Mark Scheme June 2008

GCE

GCE Mathematics (8371/8373,9371/9373)

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June 2008

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6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$2x + \frac{5}{3}x^3 + c$	M1A1A1 (3) **3**
	M1 for an attempt to integrate $x^n \to x^{n+1}$. Can be given if $+c$ is only correct term. 1st A1 for $\frac{5}{3}x^3$ or $2x + c$. Accept $1\frac{2}{3}$ for $\frac{5}{3}$. Do <u>not</u> accept $\frac{2x}{1}$ or $2x^1$ as final answer 2nd A1 for as printed (no extra or omitted terms). Accept $1\frac{2}{3}$ or $1.\dot{6}$ for $\frac{5}{3}$ but not 1.6 or 1.67 etc Give marks for the first time correct answers are seen e.g. $\frac{5}{3}$ that later becomes 1.67, the 1.67 is treated as ISW NB M1A0A1 is not possible	

Question number	Scheme	Marks
2.	$x(x^2-9)$ or $(x\pm 0)(x^2-9)$ or $(x-3)(x^2+3x)$ or $(x+3)(x^2-3x)$	B1
	$x(x-3)(x+3)$	M1A1 (3)
		3

B1 for first factor taken out correctly as indicated in line 1 above. So $x(x^2+9)$ is B0

M1 for attempting to factorise a relevant quadratic.

"Ends" correct so e.g. $(x^2-9)=(x\pm p)(x\pm q)$ where $pq=9$ is OK.

This mark can be scored for $\left(x^2-9\right)=\left(x+3\right)\left(x-3\right)$ seen anywhere.

A1 for a fully correct expression with all 3 factors.

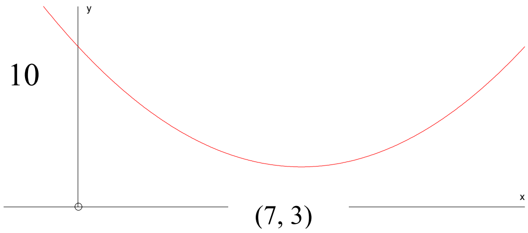
Watch out for $-x(3-x)(x+3)$ which scores A1

Treat any working to solve the equation x^3-9x as ISW.

Question number	Scheme	Marks

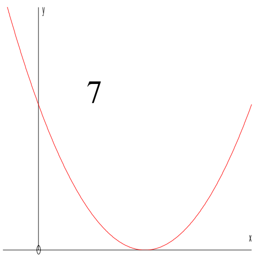
3

(a)



B1B1B1 (3)

(b)



B1B1 (2)

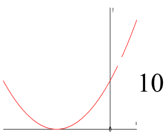
5

Allow "stopping at" (0, 10) or (0, 7) instead of "cutting"

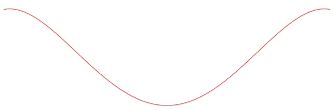
(a) 1st B1 for moving the given curve up. Must be U shaped curve, minimum in first quadrant, not touching x-axis but cutting positive y-axis. Ignore any values on axes.

2nd B1 for curve cutting y-axis at (0, 10) . Point 10(or even (10, 0) marked on positive y-axis is OK)

3rd B1 for minimum indicated at (7, 3). Must have both coordinates and in the right order.



this would score B0B1B0



If the curve flattens out to a turning point like this penalise <u>once</u> at first offence ie 1st B1 in (a) or in (b) but not in both.

The U shape mark can be awarded if the sides are fairly straight as long as the vertex is rounded.

(b) 1st B1 for U shaped curve, touching positive x-axis and crossing y-axis at (0, 7)[condone (7, 0) if marked on positive y axis] or 7 marked on y-axis

2nd B1 for minimum at (3.5, 0) or 3.5 or $\frac{7}{2}$ marked on x-axis. Do <u>not</u> condone (0, 3.5) here.

Redrawing f(x) will score B1B0 in part (b).

Points on sketch override points given in text/table.

If coordinates are given elsewhere (text or table) marks can be awarded if they are compatible with the sketch.

Question number	Scheme	Marks
4. (a)	$[\,\mathrm{f}'(x) = \,]\ 3 + 3x^2$	M1A1 (2)
(b)	$3 + 3x^2 = 15$ and start to try and simplify	M1
	$x^2 = k \rightarrow x = \sqrt{k}$ (ignore $\pm$)	M1
	$x = 2$ (ignore $x = -2$)	A1 (3)
		5

(a) M1 for attempting to differentiate $x^n \rightarrow x^{n-1}$. Just one term will do.

A poor integration attempt that gives $3x^2 + ...$ (or similar) scores M0A0

A1 for a fully correct expression. Must be 3 not $3x^0$. If there is a + c they score A0.

(b) 1st M1 for forming a correct equation and trying to rearrange their $\mathrm{f}'(x) = 15$ e.g. collect terms.

e.g. $3x^2 = 15 - 3$ or $1 + x^2 = 5$ or even 3+3x^2 $\rightarrow$ $3x^2 = \frac{15}{3}$ or $3x^{-1} + 3x^2 = 15 \rightarrow 6x = 15$

(i.e algebra can be awful as long as they try to collect terms in their $\mathrm{f}'(x) = 15$ equation)

2nd M1 this is dependent upon their $\mathrm{f}'(x)$ being of the form $a + bx^2$ and

attempting to solve $a + bx^2 = 15$

For correct processing leading to $x = \ldots$

Can condone arithmetic slips but processes should be correct so

e.g. $3 + 3x^2 = 15 \rightarrow 3x^2 = \frac{15}{3} \rightarrow x = \frac{\sqrt{15}}{3}$ scores M1M0A0

$3 + 3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow x^2 = 9 \rightarrow x = 3$ scores M1M0A0

$3 + 3x^2 = 15 \rightarrow 3x^2 = 12 \rightarrow 3x = \sqrt{12} \rightarrow x = \frac{\sqrt{12}}{3}$ scores M1M0A0

Question number	Scheme	Marks	
5. (a)	$[x_2 =]\, a-3$	B1	(1)
(b)	$[x_3 =]\ ax_2 - 3$ or $a(a-3)-3$	M1	
	$= a(a-3)-3$ } both lines needed for A1		
	$= a^2 - 3a - 3$ (*)	A1cso	(2)
(c)	$a^2 - 3a - 3 = 7$		
	$a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$	M1	
	$(a-5)(a+2) = 0$	dM1	
	$\underline{a = 5 \text{ or } -2}$	A1	(3)
			6
(a)	B1 for $a \times 1 - 3$ or better. Give for $a-3$ in part (a) or if it appears in (b) they must state $x_2 = a-3$ This must be seen in (a) or before the $a(a-3)-3$ step.		
(b)	M1 for clear show that. Usually for $a(a-3)-3$ but can follow through their x_2 and even allow $ax_2 - 3$ A1 for correct processing leading to printed answer. Both lines needed and no incorrect working seen.		
(c)	1st M1 for attempt to form a correct equation and start to collect terms. It must be a quadratic but need not lead to a 3TQ=0 2nd dM1 This mark is dependent upon the first M1. for attempt to factorize their 3TQ=0 or to solve their 3TQ=0. The “=0”can be implied. $(x \pm p)(x \pm q) = 0$, where $pq = 10$ or $(x \pm \frac{3}{2})^2 \pm \frac{9}{4} - 10 = 0$ or correct use of quadratic formula with $\pm$ They must have a form that leads directly to 2 values for *a*. Trial and Improvement that leads to only one answer gets M0 here. A1 for both correct answers. Allow $x = \ldots$ Give 3/3 for correct answers with no working or trial and improvement that gives <u>both</u> values for *a*		

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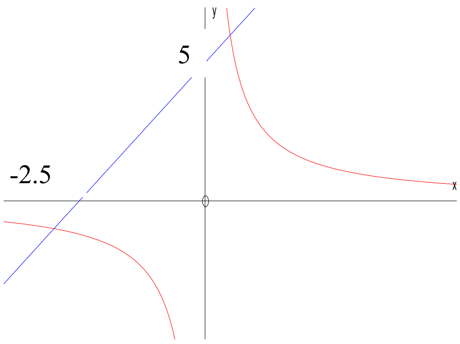
Question Number	Scheme	Marks
6. (a)	 	B1M1A1 (3)
(b)	$2x+5=\dfrac{3}{x}$	M1
	$2x^2+5x-3\,[=0]$ or $2x^2+5x=3$	A1
	$(2x-1)(x+3)\,[=0]$	M1
	$x=-3$ or $\frac{1}{2}$	A1
	$y=\dfrac{3}{-3}$ or $2\times(-3)+5$ or $y=\dfrac{3}{\frac{1}{2}}$ or $2\times\left(\frac{1}{2}\right)+5$	M1
	Points are $\underline{(-3,-1) \text{ and } (\frac{1}{2},6)}$ (correct pairings)	A1ft
		9

