow (x+3)^2 + 2$

b) TP $(-3, 2)$ y intercept at 11




c) $b^2 - 4ac = 6^2 - 4(11)$

$= 36 - 44 = \underline{-8}$

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \qquad n \geqslant 1,$$
$$a_1 = 2$$

(a) Find $a_2$ and $a_3$, leaving your answers in surd form. **(2)**

(b) Show that $a_5 = 4$ **(2)**

a) $a_1 = 2$

$a_2 = \sqrt{2^2+3} = \sqrt{7}$

$a_3 = \sqrt{(\sqrt{7})^2+3} = \sqrt{10}$

b) $a_4 = \sqrt{(\sqrt{10})^2+3} = \sqrt{13}$

$a_5 = \sqrt{(\sqrt{13})^2+3} = \sqrt{16} = \underline{4}$

6.

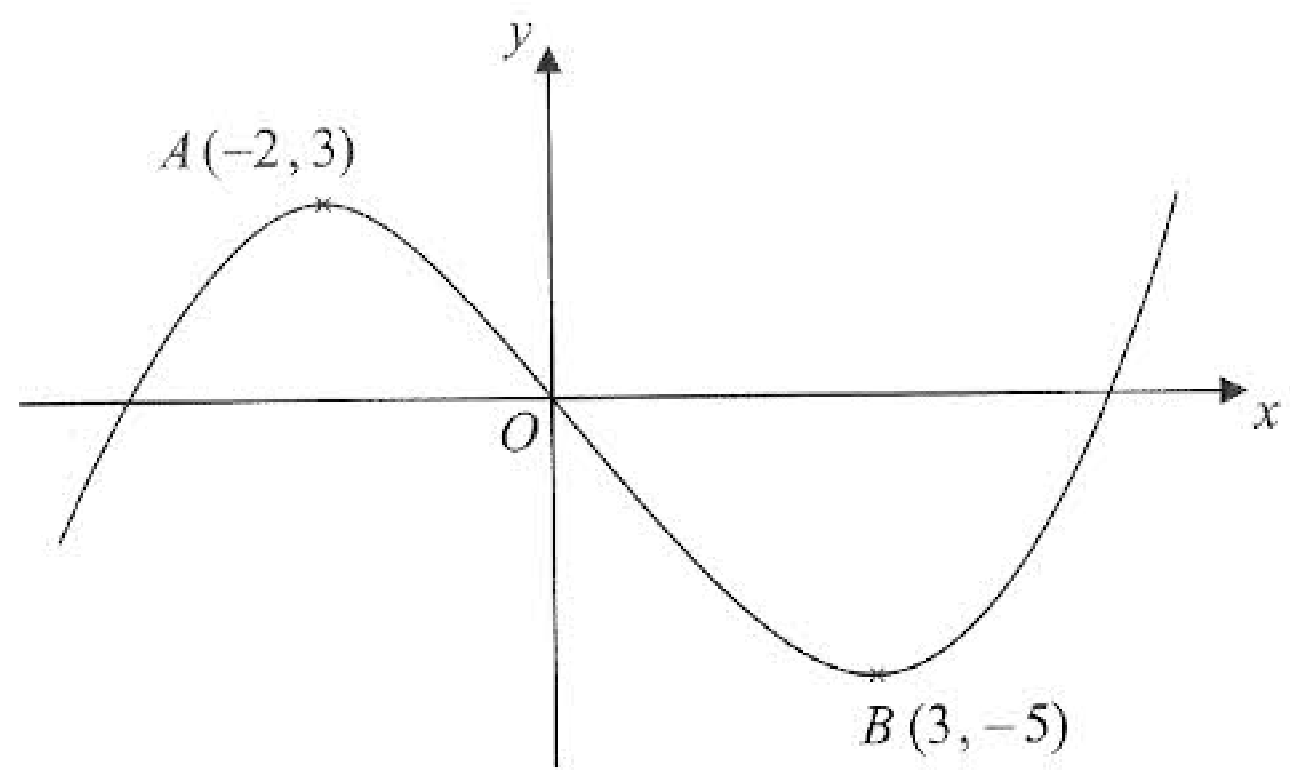



**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum point $A$ at $(-2, 3)$ and a minimum point $B$ at $(3, -5)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x+3)$    $3 \leftarrow$ **(3)**

