

Mark Scheme (Final)

January 2009

GCE

GCE Mechanics M2 (6678/01)

General Marking Guidance

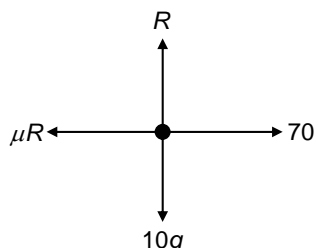
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.




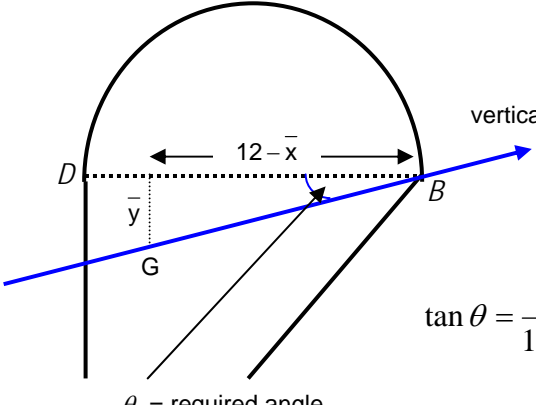
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6678 Mechanics M2
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General Note:

- For M marks, correct number of terms, dimensionally correct, all terms that need resolving are resolved.
- Omission of g from a resolution is an accuracy error, not a method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Where there is only one method mark for a question or part of a question, this is for a *complete* method.
- For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- Omission of units is not (usually) counted as an error.

Question Number	Scheme	Marks
1.	<p style="text-align: center;"> $F = ma$ parallel to the slope, $T - 1500g \sin \theta - 650 = 1500a$ Tractive force, $30000 = T \times 15$ $a = \frac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ <u>0.2</u> (m s^{-2}) </p>	M1* A1 M1* d*M1 A1 [5]
5 marks		
2.	<p>(a) $R(\uparrow) : R = 25g + 75g (= 100g)$</p> $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ $= 44g (= 431)$ <p>(b) $M(A) :$ $25g \times 2 \cos \beta + 75g \times 2.8 \cos \beta$ $= S \times 4 \sin \beta$</p> $R(\leftrightarrow) : F = S$ $176g \sin \beta = 260g \cos \beta$ $\beta = 56(^{\circ})$	B1 M1 A1 [3]
(c)	So that Reece's weight acts directly at the point C.	M1 A2,1,0 M1A1 A1 [6]
		B1 10 marks

Question Number	Scheme	Marks
<p>3. (a)</p>  <p>(b)</p>	<p>$R(\uparrow) : R = 10g$</p> <p>$F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$</p> <p>$\therefore$ WD against friction = $\frac{4}{7}(10g)(50)$</p> <p>2800(J)</p> <p>$70(50) - "2800" = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$</p> <p>$700 = 5v^2 - 20, 5v^2 = 720 \Rightarrow v^2 = 144$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p> <p>Or (b) N2L(\rightarrow): $70 - \frac{4}{7}R = 10a$</p> <p>$70 - \frac{4}{7} \times 10g = 10a, (a = 1.4)$</p> <p>AB($\rightarrow$): $v^2 = (2)^2 + 2(1.4)(50)$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>[4]</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>[4]</p>
	8 marks	
<p>4. (a)</p> <p>(b)</p>	<p>$v = 10t - 2t^2, s = \int v dt$</p> <p>$= 5t^2 - \frac{2t^3}{3} (+C)$</p> <p>$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)</p> <p>$\underline{s} = \int v dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$</p> <p>$t = 6, s = "36" \Rightarrow 36 = \frac{432}{6} + K$</p> <p>$\Rightarrow K = -36$</p> <p>At $t = 10, s = \frac{432}{10} - 36 = \underline{7.2}$ (m)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p><u>B1</u></p> <p>M1*</p> <p>A1</p> <p>d*M1</p> <p><u>A1</u></p> <p>[5]</p> <p>8 marks</p>

Question Number	Scheme				Marks
5. (a)	MR	 108	 18π	 $108 + 18\pi$	B1
	x_i (\rightarrow) from AD	4	6	\bar{x}	B1
	y_i (\downarrow) from BD	6	$-\frac{8}{\pi}$	\bar{y}	
	$AD(\rightarrow): 108(4) + 18\pi(6) = (108 + 18\pi)\bar{x}$				M1
$\bar{x} = \frac{432 + 108\pi}{108 + 18\pi} = 4.68731\dots = \underline{4.69}$ (cm) (3 sf) AG				A1 [4]	
(b)	y_i (\downarrow) from BD	6	$-\frac{8}{\pi}$	\bar{y}	B1 oe
$BD(\downarrow): 108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi)\bar{y}$				M1	
$\bar{y} = \frac{504}{108 + 18\pi} = 3.06292\dots = \underline{3.06}$ (cm) (3 sf)				A1ft A1 [4]	
(c)					M1
$\tan \theta = \frac{\bar{y}}{12 - 4.68731\dots} = \frac{3.06392\dots}{12 - 4.68731\dots}$				dM1 A1	
$\theta = 22.72641\dots = \underline{23}$ (nearest degree)				A1 [4]	
12 marks					