(a)	B1	for curve of correct shape i.e 2 branches of curve, in correct quadrants, of roughly the correct shape and no touching or intersections with axes. Condone up to 2 inward bends but there must be some ends that are roughly asymptotic.
	M1	for a straight line <u>cutting</u> the positive y-axis and the negative x-axis. Ignore any values.
	A1	for (0,5) and (-2.5,0) or points correctly marked on axes. Do not give for values in tables. Condone mixing up (x, y) as (y, x) if one value is zero and other value correct.
(b)	1st M1	for attempt to form a suitable equation and multiply by x (at least one of $2x$ or +5) should be multiplied.
	1st A1	for correct 3TQ - condone missing = 0
	2nd M1	for an attempt to solve a relevant 3TQ leading to 2 values for x =…
	2nd A1	for both x = -3 and 0.5.
	T&I for x values <u>may</u> score 1st M1A1 otherwise no marks unless both values correct. Answer only of x = -3 and $x = \frac{1}{2}$ scores 4/4, then apply the scheme for the final M1A1ft	
	3rd M1	for an attempt to find at least one y value by substituting their x in either $\dfrac{3}{x}$ or $2x + 5$
	3rd A1ft	follow through both their x values, in either equation but the same for each, correct pairings required but can be x = -3, y = -1 etc

Question number	Scheme	Marks
7. (a)	5, 7, 9, 11 or 5+2+2+2=11 or 5+6=11 use a = 5, d = 2, n = 4 and $t_4 = 5 + 3\times 2 = 11$	B1 (1)
(b)	$t_n = a + (n-1)d$ with one of a = 5 or d = 2 correct (can have a letter for the other)	M1
	$= 5 + 2(n - 1)$ or $2n + 3$ or $1 + 2(n + 1)$	A1 (2)
(c)	$S_n = \frac{n}{2}[2\times 5 + 2(n-1)]$ or use of $\frac{n}{2}(5 + \text{"their } 2n+3\text{"})$ (may also be scored in (b))	M1A1
	$= \{n(5 + n - 1)\} = n(n + 4)$ **(*)**	A1cso (3)
(d)	$43 = 2n + 3$	M1
	$[n] = 20$	A1 (2)
(e)	$S_{20} = 20\times 24$, $=$ <u>480</u> (km)	M1A1 (2)
		10
(a)	B1 Any other sum must have a convincing argument	
(b)	M1 for an attempt to use $a + (n - 1)d$ with one of a or d correct (the other can be a letter) Allow any answer of the form $2n + p$ ($p \neq 5$) to score M1. A1 for a correct expression (needn't be simplified) [**Beware** $5 + (2n-1)$ scores A0] Expression must be in n not x. Correct answers with no working scores 2/2.	
(c)	M1 for an attempt to use S_n formula with a = 5 or d = 2 or a = 5 and their "$2n + 3$" 1st A1 for a fully correct expression 2nd A1 for correctly simplifying to given answer. No incorrect working seen. Must see S_n used. Do not give credit for part (b) if the equivalent work is given in part (d)	
(d)	M1 for forming a suitable equation in n (ft their (b)) and attempting to solve leading to n =… A1 for 20 Correct answer only scores 2/2 . Allow 20 following a restart but check working. eg $43 = 2n + 5$ that leads to $40 = 2n$ and n =20 should score M1A0.	
(e)	M1 for using their answer for n in $n(n + 4)$ or S_n formula, their n must be a value. A1 for 480 (ignore units but accept 480 000 m etc)[no matter where their 20 comes from] NB "attempting to solve" eg part (d) means we will allow sign slips and slips in arithmetic but not in processes. So dividing when they should subtract etc would lead to M0. Listing in parts (d) and (e) can score 2 (if correct) or 0 otherwise in each part. Poor labelling may occur (especially in (b) and (c)) . If you see work to get $n(n + 4)$ mark as (c)	

Question number	Scheme	Marks
8. (a)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times 2q\times(-1)$	M1
	So $q^2-4\times 2q\times(-1)<0$ i.e. $q^2+8q<0$ **(*)**	A1cso (2)
(b)	$q(q+8)=0$ or $(q\pm 4)^2\pm 16=0$	M1
	$(q)=0$ or -8 (2 cvs)	A1
	$-8<q<0$ <u>or</u> $q\in(-8, 0)$ <u>or</u> $q<0$ and $q>-8$	A1ft (3)
		5

(a)	M1	for attempting b^2-4ac with one of *b* or *a* correct. < 0 not needed for M1 This may be inside a square root.
	A1cso	for simplifying to printed result with no incorrect working or statements seen. Need an intermediate step
	e.g. $q^2--8q<0$ or $q^2-4\times 2q\times -1<0$ or $q^2-4(2q)(-1)<0$ or $q^2-8q(-1)<0$ or $q^2-8q\times -1<0$ i.e. must have $\times$ or brackets on the $4ac$ term < 0 must be seen at least one line before the final answer.	
(b)	M1	for factorizing or completing the square or attempting to solve $q^2\pm 8q=0$. A method that would lead to 2 values for *q*. The "= 0" may be implied by values appearing later.
	1st A1	for $q=0$ and $q=-8$
	2nd A1	for $-8<q<0$. Can follow through their cvs but must choose "inside" region. $q<0, q>-8$ is A0, $q<0$ or $q>-8$ is A0, (-8, 0) on its own is A0 BUT "$q<0$ and $q>-8$" is A1 Do not accept a number line for final mark

Question number	Scheme	Marks
9. (a)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]3kx^2-2x+1$	M1A1 (2)
(b)	Gradient of line is $\frac{7}{2}$	B1
	When $x=-\frac{1}{2}$: $3k\times\left(\frac{1}{4}\right)-2\times\left(-\frac{1}{2}\right)+1,=\frac{7}{2}$	M1, M1
	$\frac{3k}{4}=\frac{3}{2}\Rightarrow k=2$	A1 (4)
(c)	$x=-\frac{1}{2}\Rightarrow y=k\times\left(-\frac{1}{8}\right)-\left(\frac{1}{4}\right)-\frac{1}{2}-5,=-6$	M1, A1 (2)
		8
(a)	M1 for attempting to differentiate $x^n\to x^{n-1}$ (or -5 going to 0 will do) A1 all correct. A "+ c" scores A0	
(b)	B1 for $m=\frac{7}{2}$. Rearranging the line into $y=\frac{7}{2}x+c$ does not score this mark until you are sure they are using $\frac{7}{2}$ as the gradient of the line or state $m=\frac{7}{2}$ 1st M1 for substituting $x=-\frac{1}{2}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$, some correct substitution seen 2nd M1 for forming a suitable equation in k and attempting to solve leading to $k=\ldots$ Equation must use their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and their gradient of line. Assuming the gradient is 0 or 7 scores M0 unless they have clearly stated that this is the gradient of the line. A1 for $k=2$	
(c)	M1 for attempting to substitute their k (however it was found or can still be a letter) and $x=-\frac{1}{2}$ into y (some correct substitution) A1 for - 6	

Question number	Scheme	Marks
10. (a)	$QR = \sqrt{(7-1)^2 + (0-3)^2}$	M1
	$= \sqrt{36+9}$ or $\sqrt{45}$ (condone $\pm$) A1	
	$= 3\sqrt{5}$ or $a = 3$ ($\pm 3\sqrt{5}$ etc is A0)	A1 (3)
(b)	Gradient of QR (or l_1) = $\dfrac{3-0}{1-7}$ or $\dfrac{3}{-6}$, $= -\dfrac{1}{2}$	M1, A1
	Gradient of l_2 is $-\dfrac{1}{-\frac{1}{2}}$ or 2	M1
	Equation for l_2 is: $y - 3 = 2(x-1)$ or $\frac{y-3}{x-1} = 2$ [or $y = 2x + 1$]	M1 A1ft (5)
(c)	P is (0, 1) (allow "$x = 0$, $y = 1$" but it must be clearly identifiable as P)	B1 (1)
(d)	$PQ = \sqrt{(1-x_P)^2 + (3-y_P)^2}$	M1
	$PQ = \sqrt{1^2 + 2^2} = \sqrt{5}$	A1
	Area of triangle is $\frac{1}{2}QR \times PQ = \frac{1}{2}3\sqrt{5} \times \sqrt{5}, = \dfrac{15}{2}$ or 7.5	dM1, A1 (4)
	Determinant Method e.g(0+0+7) - (1+21+0) = - 15 (o.e.) Area = $\frac{1}{2}\lvert -15 \rvert$, = 7.5	
		13

Rules for quoting formula: For an M mark, if a correct formula is quoted and some correct substitutions seen then M1 can be awarded, if no values are correct then M0. If no correct formula is seen then M1 can only be scored for a fully correct expression.

(a)
M1 for attempting QR or QR^2. May be implied by $6^2 + 3^2$
1st A1 for as printed or better. Must have square root. Condone $\pm$

(b)
1st M1 for attempting gradient of QR
1st A1 for - 0.5 or $-\frac{1}{2}$, can be implied by gradient of $l_2 = 2$
2nd M1 for an attempt to use the perpendicular rule on their gradient of QR.
3rd M1 for attempting equation of a line using Q with their changed gradient.
2nd A1ft requires all 3 Ms but can ft their gradient of QR.

$y = 2x + 1$ with no working. Send to review.

(d)
1st M1 for attempting PQ or PQ^2 follow through their coordinates of P
1st A1 for PQ as one of the given forms.
2nd dM1 for correct attempt at area of the triangle. Follow through their value of a and their PQ. This M mark is dependent upon the first M mark
2nd A1 for 7.5 or some exact equivalent. Depends on both Ms. Some working must be seen.

ALT
Use QS where S is (1, 0)
1st M1 for attempting area of $OPQS$ and QSR and OPR. Need all 3.
1st A1 for $OPQS = \frac{1}{2}(1+3) \times 1 = 2$, $QSR = 9$, $OPR = \frac{7}{2}$
2nd dM1 for $OPQS + QSR - OPR = \ldots$Follow through their values.
2nd A1 for 7.5

Determinant Method
M1 for attempt -at least one value in each bracket correct .
A1 if correct ($\pm$ 15)
M1 for correct area formula
A1 for 7.5

MR
Misreading x-axis for y-axis for P. Do NOT use MR rule as this oversimplifies the question. They can only get M marks in (d) if they use PQ and QR.