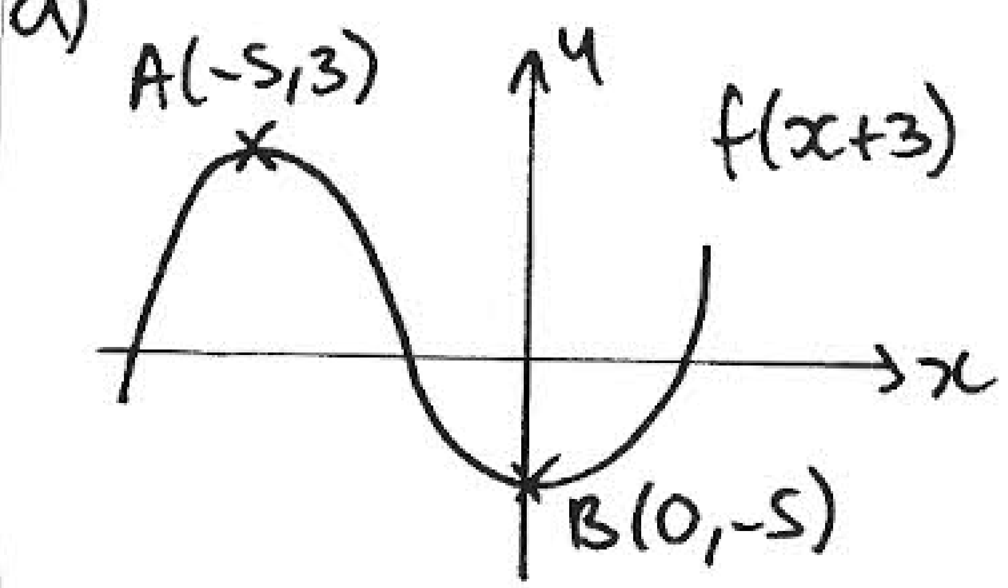
(b) $y = 2f(x)$    $2 \updownarrow$ **(3)**

On each diagram show clearly the coordinates of the maximum and minimum points.

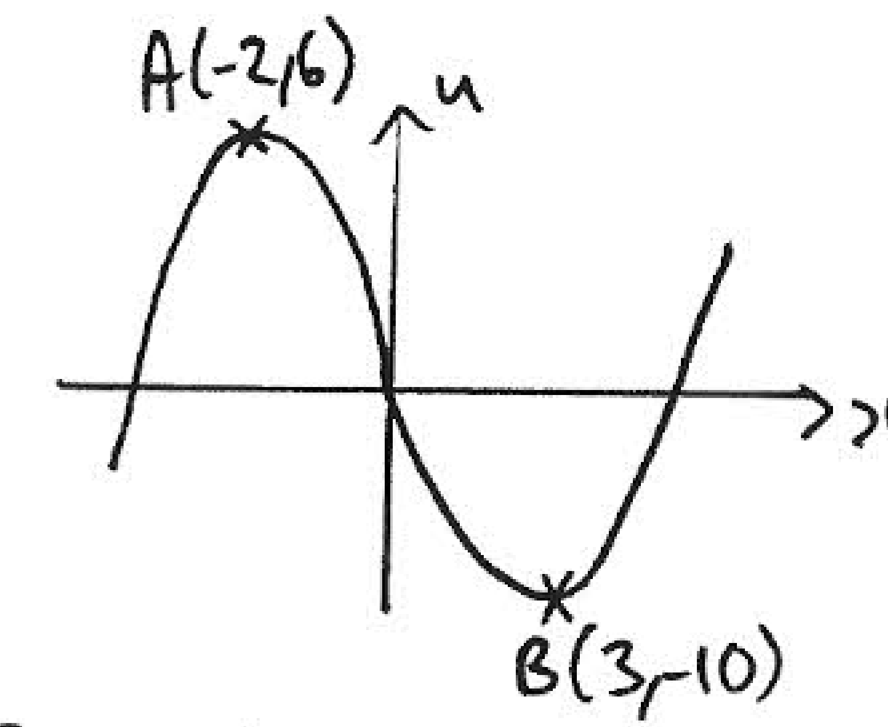
The graph of $y = f(x) + a$ has a minimum at $(3, 0)$, where $a$ is a constant.

