Mark Scheme (Final)

January 2009

GCE

GCE Further Pure Mathematics FP1(old) (6674/01)

6674/01 Further Pure FP1
January 2009 Advanced Subsidiary/Advanced Level in GCE Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.
(a)</td><td>$\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$
Simplifying this expression
$= \frac{1}{3}n(n^2-4)$ (✱)</td><td>M1, A1
M1
A1 (4) cso</td></tr>
<tr><td>(b)

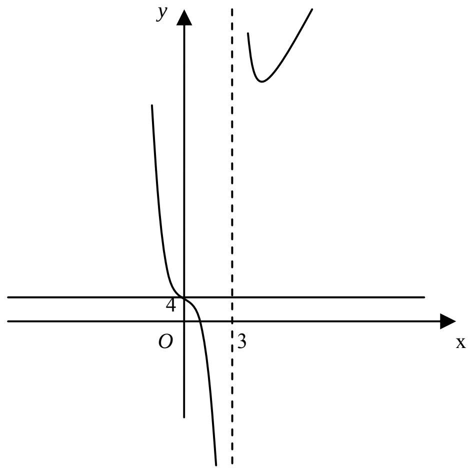
Alt. (b)</td><td>$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) = \frac{1}{3}\times 20\times(20^2-4) - \frac{1}{3}\times 9\times(9^2-4)$
$= 2409$
$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) =$
$\left(\frac{1}{6}\times 20\times 21\times 41 - \frac{1}{2}\times 20\times 21 - 20\right) - \left(\frac{1}{6}\times 9\times 10\times 19 - \frac{1}{2}\times 9\times 10 - 9\right)$ M1
$= 2409$ A1</td><td>M1
A1 (2)

(6)</td></tr>
<tr><td>Notes</td><td>(a) 1st M: Separating, substituting set results, at least TWO correct
2nd M: Either “eliminate” brackets totally or factor x [......] where any product of brackets inside [....] has been reduced to a single bracket
2nd A: ANSWER GIVEN. No wrong working seen; must have been an intermediate step, e.g. $\frac{1}{6}n(2n^2+3n+1-3n-3-6)$, so result “easy to see”
(b) M : Must be $\sum_{r=1}^{20}(....) - \sum_{r=1}^{9}(....)$ applied.
If list terms and add, allow M1 if **11 terms** with **at most two wrong**:
[89, 109, 131, 155, 181, 209, 239, 271, 305, 341, 379]</td><td></td></tr>
<tr><td>2.</td><td>3 – i is a root (seen anywhere)
Attempt to multiply out $[x-(3+i)][x-(3-i)]$ $\{= x^2-6x+10\}$
$f(x) = (x^2-6x+10)(2x^2-2x+1)$
$x = \frac{2\pm\sqrt{4-8}}{4}$, $x = \frac{1\pm i}{2}$</td><td>B1
M1
M1, A1
*M1, A1 **(6)**</td></tr>
<tr><td>Notes

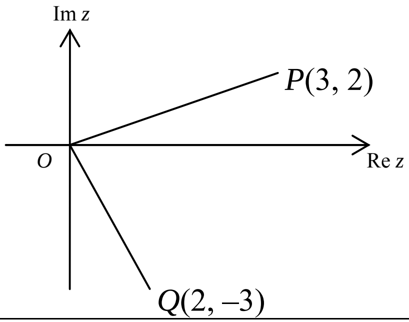
Alt:</td><td>1st M: Using the two roots to form a quadratic factor.
2nd M: Complete method to find second quadratic factor $2x^2 + ax\ (+ b)$.
(Allow remainder if appropriate, but no more marks then available)
3rd *M: Correct method, as far as $x = \ldots$, for solving candidate’s second quadratic, dependent on both previous M marks
If first quadratic has complex coeffs. can score first 2 M marks
(i)$f(x)/\{x-(3+i)\} = 2x^3 + (-8+2i)x^2 + (7-2i)x - 3 + i$ $\{=g(x)\}$
$g(x)/\{x-(3-i)\} = (2x^2-2x+1)$ Attempt at complete process M2; A1
(ii)(2)(x – a+ib)(x – a– ib)(“$x^2-6x+10$”) = f(x) and compare ≥ 1 coeff. M1 Either
–2a –6= –7, or two of $10(b^2+a^2) = 5$ or $-6(a^2+b^2) - 20a = -13$,
$20 + 2(b^2+a^2) + 24a = 33$ A1; Complete method for a **and** b, M1; AnswerA1
SC: (ii) If $(x-a)(x-b)$, a,b $\in \Re$, used and compare ≥ 1 coeff. allow first M only.</td><td>