Question number	Scheme	Marks	
11. (a)	$\left(x^2+3\right)^2 = x^4+3x^2+3x^2+3^2$	M1	
	$\dfrac{\left(x^2+3\right)^2}{x^2} = \dfrac{x^4+6x^2+9}{x^2} = x^2+6+9x^{-2}$ **(*)**	A1cso	(2)
(b)	$y = \dfrac{x^3}{3}+6x+\dfrac{9}{-1}x^{-1}\left(+c\right)$	M1A1A1	
	$20 = \dfrac{27}{3}+6\times 3-\dfrac{9}{3}+c$	M1	
	$c = -4$	A1	
	$[y =]\dfrac{x^3}{3}+6x-9x^{-1}-4$	A1ft	(6)
			8

(a) M1 for attempting to expand $\left(x^2+3\right)^2$ and having at least 3(out of the 4) correct terms.

A1 at least this should be seen and no incorrect working seen.

If they never write $\dfrac{9}{x^2}$ as $9x^{-2}$ they score A0.

(b) 1st M1 for some correct integration, one correct x term as printed or better

Trying $\dfrac{\int u}{\int v}$ loses the first M mark but could pick up the second.

1st A1 for two correct x terms, un-simplified, as printed or better

2nd A1 for a fully correct expression. Terms need not be simplified and $+c$ is not required.

No $+ c$ loses the next 3 marks

2nd M1 for using $x = 3$ and $y = 20$ in their expression for f(x) $\left[\neq \dfrac{\mathrm{d}y}{\mathrm{d}x}\right]$ to form a linear equation for c

3rd A1 for $c = -4$

4th A1ft for an expression for y with simplified x terms: $\dfrac{9}{x}$ for $9x^{-1}$ is OK .

Condone missing "$y =$ "

Follow through their numerical value of c only.

June 2008
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	(a) Attempt to find f(–4) or f(4). $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$	M1
	$\left(= -128 - 48 + 156 + 20\right)$ = 0, so $(x + 4)$ is a factor.	A1 (2)
	(b) $2x^3 - 3x^2 - 39x + 20 = (x+4)(2x^2 - 11x + 5)$	M1 A1
	.....$(2x-1)(x-5)$ (The 3 brackets need not be written together)	M1 A1cso(4)
	or .....$\left(x - \frac{1}{2}\right)(2x - 10)$ or equivalent	
		6
	(a) Long division scores no marks in part (a). The factor theorem is required. However, the first two marks in (b) can be earned from division seen in (a)… … but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b). A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.), or may be scored by a preamble, e.g. 'If f(–4) = 0, $(x + 4)$ is a factor…..' (b) First M requires use of $(x + 4)$ to obtain $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen… this could be done 'by inspection'. Second M for the attempt to factorise their three-term quadratic. Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $\lvert cd \rvert = \lvert b \rvert$ and $\lvert pq \rvert = \lvert k \rvert$. If 'solutions' appear before or after factorisation, ignore… … but factors must be seen to score the second M mark. Alternative (first 2 marks): $(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find values of a and b. [M1] $a = -11,\ b = 5$ [A1] Alternative: Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0$ $\therefore$ factor is, $(2x - 1)$ [M1, A1] Finding that $f(5) = 0$ $\therefore$ factor is, $(x - 5)$ [M1, A1] "Combining" all 3 factors is not required. If just one of these is found, score the first 2 marks M1 A1 M0 A0. Losing a factor of 2: $(x+4)\left(x - \frac{1}{2}\right)(x - 5)$ scores M1 A1 M1 A0 Answer only, one sign wrong: e.g. $(x+4)(2x-1)(x+5)$ scores M1 A1 M1 A0	

Question number	Scheme	Marks
2.	(a) 1.732, 2.058, 5.196 awrt (One or two correct B1 B0, All correct B1 B1)	B1 B1 (2)
	(b) $\frac{1}{2} \times 0.5$	B1
	......$\{(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)\}$	M1 A1ft
	= 5.899 (awrt 5.9, allowed even after minor slips in values)	A1 (4)
		6
	(a) Accept awrt (but <u>less</u> accuracy loses these marks). Also accept <u>exact</u> answers, e.g. $\sqrt{3}$ at $x = 0$, $\sqrt{27}$ or $3\sqrt{3}$ at $x = 2$. (b) For the M mark, the first bracket must contain the 'first and last' values, and the second bracket must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\frac{1}{2} \times 0.5(1.732 + 5.196) + 2(2.058 + 2.646 + 3.630)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>x values</u>: M0 if the values used in the brackets are x values instead of y values. <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.732 + 2.058) + \frac{1}{4}(2.058 + 2.646) + \frac{1}{4}(2.646 + 3.630) + \frac{1}{4}(3.630 + 5.196)\right]$	

Question number	Scheme	Marks	
3.	(a) $(1+ax)^{10} = 1+10ax......$ (<u>Not</u> unsimplified versions)	B1	
	$+\dfrac{10\times 9}{2}(ax)^2 + \dfrac{10\times 9\times 8}{6}(ax)^3$ Evidence from <u>one</u> of these terms is sufficient	M1	
	$+45(ax)^2$, $+120(ax)^3$ or $+45a^2x^2$, $+120a^3x^3$	A1, A1	(4)
	(b) $120a^3 = 2\times 45a^2$ $a=\dfrac{3}{4}$ or equiv. $\left(\text{e.g.} \dfrac{90}{120}, 0.75\right)$ Ignore a = 0, if seen	M1 A1	(2)
			6
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: 'binomial coefficient' (perhaps from Pascal's triangle) and the correct power of x. (The M mark can also be given for an expansion in <u>descending</u> powers of x). Allow 'slips' such as: $\dfrac{10\times 9}{2}ax^2$, $\dfrac{10\times 9}{3\times 2}(ax)^3$, $\dfrac{10\times 9}{2}x^2$, $\dfrac{9\times 8\times 7}{3\times 2}a^3x^3$ However, $45+a^2x^2+120+a^3x^3$ or similar is M0. $\begin{pmatrix}10\\2\end{pmatrix}$ and $\begin{pmatrix}10\\3\end{pmatrix}$ or equivalent such as ${}^{10}C_2$ and ${}^{10}C_3$ are acceptable, and even $\left(\dfrac{10}{2}\right)$ and $\left(\dfrac{10}{3}\right)$ are acceptable for the method mark. 1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (These <u>must</u> be simplified). If simplification is not seen in (a), but correct simplified terms are seen in (b), these marks can be awarded. However, if <u>wrong</u> simplification is seen in (a), this takes precedence. <u>Special case</u>: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $45ax^2$ and $120ax^3$ are <u>both</u> achieved. (b) M: Equating their coefficent of x^3 to twice their coefficient of x^2 … … <u>or</u> equating their coefficent of x^2 to twice their coefficient of x^3. (… or coefficients can be <u>correct</u> coefficients rather than their coefficients). Allow this mark even if the equation is trivial, e.g. 120a = 90a. An equation in a alone is required for this M mark, although… …condone, e.g. $120a^3x^3 = 90a^2x^2 \Rightarrow \left(120a^3 = 90a^2 \Rightarrow\right) a = \dfrac{3}{4}$. <u>Beware</u>: $a = \dfrac{3}{4}$ following 120a = 90a, which is A0.		

Question number	Scheme	Marks
4.	(a) $x = \dfrac{\log 7}{\log 5}$ or $x = \log_5 7$ (i.e. correct method up to $x =$…)	M1
	1.21 Must be this answer (3 s.f.)	A1 (2)
	(b) $(5^x - 7)(5^x - 5)$ Or another variable, e.g. $(y-7)(y-5)$, even $(x-7)(x-5)$	M1 A1
	$(5^x = 7$ or $5^x = 5)$ $x = 1.2$ (awrt) ft from the answer to (a), if used	A1ft
	$x = 1$ (allow 1.0 or 1.00 or 1.000)	B1 (4)
		6
	(a) 1.21 with no working: M1 A1 (even if it left as $5^{1.21}$). Other answers which round to 1.2 with no working: M1 A0. (b) M: Using the <u>correct</u> quadratic equation, attempt to factorise $(5^x \pm 7)(5^x \pm 5)$, or attempt quadratic formula. Allow $\log_5 7$ or $\dfrac{\log 7}{\log 5}$ instead of 1.2 for A1ft. No marks for simply substituting a decimal answer from (a) into the given equation (perhaps showing that it gives approximately zero). <u>However</u>, note the following <u>special case</u>: Showing that $5^x = 7$ satisfies the given equation, therefore 1.21 is a solution scores 0, 0, 1, 0 (and could score <u>full marks</u> if the $x = 1$ were also found). e.g. If $5^x = 7$, then $5^{2x} = 49$, and $5^{2x} - 12(5^x) + 35 = 49 - 84 + 35 = 0$, so one solution is $x = 1.21$ ('conclusion' must be seen). To score this special case mark, values substituted into the equation must be <u>exact</u>. Also, the mark would <u>not</u> be scored in the following case: e.g. If $5^x = 7$, $5^{2x} - 84 + 35 = 0 \Rightarrow 5^{2x} = 49 \Rightarrow x = 1.21$ (Showing no appreciation that $5^{2x} = (5^x)^2$) B1: Do not award this mark if $x = 1$ clearly follows from <u>wrong</u> working.	