(c) Write down the value of $a$. **(1)**

a)




b)




c) $f(x)+a$ shifted 5 up to $(3,0)$ from $(3,-5)$

$\Rightarrow \underline{a = 5}$

7. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x}, \qquad x > 0$$

find $\dfrac{dy}{dx}$. **(6)**

$y = 8x^3 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$

$\dfrac{dy}{dx} = 24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$

8. (a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c=0$, where $a$, $b$ and $c$ are integers.

(3)

(b) Find the length of $AB$, leaving your answer in surd form.

(2)

The point $C$ has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of $t$.

(1)

(d) Find the area of triangle $ABC$.

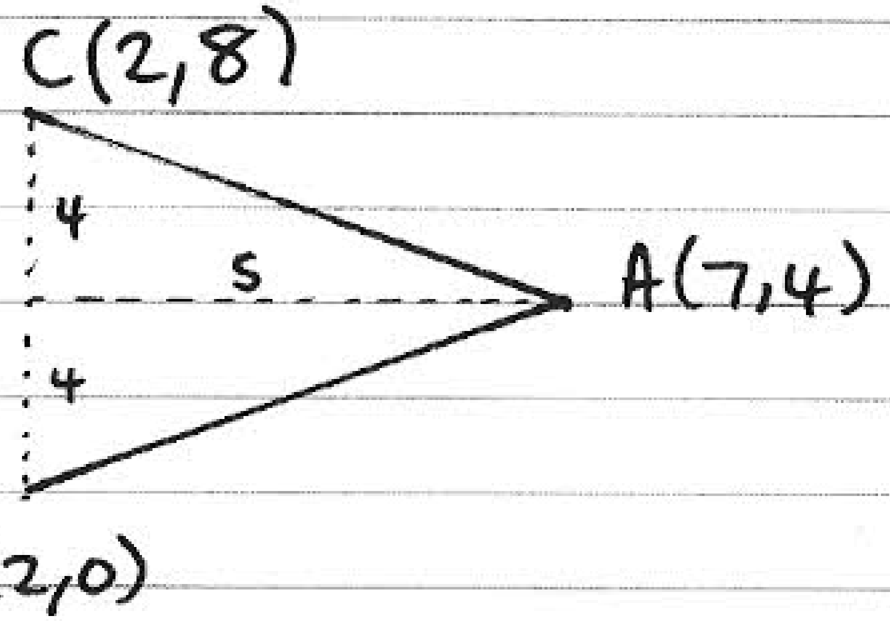
(2)

a) $m_{BA} = \frac{0-4}{2-7} = \frac{-4}{-5} = \frac{4}{5}$

$y - 0 = \frac{4}{5}(x-2) \Rightarrow 5y = 4x - 8 \Rightarrow \underline{4x - 5y - 8 = 0}$

b) $AB = \sqrt{(7-2)^2 + (4-0)^2} = \sqrt{5^2+4^2} = \underline{\sqrt{41}}$

c)




d) Area $\triangle ABC$

$= \frac{1}{2}(8)(5)$

$= \underline{20}$

9. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £$a$ for their first day, £$(a+d)$ for their second day, £$(a+2d)$ for their third day, and so on, thus increasing the daily payment by £$d$ for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in $a$ and $d$.