Question Number	Scheme	Marks
6. (a)	Horizontal distance: $57.6 = p \times 3$ $p = 19.2$	M1 A1 [2]
(b)	Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. $-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ $-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$ $q = \frac{43.2}{3} = 14.4$ *AG*	M1 A1 A1 cso [3]
(c)	initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) $= \sqrt{576} = \underline{24}$ (m s ⁻¹)	M1 A1 cao [2]
(d)	$\tan \alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)	B1 [1]
(e)	When the ball is 4 m above ground: $3.1 = ut + \frac{1}{2}at^2$ used $3.1 = 14.4t - \frac{1}{2}gt^2$ o.e ($4.9t^2 - 14.4t + 3.1 = 0$) $\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied $t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389\dots$ or $2.70488\dots$ awrt 0.23 and 2.7 duration = $2.70488\dots - 0.023389\dots$ $= 2.47$ or 2.5 (seconds)	M1 A1 M1 A1 [6]
or 6. (e)	M1A1M1 as above $t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. $= 2.47$ or 2.5 (seconds)	A1 M1 A1 [6]
(f)	Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.	B1 [1] 15 marks

Question Number	Scheme	Marks
7. (a)	<p style="text-align: center;"> Before $\xrightarrow{2u}$ \xleftarrow{u} $P \begin{matrix} \circ \\ 3m \end{matrix}$ $\begin{matrix} \circ \\ 2m \end{matrix} Q$ After \xrightarrow{x} \xrightarrow{y} </p> <p style="text-align: right;">Correct use of NEL</p> <p style="text-align: right;">$y - x = e(2u + u)$ o.e.</p> <p>CLM (\rightarrow): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$) Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$) Hence, speed of $Q = \frac{1}{5}(9e + 4)u$ AG</p>	<p>M1*</p> <p>A1</p> <p>B1</p> <p>d*M1</p> <p>A1 cso</p> <p style="text-align: right;">[5]</p>
(b)	<p>$x = y - 3eu = \frac{1}{5}(9e + 4)u - 3eu$</p> <p>Hence, speed P = $\frac{1}{5}(4 - 6e)u = \frac{2u}{5}(2 - 3e)$ o.e.</p> <p>$x = \frac{1}{2}u = \frac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e</p> <p>gives, $e = \frac{3}{12} \Rightarrow e = \frac{1}{4}$ AG</p>	<p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p style="text-align: right;">[4]</p>
or (b)	<p>Using NEL correctly with given speeds of P and Q</p> <p>$3eu = \frac{1}{5}(9e + 4)u - \frac{1}{2}u$</p> <p>$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e</p> <p>$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.</p>	<p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p style="text-align: right;">[4]</p>
(c)	<p>Time taken by Q from A to the wall = $\frac{d}{y} = \left\{ \frac{4d}{5u} \right\}$</p> <p>Distance moved by P in this time = $\frac{u}{2} \times \frac{d}{y} = \left(\frac{u}{2} \left(\frac{4d}{5u} \right) \right) = \frac{2}{5}d$</p> <p>Distance of P from wall = $d - x \left(\frac{d}{y} \right) = d - \frac{2}{5}d = \frac{3}{5}d$ AG</p>	<p>M1†</p> <p>A1</p> <p>d†M1;</p> <p>A1 cso</p> <p style="text-align: right;">[4]</p>
or (c)	<p>Ratio speed P: speed Q = $x:y = \frac{1}{2}u : \frac{1}{5} \left(\frac{9}{4} + 4 \right)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$</p> <p>So if Q moves a distance d, P will move a distance $\frac{2}{5}d$</p> <p>Distance of P from wall = $d - \frac{2}{5}d = \frac{3}{5}d$ AG</p> <p>cso</p>	<p>M1†</p> <p>A1</p> <p>d†M1;</p> <p>A1</p> <p style="text-align: right;">[4]</p>

(d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y</p> <p>Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$</p> <p>gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \frac{3d}{5} \Rightarrow x = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1 cao</p> <p>[4]</p>
or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y</p> <p>Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5} ; = \frac{d}{5}$ their $\frac{1}{2+1}$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>[4]</p>
2 nd or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Combined speed of P and Q = $\frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$</p> <p>Time from wall to 2nd collision = $\frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y</p> <p>Distance of B from wall = (their speed)x(their time) = $\frac{u}{4} \times \frac{4d}{5u} ; = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>[4]</p>
		17 marks