Lines 2 and 3

Lines 3 and 4</td></tr>
</table>

Question Number	Scheme	Marks
3.	Identifying 3 as critical value e.g. used in soln Identifying 0 as critical value e.g. used in soln $\dfrac{x^3+5x-12-4(x-3)}{x-3}>0$ or $(x^3+5x-12)(x-3)>4(x-3)^2$ o.e. $\dfrac{x(x^2+1)}{x-3}>0$ or $(x-3)(x^3+x)>0$ If mult. by (x-3), needs to be consideration of x>3 and x<3 for this M Using their critical values to **obtain** one or more inequalities. $x<0$ or $x>3$	B1 B1 M1 A1 M1 A1 cso **(6)**
Notes	Allow the B marks wherever seen, even if other spurious values too. 1st M must be a valid opening strategy. Sketching y = $\dfrac{x}{x-3}$ or y = $\dfrac{x(x^2+1)}{x-3}$ should mark as scheme. The result 0 > x > 3 (poor notation) can gain final M but not A.	
Alt.	 Identifying 3 as critical value e.g. x = 3 seen as asymp. (x = 3 on axis,allow) Identifying 0 as critical value e.g. pt of intersection on y-axis of $y=\dfrac{x^3+5x-12}{x-3}$ and y = 4 M1 $y=\dfrac{x^3+5x-12}{x-3}$ sketched for $x<3$ or $y=\dfrac{x^3+5x-12}{x-3}$ sketched for $x>3$ A1 All correct including y = 4 drawn Using the graph values to obtain one or more inequalities $x<0$ or $x>3$	B1 B1 M1, A1 M1 A1

Question Number	Scheme	Marks
4. (a)	At st. pt $f'(x) = 0$, $\therefore x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ is undefined or at st. pt, **tan.** // to x-axis, or **tan.** does not cross x-axis, o.e.	B1 (1)
(b)	$f'(x) = -1 - 2x\cos(x^2)$ (may be seen in body of work)	M1, A1
	$f(0.6) = 0.0477\ldots$, $f'(0.6) = -2.123\ldots$ (may be implied by correct answer)	A1
	Attempt to use $(x_1) = 0.6 - \dfrac{f(0.6)}{f'(0.6)}$ $\quad \left[0.6 - \dfrac{0.0477...}{-2.123...}\right]$	M1
	$= 0.622$ (3 dp) $\quad$ (0.6224795…)	A1 (5)
(c)	$f(0.6215) = 1.77\ldots\times10^{-3} > 0$, $f(0.6225) = -3.807\times\ldots\times10^{-4} < 0$	M1
	Change of sign in $f(x)$ in (0.6215, 0.6225) "so 0.622 correct"	A1 (2)
		(8)
Notes	(b) 1st M: Evidence of differentiation with at least $\sin(x^2) \to \cos(x^2)$ or $\sin(x^2) \to 2x\cos(....)$ 2ndM: If the N-R statement applied to 0.6 **not** seen, can be implied if answer correct; otherwise M0 If the statement seen, allow M1 if an answer given If no values for f(0.6), $f'(0.6)$ seen, they can be implied if final answer correct.	
	(c) M: For candidate's x_1, calculate $f(x_1 - 0.0005)$ and $f(x_1 + 0.0005)$ (or a tighter interval) A: Requires correct values of f(0.6215) and f(0.6225) (or their acceptable values) [may be rounded, e.g. 2×10^{-3}, or truncated, e.g -3.80×10^{-4}], sign change stated or >0, <0 seen, and conclusion.	