Question number	Scheme	Marks
5.	(a) $(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (*k* a positive <u>value</u>)	M1
	$(x-3)^2+(y-1)^2=29$ (<u>Not</u> $\left(\sqrt{29}\right)^2$ or 5.39^2)	A1 (4)
	(b) Gradient of radius = $\frac{2}{5}$ (or exact equiv.) Must be seen or used in (b)	B1
	Gradient of tangent = $\frac{-5}{2}$ (Using perpendicular gradient method)	M1
	$y-3=\frac{-5}{2}(x-8)$ (ft gradient of radius, dependent upon <u>both</u> M marks)	M1 A1ft
	$5x+2y-46=0$ (Or equiv., equated to zero, e.g. $92-4y-10x=0$) (Must have <u>integer</u> coefficients)	A1 (5) **9**
	(a) For the M mark, condone <u>one</u> slip <u>inside</u> a bracket, e.g. $(8-3)^2+(3+1)^2$, $(8-1)^2+(1-3)^2$ The first two marks may be gained implicitly from the circle equation. (b) 2nd M: Eqn. of line through (8, 3), in any form, with any grad.(except 0 or ∞). If the 8 and 3 are the 'wrong way round', this M mark is only given if a correct general formula, e.g. $y-y_1=m(x-x_1)$, is quoted. <u>Alternative:</u> 2nd M: Using (8, 3) and an *m* value in $y=mx+c$ to find a value of *c*. A1ft: as in main scheme. (Correct substitution of 8 and 3, then a wrong *c* value will still score the A1ft) (b) <u>Alternatives for the first 2 marks</u>: (but in these 2 cases the 1st A mark is <u>not</u> ft) (i) Finding gradient of tangent by <u>implicit</u> differentiation	
	$2(x-3)+2(y-1)\frac{dy}{dx}=0$ (or equivalent)	B1
	Subs. $x=8$ <u>and</u> $y=3$ into a 'derived' expression to find a value for dy/dx	M1
	(ii) Finding gradient of tangent by differentiation of $y=1+\sqrt{20+6x-x^2}$	
	$\frac{dy}{dx}=\frac{1}{2}\left(20+6x-x^2\right)^{-\frac{1}{2}}(6-2x)$ (or equivalent)	B1
	Subs. $x=8$ into a 'derived' expression to find a value for dy/dx	M1
	<u>Another alternative</u>: Using $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $x^2+y^2-6x-2y-19=0$ B1 $8x+3y, \quad -3(x+8)-(y+3)-19=0$ M1, M1 A1ft (ft from circle eqn.) $5x+2y-46=0$ A1	

Question number	Scheme	Marks
6.	(a) $T_{20} = 5 \times \left(\frac{4}{5}\right)^{19} = 0.072$ (Accept awrt) Allow $5 \times \frac{4}{5}^{19}$ for M1 M1 A1	(2)
	(b) $S_\infty = \frac{5}{1-0.8} = 25$	M1 A1 (2)
	(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$ (Allow with = or <)	M1
	$1-0.8^k > 0.998$ (or equiv., see below) (Allow with = or <)	A1
	$k \log 0.8 < \log 0.002$ or $k > \log_{0.8} 0.002$ (Allow with = or <)	M1
	$k > \frac{\log 0.002}{\log 0.8}$ (*)	A1cso (4)
	(d) k = 28 (Must be this integer value) <u>Not</u> $k > 27$, or $k < 28$, or $k > 28$ B1	(1)
		9
	(a) and (b): Correct answer without working scores both marks. (a) M: Requires use of the correct formula ar^{n-1}. (b) M: Requires use of the correct formula $\frac{a}{1-r}$ (c) 1st M: The sum may have already been 'manipulated' (perhaps wrongly), but this mark can still be allowed. 1st A: A 'numerically correct' version that has dealt with $(1-0.8)$ denominator, e.g. $1-\left(\frac{4}{5}\right)^k > 0.998$, $5(1-0.8^k) > 4.99$, $25(1-0.8^k) > 24.95$, $5-5(0.8^k) > 4.99$. In any of these, $\frac{4}{5}$ instead of 0.8 is fine, and condone $\frac{4}{5}^k$ if correctly treated later. 2nd M: Introducing logs and using laws of logs correctly (this must include dealing with the power k so that $p^k = k \log p$). 2nd A: An <u>incorrect</u> statement (including equalities) at any stage in the working loses this mark (this is often identifiable at the stage $k \log 0.8 > \log 0.002$). (So a fully correct method with inequalities is required.)	

Question number	Scheme	Marks	
7.	(a) $r\theta = 7 \times 0.8 = 5.6$ (cm)	M1 A1	(2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = 19.6$ (cm^2)	M1 A1	(2)
	(c) $BD^2 = 7^2 + (\text{their } AD)^2 - (2 \times 7 \times (\text{their } AD) \times \cos 0.8)$	M1	
	$BD^2 = 7^2 + 3.5^2 - (2 \times 7 \times 3.5 \times \cos 0.8)$ (or awrt 46° for the angle)	A1	
	($BD =$ 5.21) Perimeter = (their DC) + "5.6" + "5.21" = 14.3 (cm) (Accept awrt)	M1 A1	(4)
	(d) $\Delta ABD = \frac{1}{2} \times 7 \times (\text{their } AD) \times \sin 0.8$ (or awrt 46° for the angle) (ft their AD)	M1 A1ft	
	(= 8.78…) (If the correct formula $\frac{1}{2}ab\sin C$ is quoted the use of any two of the sides of ΔABD as a and b scores the M mark).		
	Area = "19.6" – "8.78…" = 10.8 (cm^2) (Accept awrt)	M1 A1	(4)
			12
	Units (cm or cm^2) are not required in any of the answers. (a) and (b): Correct answers without working score both marks. (a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). (c) 1st M: Use of the (correct) cosine rule formula to find BD^2 or BD. Any other methods need to be complete methods to find BD^2 or BD. 2nd M: Adding their DC to their arc BC and their BD. <u>Beware</u>: If 0.8 is used, but calculator is in degree mode, this can still earn M1 A1 (for the required expression), but this gives BD = 3.50… so the perimeter may appear as 3.5 + 5.6 + 3.5 (earning M1 A0). (d) 1st M: Use of the (correct) area formula to find ΔABD. Any other methods need to be complete methods to find ΔABD. 2nd M: Subtracting their ΔABD from their sector ABC. Using segment formula $\frac{1}{2}r^2(\theta - \sin\theta)$ scores no marks in part (d).		

Question number	Scheme	Marks
8.	(a) $\left(\frac{dy}{dx}=\right)8+2x-3x^2$ (M: $x^n \to x^{n-1}$ for one of the terms, <u>not</u> just $10 \to 0$)	M1 A1
	$3x^2-2x-8=0$ $(3x+4)(x-2)=0$ $x=2$ (Ignore other solution) (*)	A1cso (3)
	(b) Area of triangle = $\frac{1}{2}\times 2\times 22$ (M: Correct method to find area of triangle)	M1 A1
	(Area = 22 with no working is acceptable)	
	$\int 10+8x+x^2-x^3\,dx = 10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (M: $x^n \to x^{n+1}$ for one of the terms)	M1 A1 A1
	Only one term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 $\boxed{\text{Integrating the gradient function loses this M mark.}}$	
	$\left[10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	$\left(=20+16+\frac{8}{3}-4\right)$ (This M can be awarded even if the other limit is wrong)	
	Area of $R=34\frac{2}{3}-22=\frac{38}{3}\left(=12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
	M: <u>Dependent on use of calculus in (b) and correct overall 'strategy'</u>: subtract either way round. A: Must be <u>exact</u>, not 12.67 or similar. A negative area at the end, even if subsequently made positive, loses the A mark.	
		11
	(a) The final mark may also be scored by <u>verifying</u> that $\frac{dy}{dx}=0$ at $x=2$.	
	(b) <u>Alternative</u>:	
Eqn. of	line $y=11x$. (Marks dependent on subsequent use in integration) (M1: Correct method to find equation of line. A1: Simplified form $y=11x$)	M1 A1
	$\int 10+kx+x^2-x^3\,dx = 10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$ (k perhaps −3)	M1 A1 A1
	$\left[10x+\frac{kx^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = \ldots\ldots$ (Substitute limit 2 into a 'changed function')	M1
	Area of $R=\left[10x-\frac{3x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = 20-6+\frac{8}{3}-4=\frac{38}{3}$ $\left(=12\frac{2}{3}\right)$	M1 A1 (8)
	Final M1 for $\int(\text{curve})-\int(\text{line})$ or $\int(\text{line})-\int(\text{curve})$.	