(2)

A picker who works for all 30 days will earn a total of £1005

(b) Show that $15(a+40.75) = 1005$

(2)

(c) Hence find the value of $a$ and the value of $d$.

(4)

a) $U_{30} = a + 29d \Rightarrow a + 29d = 40.75$

b) $S_{30} = \frac{1}{2}n(a+L)$ $\quad L = 40.75 = U_{30}$

$S_{30} = 15(a + 40.75) = 1005$

c) $a + 40.75 = 67$ $\quad (\div 15)$

$\underline{a = £26.25}$

$26.25 + 29d = 40.75 \Rightarrow 29d = 14.50$

$\Rightarrow \underline{d = \frac{1}{2}}$

**10.** (a) On the axes below sketch the graphs of

(i) $y = x(4-x)$ Cuts 0, 4

(ii) $y = x^2(7-x)$ touches 0, cuts 7 $x \to \infty$ $y \to -\infty$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

**(5)**

(b) Show that the $x$-coordinates of the points of intersection of

$$y = x(4-x) \quad \text{and} \quad y = x^2(7-x)$$

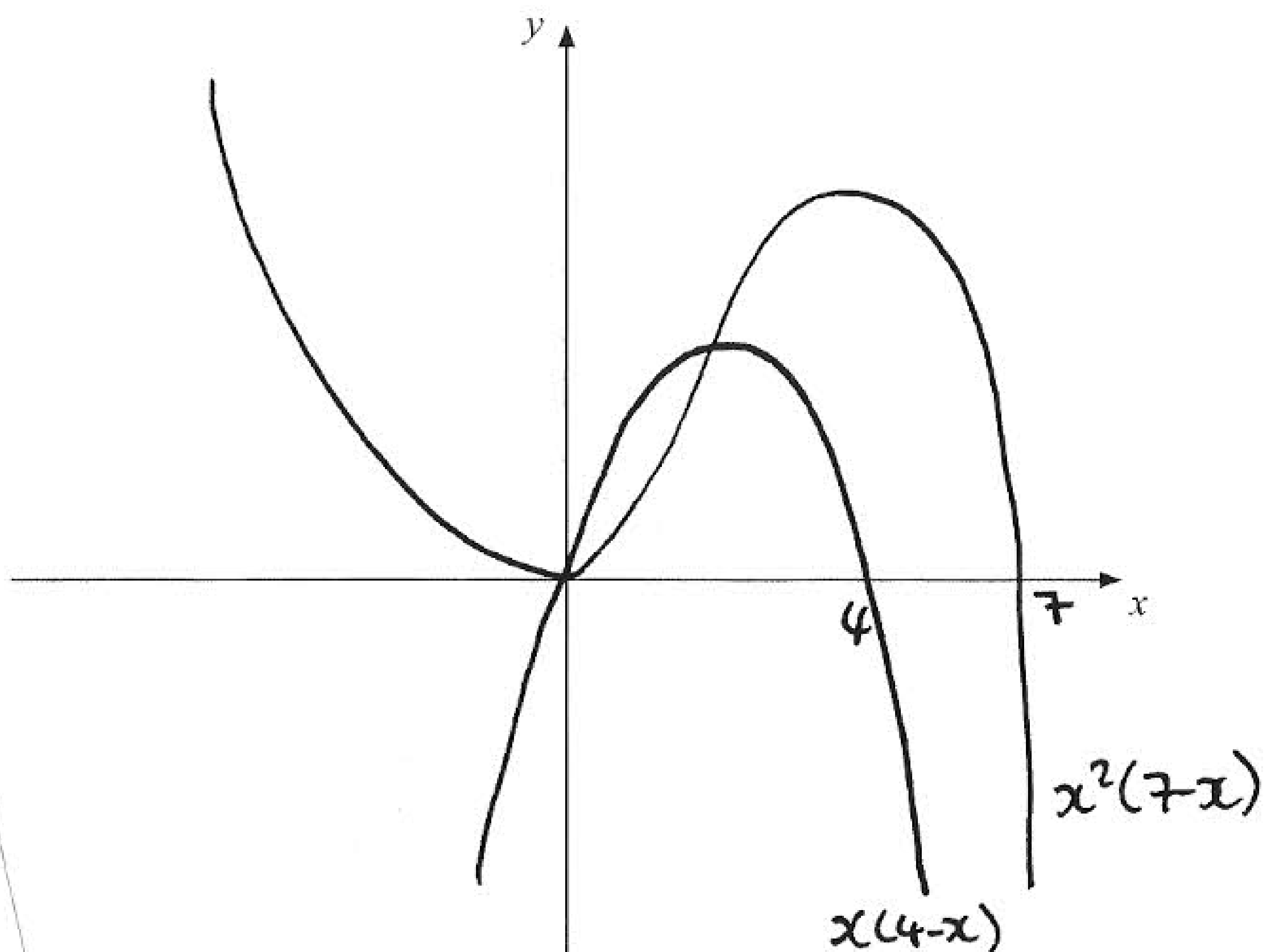
are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