Question Number	Scheme	Marks
5. (a)	$z_2 = \frac{12-5\mathrm{i}}{3+2\mathrm{i}} \times \frac{3-2\mathrm{i}}{3-2\mathrm{i}} = \frac{36-24\mathrm{i}-15\mathrm{i}-10}{13}$ $= 2 - 3\mathrm{i}$	M1 A1 (2)
(b)	 *P*: B1, *Q*: B1ft from (a)	B1, B1ft (2)
(c)	grad. *OP* × grad. *OQ* $= (\frac{2}{3} \times -\frac{3}{2})$ $= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2}$ (✱)	M1 A1 (2)
Alt. (c)	(i) $\angle \mathrm{POX} = \tan^{-1}\frac{2}{3}$, $\angle \mathrm{QOX} = \tan^{-1}\frac{3}{2}$ $\mathrm{Tan}(\angle \mathrm{POQ}) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1 $\Rightarrow \angle POQ = \frac{\pi}{2}$ (✱) A1	
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}\mathrm{i}$ $= \frac{5}{2} - \frac{1}{2}\mathrm{i}$	M1 A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$ $= \frac{\sqrt{26}}{2}$ or exact equivalent	M1 A1 (2) **(10)**
Notes	(a) M: Multiplying num. and den. by 3–2i and attempt to simplify num. and denominator (however badly) If (c + id)(3 + 2i) = 12 –5i used, need to find 2 equations in c and d and then solve for c and d. (b) Coords seen or clear from labelled axes. S.C: If only P and Q seen(no coords) or correct coords given but P and Q interchanged allow B1B0 (c) If separate arguments are found and then added, allow M1 but **not** A1for e.g.1.570796327..= ½π, $\tan^{-1}(2/3) + \tan^{-1}(3/2) = ½\pi$ A1 if no dec. Alts: Appropriate transformation matrix applied to one point M1; A1 Scalar product used correctly M1; 0 and conclusion A1 Pythagoras' theorem, congruent triangles are other methods seen. (d) M: Any complete method for finding centre. A: Must be complex number; coordinates not sufficient. (e) M: Correct method for radius, or diameter, for candidate's answer to (d) Disregard labelling for (d) and (e); i.e.allow marks seen in "wrong" part	

Question Number	Scheme	Marks
6. (a)	$r=\sqrt{x^2+y^2}$, $y=r\sin\theta$ $\therefore \sqrt{x^2+y^2}=\dfrac{6y}{\sqrt{x^2+y^2}}$ or $x^2+y^2=6y$ o.e.	M1, A1 (2)
(b)	$r=9\sqrt{6}(1-2\sin^2\theta)$ o.e.	B1 (1)
(c)	$y=r\sin\theta=9\sqrt{6}(\sin\theta-2\sin^3\theta)\Rightarrow\dfrac{dy}{d\theta}=;\ 9\sqrt{6}\cos\theta(1-6\sin^2\theta)$ o.e. Or $y=9\sqrt{6}\sin\theta\cos2\theta\Rightarrow\dfrac{dy}{d\theta}=9\sqrt{6}(\cos2\theta\cos\theta-2\sin\theta\sin2\theta)$ o.e. $\dfrac{dy}{d\theta}=0$ [$\Rightarrow\cos\theta(1-6\sin^2\theta)=0$] **and** attempt to solve $(0\le\theta\le\dfrac{\pi}{4})$ $\therefore\sin\theta=\dfrac{1}{\sqrt{6}}$ (✱)	M1;A1 M1 A1 (4)
(d)	$r=9\sqrt{6}\left(1-2\times\dfrac{1}{6}\right)$ $=6\sqrt{6}$ or 14.7 (awrt)	M1 A1 (2)
(e)	C_2: tan. // to initial line is $y=r\sin\theta=6\sqrt{6}\times\dfrac{1}{\sqrt{6}}=6$ C_1: Circle, centre (0, 3) (cartesian) or $(3,\frac{\pi}{2})$ (polar), passing through (0,0). $\therefore$ tangent // to initial line has eqn $y=6\Rightarrow y=6$ is a common tangent	B1 M1 A1 (3) **(12)**
Notes	(a) M1: Use of $r=\sqrt{x^2+y^2}$ or $r^2=x^2+y^2$, and $y=r\sin\theta$ (allow $x=r\sin\theta$) to form cartesian equation. (b) May be scored in (c) (c) 1st M: Finds y and attempts to find $\dfrac{dy}{d\theta}$ **Working with $r\cos\theta$ instead of $r\sin\theta$, can score the M marks.** If $\dfrac{dy}{dx}=\dfrac{dy}{d\theta}/\dfrac{dx}{d\theta}$ used throughout, $\dfrac{dy}{dx}=0$ etc. all marks may be gained (d) M: Using $\sin\theta=\dfrac{1}{\sqrt{6}}$ and correct formula for $\cos2\theta$ to find r (e) Alt. for C_1: M:Find $y=6\sin^2\theta$, ($\dfrac{dy}{d\theta}=12\sin\theta\cos\theta$) and solve $\dfrac{dy}{d\theta}=0$ A: Find $\theta=\dfrac{\pi}{2}$ and conclude that $y=6$, so common tangent	