Question number	Scheme	Marks
9.	(a) 45 (α) (This mark can be implied by an answer 65)	B1
	$180-\alpha$, Add 20 (for at least one angle)	M1, M1
	65 155	A1 (4)
	(b) 120 or 240 (β): (This mark can be implied by an answer 40 or 80)	B1
	(Could be achieved by working with 60, using $180-60$ and/or $180+60$)	
	$360-\beta$, $360+\beta$ (or 120 + an angle that has been divided by 3)	M1, M1
	Dividing by 3 (for at least one angle)	M1
	40 80 160 200 280 320 First A1: at least 3 correct	A1 A1(6)
		10
	(a) Extra solution(s) in range: Loses the A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 65 (only correct solution) will score B1 M0 M1 A0 (2 marks) 65 and 115 will score B1 M0 M1 A0 (2 marks) 44.99 (or similar) for α is B0, and 64.99, 155.01 (or similar) is A0. (b) Extra solution(s) in range: Loses the final A mark. Extra solutions outside range: Ignore (whether correct or not). Common solutions: 40 (only correct solution) will score B1 M0 M0 M1 A0 A0 (2 marks) 40 and 80 (only correct solutions) B1 M1 M0 M1 A0 A0 (3 marks) 40 and 320 (only correct solutions) B1 M0 M0 M1 A0 A0 (2 marks) Answers without working: Full marks can be given (in both parts), B and M marks by implication. Answers given in radians: Deduct a maximum of 2 marks (misread) from B and A marks. (Deduct these at first and second occurrence.) Answers that begin with statements such as $\sin(x-20)=\sin x-\sin 20$ or $\cos x=-\frac{1}{6}$, then go on to find a value of 'α' or 'β', however badly, can continue to earn the first M mark in either part, but will score no further marks. Possible misread: $\cos 3x=\frac{1}{2}$, giving 20, 100, 140, 220, 260, 340 Could score up to 4 marks B0 M1 M1 M1 A0 A1 for the above answers.	

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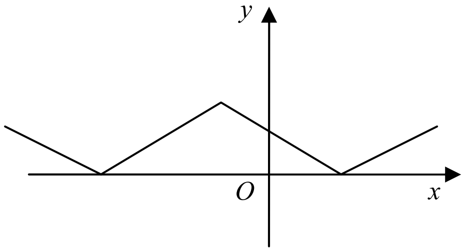
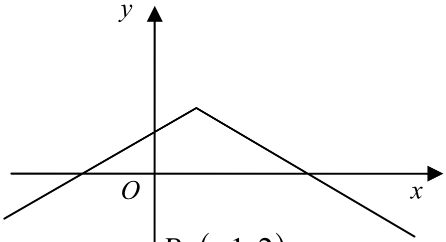
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x+1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \frac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x-\alpha) = \frac{6}{13}$		M1	
	$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x - \alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3)
				[12]

Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1,2)$	B1
	$Q:(0,1)$	B1
	$R:(1,0)$	B1 (3)
	(d) $x>-1$; $2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x<-1$; $2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1 (5)
		[12]

Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$\text{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)} \right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ * cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\text{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=\text{f}(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\text{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $\text{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta + \cos^2\theta = 1$	
	$\div \sin^2\theta$ $\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ∗ cso	A1 (2)
	Alternative for (a)	
	$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$= \operatorname{cosec}^2\theta$ ∗ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$	M1
	$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$	M1
	$\left(2\operatorname{cosec}\theta + 1\right)\left(\operatorname{cosec}\theta - 5\right) = 0$ *or* $\left(5\sin\theta - 1\right)\left(\sin\theta + 2\right) = 0$	M1
	$\operatorname{cosec}\theta = 5$ *or* $\sin\theta = \dfrac{1}{5}$	A1
	$\theta = 11.5°, 168.5°$	A1 A1 (6) **[8]**

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(\sin x + 2\cos x)\right) = 3\mathrm{e}^{3x}(\sin x + 2\cos x) + \mathrm{e}^{3x}(\cos x - 2\sin x)$	M1 A1 A1 (3)
	$\left(= \mathrm{e}^{3x}(\sin x + 7\cos x)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$= \frac{20}{(x+1)^3}$ * cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{60}{(x+1)^4} = -\frac{15}{4}$	M1
	$(x+1)^4 = 16$	M1
	$x = 1, -3$ both	A1 (3) **[14]**
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	
	$= \frac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$	
	$= \frac{20x+20}{(x+1)^4} = \frac{20(x+1)}{(x+1)^4} = \frac{20}{(x+1)^3}$	M1 A1

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(1.4) = -0.568 \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245 \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	A1 (2)
	(b) $3x^3 = 2x + 6$	
	$x^3 = \frac{2x}{3} + 2$	
	$x^2 = \frac{2}{3} + \frac{2}{x}$	M1 A1
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ * cso	A1 (3)
	(c) $x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01 \ldots$	
	$\mathrm{f}(1.4355) = 0.003 \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso	A1 (3) **[11]**
	Note: $\alpha = 1.435\,304\,553 \ldots$	

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6666 Core Mathematics C4
Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2
or y	1	1.08329…	1.37713…	2.05443…	3.59664…	7.38906…

Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)</td><td>B1

[1]</td></tr>
<tr><td>(b)
Way 1</td><td>

$\text{Area} \approx \frac{1}{2} \times 0.4\ ; \times \underline{\left[\ e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2\ \right]}$

Outside brackets $\frac{1}{2} \times 0.4$ or 0.2

<u>For structure of trapezium rule</u> $\underline{[\ \ldots\ldots\ldots\ldots\ ]}$;

$= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf) $\underline{4.922}$</td><td>B1;

<u>M1</u> $\sqrt{}$

A1 cao
[3]</td></tr>
<tr><td>Aliter
(b)
Way 2</td><td>

$\text{Area} \approx 0.4 \times \left[\frac{e^0+e^{0.08}}{2} + \frac{e^{0.08}+e^{0.32}}{2} + \frac{e^{0.32}+e^{0.72}}{2} + \frac{e^{0.72}+e^{1.28}}{2} + \frac{e^{1.28}+e^2}{2}\right]$

0.4 and a divisor of 2 on all terms inside brackets.

which is equivalent to:

$\text{Area} \approx \frac{1}{2} \times 0.4\ ; \times \underline{\left[\ e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2\ \right]}$

One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.

$= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf) $\underline{4.922}$</td><td>B1

<u>M1</u> $\sqrt{}$

A1 cao
[3]</td></tr>
<tr><td></td><td></td><td>4 marks</td></tr>
</table>

Note an expression like $\text{Area} \approx \frac{1}{2} \times 0.4 + e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the $(\quad)$ brackets for M1.

The M1 mark for structure is for the material found in the curly brackets ie

$\underline{\left[\ \text{first } y \text{ ordinate} + 2\left(\text{intermediate ft } y \text{ ordinate}\right) + \text{final } y \text{ ordinate}\ \right]}$

Question Number	Scheme		Marks
2. (a)	$\left\{\begin{matrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int xe^x \, dx = xe^x - \int e^x.1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.) Correct expression. (Ignore dx)	M1 A1
	$= xe^x - \int e^x \, dx$		
	$= xe^x - e^x \, (+c)$	Correct integration with/without + c	A1
			[3]
(b)	$\left\{\begin{matrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int x^2e^x \, dx = x^2 e^x - \int e^x.2x \, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore dx)	M1 A1
	$= x^2e^x - 2\int xe^x \, dx$		
	$= x^2e^x - 2\left(xe^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW**
			[3]
	$\left\{\begin{matrix} = x^2e^x - 2xe^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{matrix}\right\}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example.

+ c is not required in part (a).
+ c is required in part (b).

Question Number	Scheme		Marks
3. (a)	From question, $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 0.032$	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = 0.032$ seen or implied from working.	B1
	$\left\{ A = \pi x^2 \Rightarrow \dfrac{\mathrm{d}A}{\mathrm{d}x} = \right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}t} \div \dfrac{\mathrm{d}A}{\mathrm{d}x} = (0.032)\dfrac{1}{2\pi x}; \left\{ = \dfrac{0.016}{\pi x} \right\}$	$0.032 \div$ Candidate's $\dfrac{\mathrm{d}A}{\mathrm{d}x}$;	M1;
	When $x = 2\,\text{cm}$, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{0.016}{2\pi}$		
	Hence, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 15\pi x^2$	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t} = 15\pi x^2.\left(\dfrac{0.016}{\pi x}\right); \{= 0.24x\}$	Candidate's $\dfrac{\mathrm{d}V}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t}$;	M1 $\sqrt{}$
	When $x = 2\,\text{cm}$, $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\,\text{s}^{-1})$	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Question Number	Scheme		Marks
4. (a)	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$		
		Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} =\right)$)	M1
	$\left\{\cancel{\frac{dy}{dx}} \cancel{=}\right\} \quad \underline{6x - 2y\frac{dy}{dx}} + \underline{\underline{\left(y + x\frac{dy}{dx}\right)}} = \underline{0}$	Correct application ( ) of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	A1
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1 *
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1 *
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b)	At P & Q, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the y terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	A1
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	$\boxed{\text{Both } \underline{(2,4)} \text{ and } \underline{(-2,-4)}}$	A1
			[3]
			9 marks

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
5. (a)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	<u>B1</u>
		Expands $(1+{*}{*}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({*}{*}x)$;	M1;
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})({*}{*}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({*}{*}x)^2+\ldots}\right]$ **with** ${*}{*} \neq 1$	A correct simplified or an un-simplified $[\ \underline{\ldots\ldots\ldots\ldots}\ ]$ expansion with candidate's followed through $({*}{*}x)$	A1 $\sqrt{}$
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+\ldots}\right]$	Award SC M1 if you see $(-\frac{1}{2})({*}{*}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({*}{*}x)^2$	
		$\frac{1}{2}[\ 1+\frac{3}{8}x;\ldots\ ]$	A1 isw
	$=\frac{1}{2}[\ 1+\frac{3}{8}x;+\frac{27}{128}x^2+\ldots\ ]$	SC: $K[\ 1+\frac{3}{8}x+\frac{27}{128}x^2+\ldots\ ]$	
		$\frac{1}{2}[\ \ldots\ldots\ldots;\frac{27}{128}x^2\ ]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+\ldots\right\}$	*Ignore subsequent working*	
			[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\ldots\right)$	Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+\ldots\ldots$ $+\ 4+\frac{3}{2}x+\frac{27}{32}x^2+\ldots\ldots$	Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$= 4+2x;+\frac{33}{32}x^2+\ldots$	Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
			[4]
			9 marks