**(3)**

The point $A$ lies on both of the curves and the $x$ and $y$ coordinates of $A$ are both positive.

(c) Find the exact coordinates of $A$, leaving your answer in the form $(p + q\sqrt{3},\ r + s\sqrt{3})$, where $p$, $q$, $r$ and $s$ are integers.

**(7)**




3 points of intersection

b) $x(4-x) = x^2(7-x) \Rightarrow 4x - x^2 = 7x^2 - x^3$

$\Rightarrow x^3 - 8x^2 + 4x = 0 \Rightarrow x(x^2 - 8x + 4) = 0$ qed

c) $x^2 - 8x + 4 = 0$ $\quad x^2 - 8x = -4$

$(x-4)^2 - 16 = -4$ $\quad (x-4)^2 = 12$

$x - 4 = \pm\sqrt{12} = \pm 2\sqrt{3}$ $\quad x = 4 \pm 2\sqrt{3}$

when $x = 4 + 2\sqrt{3}$ $y < 0 \Rightarrow$ not A

$\underline{x = 4 - 2\sqrt{3}}$ at A

$y = (4 - 2\sqrt{3})(4 - (4 - 2\sqrt{3})) = (4 - 2\sqrt{3})(2\sqrt{3})$

$= \underline{8\sqrt{3} - 12}$ $\quad \underline{A(4 - 2\sqrt{3}, -12 + 8\sqrt{3})}$

**11.** The curve $C$ has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on $C$, find

(a) $\mathrm{f}(x)$,

**(5)**

(b) an equation of the tangent to $C$ at the point $P$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(4)**

(a) $\frac{dy}{dx} = 3x - 5x^{-\frac{1}{2}} - 2$

$y = \frac{3}{2}x^2 - \frac{5x^{\frac{1}{2}}}{(\frac{1}{2})} - 2x + c$

$y = \frac{3}{2}x^2 - 10x^{\frac{1}{2}} - 2x + c$

$5 = \frac{3}{2} \times 4^2 - 10\sqrt{4} - 2(4) + c$

$5 = 24 - 20 - 8 + c$ $\quad \underline{c = 9}$

$y = \frac{3}{2}x^2 - 10\sqrt{x} - 2x + 9$

b) $x = 4$ $m_t = 3(4) - \frac{5}{\sqrt{4}} - 2 = 10 - \frac{5}{2} = \frac{15}{2}$

$y - 5 = \frac{15}{2}(x - 4) \Rightarrow 2y - 10 = 15x - 60$

$\underline{15x - 2y - 50 = 0}$