Question Number	Scheme	Marks
7. (a)	$\dfrac{dy}{dx} = \lambda x e^x + \lambda e^x$ Use of the product rule $\dfrac{d^2y}{dx^2} = \lambda x e^x + \lambda e^x + \lambda e^x$ $\lambda x e^x + 2\lambda e^x + 4\lambda x e^x + 4\lambda e^x - 5\lambda x e^x = 4e^x$ $\lambda = \dfrac{2}{3}$ $(\therefore$ P.I. is $\dfrac{2}{3}xe^x)$	M1 A1 *M1 A1 (4)
(b)	Aux. eqn. $m^2 + 4m - 5 = 0$ $(m-1)(m+5) = 0$ $m = 1$ or $m = -5$ C.F. is $y = Ae^x + Be^{-5x}$ Gen. soln. is $(y =)\ \dfrac{2}{3}xe^x + Ae^x + Be^{-5x}$ [f.t: Candidate's C.F + P.I.]	 M1 M1 A1 A1ft (4)
(c)	$-\dfrac{2}{3} = A + B$ $\dfrac{dy}{dx} = \dfrac{2}{3}xe^x + \dfrac{2}{3}e^x + Ae^x - 5Be^{-5x}$ $-\dfrac{4}{3} = \dfrac{2}{3} + A - 5B$ A1 two correct unsimplified eqns. $-2 = A - 5B$ $\dfrac{4}{3} = 6B$ $B = \dfrac{2}{9}, A = -\dfrac{8}{9}$ $y = \dfrac{2}{3}xe^x - \dfrac{8}{9}e^x + \dfrac{2}{9}e^{-5x}$	M1 M1 A1 M1 A1 (5) **(13)**
Notes	(a) 2^{nd} M dependent on first M. (b) 1^{st} M: Attempt to solve A.E. (usual rules for factorising) 2^{nd} M: Only allow C.F. of form $Ae^{ax} + Be^{bx}$, where a and b are real. If seen in (a), award marks there. PI must be of form λxe^x ($\lambda \neq 0$) to gain final A1 f.t. (c) 1^{st} M: Using $x = 0, y = -⅔$ in their **general solution**. (CF + PI $\neq 0$) 2^{nd} M: Differentiating their **general solution** (must have term in λxe^x and attempt product rule) (condone slips) and using $x = 0, \dfrac{dy}{dx} = -\dfrac{4}{3}$ to find an equation in A and B. 3^{rd} M: Solving simultaneous equations to find a value of A and a value of B. Can be awarded if only C.F. found. Insist on $y =$ .... in this part.	

Question Number	Scheme	Marks
8. (a)	$\frac{dy}{dx} = -\frac{1}{t^2} \times \frac{dt}{dx}$ o.e.	M1, A1
	$\sin x \times -\frac{1}{t^2} \times \frac{dt}{dx} + \frac{1}{t}\cos x = \frac{1}{t^2}$	M1
	$\frac{dt}{dx} - t\cot x = -\operatorname{cosec} x$ (✱)	A1 cso(4)
(b)	$I = e^{\int -\cot x\, dx}$	M1
	$= e^{-\ln \sin x}$	
	$= \frac{1}{\sin x}$ or cosec$x$	A1
	$\frac{1}{\sin x}\frac{dt}{dx} - t\frac{\cos x}{\sin^2 x} = -\operatorname{cosec}^2 x$	M1
	$\frac{t}{\sin x} = \int -\operatorname{cosec}^2 x\, dx$ or $\frac{d}{dx}\left(\frac{t}{\sin x}\right) = -\operatorname{cosec}^2 x$	A1f.t.
	$\frac{t}{\sin x} = \cot x \ (+c)$ o.e.	A1 cso(5)
(c)	$t = \cos x + c\sin x \Rightarrow y = \frac{1}{\cos x + c\sin x}$ (✱)	M1, A1 (2)
(d)	$\frac{\sqrt{2}}{3} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}}$	M1
	$\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}\right) = 3$	
	$c = 2$	A1
	$x = \frac{\pi}{2}, \quad y = \frac{1}{2}$ ft on their c	A1ft (3)
		(14)
Notes	(a) 1st M: Use of $\frac{dy}{dt}.\frac{dt}{dx}$ (even if integrated 1/t) 2nd M: Substituting for $\frac{dy}{dx}, y, y^2$ to form d.e. in x and t only (b) 1st M: For $e^{\int -\cot x\, dx}$ (allow $e^{\int \cot x\, dx}$) and attempt at integrating 2nd* M: Multiplying by their integrating factor (requires at least two terms "correct" for their IF.) (can be implied) 3rdA1f.t: (**M on Epen**): **is only for I.F. = sin*x* or $-$sin*x*** and have $\frac{d}{dx}(t\sin x) = -1$ or equivalent integral (c) M: Substituting to find t = 1/y **in their solution** to (b) (d) M: Using y = $\frac{\sqrt{2}}{3}, x = \frac{\pi}{4}$ to find a value for c. (Allow sign errors only)	