Question Number	Scheme		Marks
6. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\mathbf{i}: \ -9+2\lambda = 3+3\mu \quad (1)$ $\mathbf{j}: \ \lambda = 1-\mu \quad (2)$ $\mathbf{k}: \ 10-\lambda = 17+5\mu \quad (3)$	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) – 2**(2)** gives: $-9 = 1+5\mu \ \Rightarrow \ \mu = -2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda = 1 - -2 = 3$	Both $\underline{\lambda = 3}$ & $\underline{\mu = -2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively**. This mark can be implied by any two correct components of $(-3, 3, 7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3, 3, 7)$	A1
	Either check **k:** $\lambda = 3$: LHS $= 10-\lambda = 10-3 = 7$ $\mu = -2$: RHS $= 17+5\mu = 17-10 = 7$ (As LHS = RHS then the lines intersect.)	**Either** check that $\lambda = 3$, $\mu = -2$ in a third equation **or** check that $\lambda = 3$, $\mu = -2$ give the same coordinates on the other line. Conclusion not needed.	B1 **[6]**
(b)	$\mathbf{d}_1 = 2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix} = \underline{(2\times 3)+(1\times -1)+(-1\times 5)} = 0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times 3)+(1\times -1)+(-1\times 5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1 **[2]**

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>6. (c)
Way 1</td><td>Equating i ; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$

$\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$
$(= \overrightarrow{OA}$. Hence the point A lies on l_1.)</td><td>Substitutes candidate's $\lambda = 7$ into the line l_1 and finds $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.
The conclusion on this occasion is not needed.</td><td>B1

[1]</td></tr>
<tr><td>(d)</td><td>Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection

$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \underline{\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$</td><td>Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$.
$\overrightarrow{AX} = \pm\left(\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}\right)$</td><td>M1 $\sqrt{\ }$ $\pm$</td></tr>
<tr><td></td><td>$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$

$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$</td><td>$\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$</td><td>dM1 $\sqrt{\ }$</td></tr>
<tr><td></td><td>Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$</td><td>$\underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$
or $\underline{(-11, -1, 11)}$</td><td>A1

[3]</td></tr>
<tr><td></td><td></td><td></td><td>12 marks</td></tr>
</table>

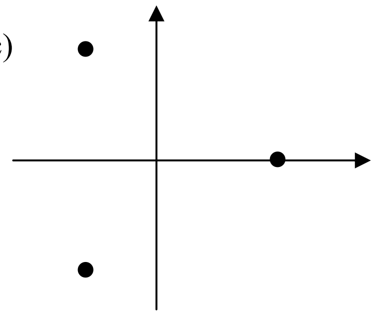
Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A = \frac{1}{2}$ or $B = \frac{1}{2}$	A1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	$\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$, aef	A1 **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of A or B is incorrect then M0A0A0.)		**[3]**

Question Number	Scheme		Marks
7. (b)	$\displaystyle\int \frac{2}{4-y^2}\,\mathrm{d}y = \int \frac{1}{\cot x}\,\mathrm{d}x$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\displaystyle\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,\mathrm{d}y = \int \tan x\,\mathrm{d}x$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$	their $\int \frac{1}{\cot x}\,\mathrm{d}x =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$	A1 $\sqrt{\ }$
	$y=0,\ x=\frac{\pi}{3}\ \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\left\{0 = \ln 2 + c\ \Rightarrow\ \underline{c = -\ln 2}\right\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\displaystyle\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\displaystyle\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\displaystyle\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\displaystyle\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\displaystyle\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\displaystyle\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

Question Number	Scheme		Marks
8. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or awrt 1.05 (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
			[2]
(b)	$x = 8\cos t, \quad y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence m(**N**) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses m(**N**) $= -\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their m}_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
8. (c)	$A = \int_0^4 y\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.\left(-8\sin t\right)\,\mathrm{d}t$	attempt at $A = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$ correct expression (ignore limits and $\mathrm{d}t$)	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,\mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\left(2\sin t\cos t\right).\sin t\,\mathrm{d}t$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,\mathrm{d}t$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,\mathrm{d}t$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$ } {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$ }		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$ Correct integration ignoring limits.	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = 1 \text{ and } u = \frac{\sqrt{3}}{2}\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw** **[4]**
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks

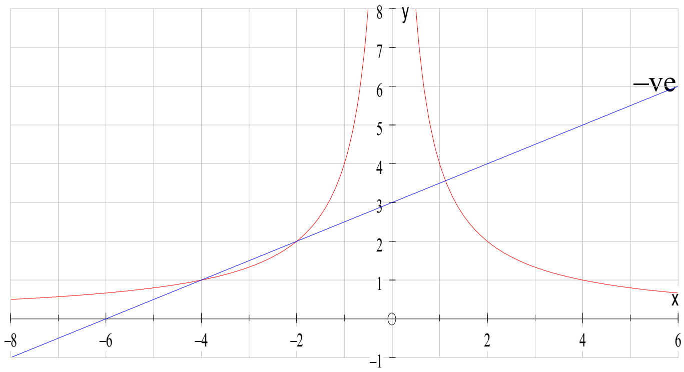
June 2008
Further Pure Mathematics FP1
Mark Scheme

Question number	Scheme	Marks
1.	(a) 4	B1 (1)
	(b) $(x-4)(x^2+4x+16)$	M1 A1
	$x=\dfrac{-4\pm\sqrt{16-64}}{2}$, $x=-2\pm2\sqrt{3}\,\mathrm{i}$ (or equiv. surd for $2\sqrt{3}$)	M1, A1 (4)
	(c)  Root on + ve real axis, one other in correct quad.	B1
	Third root in conjugate complex position	B1ft (2)
		7
	M1 in part (b) needs (x-“their 4”) times quadratic ($x^2+ax+..$) or times (x^2+16) M1 needs solution of three term quadratic So (x^2+16) special case, results in B1M1A0M0A0B0B1 possibly Alternative scheme for (b)	
	$(a+ib)^3=64$, so $a^3+3a^2ib+3a(ib)^2+(ib)^3=64$ and equate real, imaginary parts	M1
	so $a^3-3ab^2=64$ and $3a^2b-b^3=0$	A1
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1
	Alternative ii	
	$(x-4)(x-a-ib)(x-a+ib)=0$ expand and compare coefficients	M1
	two of the equations $-2a-4=0,\ 8a+a^2+b^2=0,\ 4(a^2+b^2)=64$	A1
	Solve to obtain $a=-2,\ b=\sqrt{12}$	M1A1
	(c)Allow vectors, line segments or points in Argand diagram. Extra points plotted in part (c) – lose last B mark Part (c) answers are independent of part (b)	

Question number	Scheme	Marks	
2.	(a) $f(1.6) = ...$ $\quad f(1.7) = ...$ $\quad$ (Evaluate both)	M1	
	0.08… (or 0.09), $\quad$ –0.3… $\quad$ One +ve, one –ve or Sign change, $\therefore$ root	A1	(2)
	(b) $f'(x) = -4\sin x - e^{-x}$	B1	
	$1.6 - \dfrac{f(1.6)}{f'(1.6)}$	M1	
	$= 1.6 - \dfrac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})} \quad \left(= 1.6 - \dfrac{0.085...}{-4.2...}\right)$	A1	
	$= 1.62$	A1	(4)
			6
	(a) Any errors seen in evaluation of f(1.6) or f(1.7) lose A mark so –0.32 is A0 Values are 0.0851 and –0.3327 Need concluding statement also. (b) B1 may be awarded if seen in N-R as $-4\sin 1.6 - e^{-1.6}$ or as –4.2 M1 for statement of Newton Raphson (sign error in rule results in M 0) First A1 may be implied by correct work previously followed by correct answer Do not accept 1.620 for final A1. It must be given and correct to 3sf. 1.62 may follow incorrect work and is A0 No working at all in part (b) is zero marks.		

Question number	Scheme	Marks
3.	(a) $z = \dfrac{(a+2\mathrm{i})(a+\mathrm{i})}{(a-\mathrm{i})(a+\mathrm{i})} = \dfrac{a^2+3a\mathrm{i}-2}{a^2+1}$	M1 A1
	$\dfrac{a^2-2}{a^2+1} = \dfrac{1}{2}$, $\quad 2a^2-4 = a^2+1 \quad\quad a=\sqrt{5}$ (presence of $-\sqrt{5}$ also is A0)	M1, A1 (4)
	(b) Evaluating their "$\dfrac{3a}{a^2+1}$", or "$3a$" $\left(\dfrac{\sqrt{5}}{2} \text{ or } 3\sqrt{5}\right)$ (ft errors in part *a*)	B1ft
	$\tan\theta = \dfrac{3a}{a^2-2}$ $\left(= \dfrac{3\sqrt{5}}{3}\right)$, arg z = 1.15 (accept answers which round to 1.15)	M1, A1 (3)
	(b) B mark is treated here as a method mark The M1 is for tan (argz) = Imaginary part / real part answer in degrees is A0 <u>Alternative method</u>: (a) $\left(\dfrac{1}{2}+\mathrm{i}y\right)(a-\mathrm{i}) = a+2\mathrm{i} \;\Rightarrow\; \dfrac{1}{2}a+y=a$ and $ay-\dfrac{1}{2}=2$ $y=\dfrac{1}{2}a$ and $ay=\dfrac{5}{2} \;\Rightarrow\; \dfrac{1}{2}a^2=\dfrac{5}{2} \;\Rightarrow\; a=\sqrt{5}$ (b) $y=\dfrac{\sqrt{5}}{2}$ (May be seen in part (a)) $\tan\theta=\sqrt{5}$ arg z = 1.15 <u>Further Alternative method in (b)</u> Use $\arg(a+2\mathrm{i})-\arg(a-\mathrm{i})$ $=0.7297-(-0.4205)=1.15$	 M1 A1 M1 A1 (4) B1ft M1 A1 (3) B1 M1A1 (3) **7**

Question number	Scheme	Marks
4.	(a) $m^2 + 4m + 3 = 0$ $\quad m = -1,\ m = -3$	M1 A1
	C.F. $(x =)Ae^{-t} + Be^{-3t}$ $\quad$ must be function of t, not x	A1
	P.I. $x = pt + q$ $\quad$ **(or** $\quad x = at^2 + bt + c$)	B1
	$4p + 3(pt + q) = kt + 5$ $\quad 3p = k$ $\quad$ (Form at least one eqn. in p and/or q)	M1
	$4p + 3q = 5$	
	$p = \frac{k}{3}$, $\quad q = \frac{5}{3} - \frac{4k}{9}\ \left(= \frac{15-4k}{9}\right)$	A1
	General solution: $x = Ae^{-t} + Be^{-3t} + \frac{kt}{3} + \frac{15-4k}{9}$ must include x = and be function of t	A1 ft
	(7) $\quad$ (b) When $k = 6$, $\quad x = 2t - 1$	M1 A1cao (2)
		9
	(a) M1 for auxiliary equation substantially correct B1 not awarded for $x = kt$+constant (b) M mark for using $k = 6$ to derive a linear expression in t. (cf must have involved negative exponentials only) so e.g. $y = 2t - 1$ is M1 A0	

Question number	Scheme	Marks
5.	(a) $\frac{4}{x}=\frac{x}{2}+3$ $\quad x^2+6x-8=0$ $x=\ldots,\left(\frac{-6\pm\sqrt{68}}{2}\right)$ $\quad -3\pm\sqrt{17}$ - root not needed	M1, A1
	$-\frac{4}{x}=\frac{x}{2}+3,$ $\quad x^2+6x+8=0$ $\quad x=-4$ and -2	M1, A1
	Three correct solutions (and no extras): $-4,\ -2,\ -3+\sqrt{17}$	A1 (5)
	(b)  Line through point on −ve x axis and + y axis	B1
	Curve	B1
	3 Intersections in correct quadrants	B1 (3)
	(c) $-4<x<-2,$ $\quad x>-3+\sqrt{17}$ $\quad$ o.e.	B1, B1(2)
		10
	(a) <u>Alternative using squaring method</u> Square both sides and attempt to find roots M1 $x^4+12x^3+36x^2-64=0$ gives x = -2 and x = -4 A1 Obtain quadratic factor, divide find solutions of quadratic and obtain $\left(-3\pm\sqrt{17}\right)$ M1 A1 Last mark as before (c) Use of $\leq$ instead of < lose last B1 Extra inequalities lose last B1	

Question number	Scheme	Marks
6.	(a) $\frac{2}{(r+1)(r+3)} = \frac{1}{r+1} - \frac{1}{r+3}$ M: $\frac{2}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$	M1 A1 (2)
	(b) $r = 1$: $\left(\frac{2}{2\times 4}\right) = \frac{1}{2} - \frac{1}{4}$	M1
	$r = 2$: $\left(\frac{2}{3\times 5}\right) = \frac{1}{3} - \frac{1}{5}$	
	… $r = n - 1$: $\left(\frac{2}{n(n+2)}\right) = \frac{1}{n} - \frac{1}{n+2}$	
	$r = n$: $\left(\frac{2}{(n+1)(n+3)}\right) = \frac{1}{n+1} - \frac{1}{n+3}$	A1 ft
	Summing: $\sum = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$	M1 A1
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{6(n+2)(n+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$	**d** M1A1cso 6
	(c) $\sum_{21}^{30} = \sum_{1}^{30} - \sum_{1}^{20} = \frac{30\times 163}{6\times 32\times 33} - \frac{20\times 113}{6\times 22\times 23}$, $= 0.02738$	M1A1ftA1cso3
		(11)
	(b) The first M1 requires list of first two and last two terms The A1 must be correct but ft on their A and B The second M1 requires terms to be eliminated and A1 is cao (c) The M mark is also allowed for $\sum_{1}^{30} - \sum_{1}^{21}$ applied with numbers included Using $u_{30} - u_{20}$ scores M0 A0 A0 The first A1 is ft their A and B or could include A and B, but final A1 is cao but accept 0.027379775599 to 5 or more decimal places..	

Question number	Scheme	Marks
7.	(a) $\frac{dy}{dx} = v + x\frac{dv}{dx}$	B1
	$\left(v + x\frac{dv}{dx}\right) = \frac{x}{vx} + \frac{3vx}{x} \Rightarrow x\frac{dv}{dx} = 2v + \frac{1}{v}$ (*)	M1 A1 (3)
	(b) $\int \frac{v}{1+2v^2}\,dv = \int \frac{1}{x}\,dx$	M1
	$\frac{1}{4}\ln(1+2v^2), \quad = \ln x \ (+C)$	dM1 A1, B1
	$Ax^4 = 1 + 2v^2$	d M1
	$Ax^4 = 1 + 2\left(\frac{y}{x}\right)^2$ so $y = \sqrt{\frac{Ax^6 - x^2}{2}}$ or $y = x\sqrt{\frac{Ax^4 - 1}{2}}$ or $y = x\sqrt{\left(\frac{1}{2}e^{4\ln x + 4c} - \frac{1}{2}\right)}$	M1 A1 (7)
	(c) $x = 1$ at $y = 3$: $3 = \sqrt{\frac{A-1}{2}}$ $A = \ldots$	M1
	$y = \sqrt{\frac{19x^6 - x^2}{2}}$ or $y = x\sqrt{\frac{19x^4 - 1}{2}}$	A1 (2) **12**
	(a) B1 for statement printed or for $\frac{dy}{dx} = (x + v\frac{dx}{dv})\frac{dv}{dx}$ First M1 is for RHS of equation only but for A1 need whole answer correct . (b) First M1 accept $\int \frac{1}{2v + \frac{1}{v}}\ dv = \int \frac{1}{x}\ dx$ Second M1 requires an integration of correct form ¼ may be missing A1 for LHS correct with ¼ and B1 is independent and is for ln$x$ Third M1 is **dependent** and needs correct application of log laws Fourth M1 is independent and merely requires return to y/x for v **N.B. There is an IF method possible after suitable rearrangement – see note**.	

Question number	Scheme	Marks
8.	(a) $r\cos\theta = 4(\cos\theta - \cos^2\theta)$ or $r\cos\theta = 4\cos\theta - 2\cos 2\theta - 2$	B1
	$\dfrac{d(r\cos\theta)}{d\theta} = 4(-\sin\theta + 2\cos\theta\sin\theta)$ or $\dfrac{d(r\cos\theta)}{d\theta} = 4(-\sin\theta + \sin 2\theta)$	M1 A1
	$4(-\sin\theta + 2\cos\theta\sin\theta) = 0 \Rightarrow \cos\theta = \dfrac{1}{2}$ which is satisfied by $\theta = \dfrac{\pi}{3}$ and $r = 2$(*)	d M1 A1 (5)
	(b) $\dfrac{1}{2}\int r^2\, d\theta = (8)\int\left(1 - 2\cos\theta + \cos^2\theta\right)d\theta$	M1
	$= (8)\left[\theta - 2\sin\theta + \dfrac{\sin 2\theta}{4} + \dfrac{\theta}{2}\right]$	M1 A1
	$8\left[\dfrac{3\theta}{2} - 2\sin\theta + \dfrac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi/2} = 8\left(\left(\dfrac{3\pi}{4} - 2\right) - \left(\dfrac{\pi}{2} - \sqrt{3} + \dfrac{\sqrt{3}}{8}\right)\right) = 2\pi - 16 + 7\sqrt{3}$	M1
	Triangle: $\dfrac{1}{2}(r\cos\theta)(r\sin\theta) = \dfrac{1}{2}\times 1\times\sqrt{3} = \dfrac{\sqrt{3}}{2}$	M1 A1
	Total area: $\left(2\pi - 16 + 7\sqrt{3}\right) + \dfrac{\sqrt{3}}{2} = \left(2\pi - 16\right) + \dfrac{15\sqrt{3}}{2}$	(A1) A1 (8)
		13
	(a) Alternative for first 3 marks: $\dfrac{dr}{d\theta} = 4\sin\theta$ B1 $\dfrac{dx}{d\theta} = -r\sin\theta + \cos\theta\dfrac{dr}{d\theta} = -4\sin\theta + 8\sin\theta\cos\theta$ M1 A1 Substituting r = 2 and $\theta = \frac{\pi}{3}$ into **original equation** scores 0 marks. (b) M1 needs attempt to expand $(1-\cos\theta)^2$ giving three terms (allow slips) Second M1 needs integration of $\cos^2\theta$ using $\cos 2\theta \pm 1$ Third M1 needs correct limits- may evaluate two areas and subtract M1 needs attempt at area of triangle and A1 for cao Next A1 is for value of area within curve, then final A1 is cao, must be exact but allow 4 terms and isw for incorrect collection of terms Special case for use of rsinθ gives B0M1A0M0A0	

June 2008
Further Pure Mathematics FP2
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\tanh x)) = \frac{\operatorname{sech}^2 x}{\tanh x}$	M1 A1
	$= \frac{1}{\sinh x \cosh x} = \frac{2}{\sinh 2x} = 2\operatorname{cosech} 2x$ (*)	M1 A1 (4)
		4
	Notes **1M1** Any valid differentiation attempt including $\ln(e^x - e^{-x}) - \ln(e^x + e^{-x})$ **1A1** c.a.o. (o.e e.g. $\frac{\cosh x}{\sinh x} - \frac{\sinh x}{\cosh x}$) **2M1** Proceeding to a hyperbolic expression in $2x$ **2A1** c.s.o.	

Question number	Scheme	Marks	
2.	$8\left(\dfrac{e^x+e^{-x}}{2}\right)-4\left(\dfrac{e^x-e^{-x}}{2}\right)=13$	B1	
	$4e^x+4e^{-x}-2e^x+2e^{-x}=13$		
	$2e^{2x}-13e^x+6=0$ (or equiv.)	M1 A1	
	$(2e^x-1)(e^x-6)=0$		
	$e^x=\dfrac{1}{2},\quad e^x=6$	M1 A1ft	
	$x=\ln\dfrac{1}{2}\ (\text{or } -\ln 2),\quad x=\ln 6$	A1	(6)
			6
	Notes		
	B1 Correctly substituting exponentials for all hyperbolics		
	1M1 To a three term quadratic in e^x		
	1A1 c.a.o. (o.e.)		
	2M1 Solving their equation to $e^x=$		
	2A1ft f.t. their equation.		
	3A1 c.a.o.		

Question number	Scheme	Marks
3.	$\int \frac{3}{\sqrt{x^2-9}}\,dx + \int \frac{x}{\sqrt{x^2-9}}\,dx$	B1
	$= \left[3\text{arcosh}\,\frac{x}{3} + \sqrt{x^2-9}\right]$	M1 A1 A1
	$= \left[3\ln\left(\frac{x+\sqrt{x^2-9}}{(3)}\right) + \sqrt{x^2-9}\right]_5^6$	
	$= \left(3\ln\left(\frac{6+\sqrt{27}}{3}\right) + \sqrt{27}\right) - \left(3\ln\left(\frac{5+4}{3}\right) + 4\right)$	M1 A1
	$= 3\ln\frac{6+\sqrt{27}}{9} + \sqrt{27} - 4 = 3\ln\frac{2+\sqrt{3}}{3} + 3\sqrt{3} - 4$ (*)	A1(7)
	Notes	**7**
	B1 Correctly changing to an integrable form. **1M1** Complete attempt to integrate at least one bit. **1A1** One term correct **2A1** All correct **2DM1** Substituting limits in all. **Must have got first M1** **3A1** Correctly (no follow through) **4A1** c.s.o.	

Question number	Scheme	Marks
4.	(a) $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}$, At $x = \sqrt{2}$ $\frac{dy}{dx} = \frac{6}{3} = 2$	M1 A1, A1
	$y - \text{arsinh}\,(2\sqrt{2}) = 2(x - \sqrt{2})$	M1
	$y = 2x - 2\sqrt{2} + \ln\left(3 + 2\sqrt{2}\right)$ (*)	A1 (5)
	(b) $\frac{3a^2}{\sqrt{1+a^6}} = 2$ $9a^4 = 4(1+a^6)$	M1 A1
	$4a^6 - 9a^4 + 4 = 0$ $(a^2-2)(4a^4 - a^2 - 2) = 0$	A1
	$a^2 = \frac{1 \pm \sqrt{1+32}}{8}$ $a = \sqrt{\frac{1+\sqrt{33}}{8}} \approx 0.92$	M1 A1 (5)
		10
	Notes **(a)1M1** Attempt to differentiate need $\left(1+x^6\right)^{-\frac{1}{2}}$ at least **1A1** correct **2A1** c.a.o. **2M1** Substituting into straight line equation (linear). Must use $x = \sqrt{2}$ **3A1** c.s.o. **(b)1M1** Their derivative = their gradient (condone x throughout) **2M1= A mark cao, any form** **1A1** quartic cao **3M1** Solving their quartic to 'a' = **2A1** c.a.o. (a.w.r.t. 0.92 to 2dp)	

Question number	Scheme	Marks
5.	(a) $I_n = \int_0^{\pi} e^x \sin^n x \, dx = \left[e^x \sin^n x\right] - \int e^x n \sin^{n-1} x \cos x \, dx$	M1 A1
	$\left[e^x \sin^n x - n e^x \sin^{n-1} x \cos x\right] + n \int e^x (-\sin^n x + (n-1)\cos x \sin^{n-2} x \cos x) \, dx$	M1 A1
	$\left[e^x \sin^n x - n e^x \sin^{n-1} x \cos x\right]_0^{\pi} = 0$	B1
	$I_n = -n \int e^x \sin^n x \, dx + n(n-1) \int \sin^{n-2} x (1 - \sin^2 x) \, dx$	M1
	$I_n = -nI_n + n(n-1)I_{n-2} - n(n-1)I_n \qquad I_n = \dfrac{n(n-1)}{n^2+1} I_{n-2}$ (*)	M1 A1 (8)
	(b) $I_4 = \dfrac{4 \times 3}{17} I_2, \qquad = \dfrac{12}{17} \times \dfrac{2}{5} I_0$	M1, A1
	$I_0 = \int_0^{\pi} e^x \, dx = \left[e^x\right]_0^{\pi} = ...$, $\qquad I_4 = \dfrac{24}{85}\left(e^{\pi} - 1\right)$	M1, A1 (4)
		12

(a)1M1 Complete attempt to use parts once in the right direction need $\sin^{n-1} x$

1A1 cao

2M1 Attempt to use parts again with sensible choice of parts, not reversing. Need to be differentiating a product.

2A1 cao

1B1 both = 0 at some point. (doesn't need to be correct, must must =0)

3DM1 $I_n =$ expressions in $\int e^x \sin^k x \, dx$ **Depends on 2nd M**

4DM1 Expresssion in I_n and I_{n-2} to $I_n =$. **Depends on 3rd M**

3A1 c.s.o.

(b)1M1 I_4 in terms of I_2

1A1 I_4 correctly in terms of I_0 [o.e.]

2M1 $\int e^x dx$

2A1 c.a.o for I_4 .

Question number	Scheme	Marks
6.	(a) $\int \cosh x \arctan(\sinh x)\,dx = \sinh x \arctan(\sinh x) - \int \sinh x \frac{\cosh x}{1+\sinh^2 x}\,dx$	M1 A1 A1
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)\ \ (+C)$	M1 A1 (5)
	Or: .... $-\int \tanh x\,dx$	
	$= \sinh x \arctan(\sinh x) - \ln(\cosh x)\ \ (+C)$ M1 A1	
	Alternative:	
	Let $t = \sinh x$, $\frac{dt}{dx} = \cosh x$, $\int \arctan t\,dt = t\arctan t - \int \frac{t}{1+t^2}\,dt$ M1 A1 A1	
	$= \ldots\ldots - \frac{1}{2}\ln(1+t^2)$ M1	
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)\ \ (+C)$ (or equiv.) A1	
	(b) $\frac{1}{10}\left[\sinh x \arctan(\sinh x) - \ln(\cosh x)\right]_0^2 = \ldots..,$ 0.34 (*)	M1, A1 (2)
		7
	(a) Alternative: Let $\tan t = \sinh x$, $\sec^2 t\frac{dt}{dx} = \cosh x$, $\int t\sec^2 t\,dt = t\tan t - \int \tan t\,dt$ M1 A1 A1 $= \ldots\ldots - \ln(\sec t)$ M1 $= \sinh x \arctan(\sinh x) - \ln\sqrt{1+\sinh^2 x}\ \ (+C)$ (or equiv.) A1 **Notes** **(a)1M1** Complete attempt to use parts **1A1** One term correct. **2A1** All correct. **2M1** All integration completed. Need a ln term. **3A1** c.a.o. (in x) o.e, any correct form, simplified or not **(b)1M1** Use of limits 0 and 2 and 1/10. **1A1** c.s.o.	

Question number	Scheme	Marks	
7.	(a) $\frac{2x}{16}-\frac{2y}{9}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\left[\frac{\mathrm{d}x}{\mathrm{d}t}=4\sec t\tan t,\ \frac{\mathrm{d}y}{\mathrm{d}t}=3\sec^2 t\right]$	M1 A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{9x}{16y}=\frac{36\sec t}{48\tan t}=\frac{3}{4\sin t}$	M1 A1	
	$y-3\tan t=\frac{-4\sin t}{3}(x-4\sec t)$	M1	
	$4x\sin t+3y=25\tan t$ (*)	A1	(6)
	(b) Using $b^2=a^2(e^2-1)$: $ae=\sqrt{a^2+b^2}=5$ or $e=\frac{5}{4}$	M1 A1	
	P: $4\sec t=5$ $\cos t=\frac{4}{5}$	M1	
	Coordinates of P: $(4\sec t,3\tan t)=\left(5,\frac{9}{4}\right)$	M1 A1	(5)
	(c) R: $x=\frac{25\tan t}{4\sin t}=\frac{125}{16}$	M1	
	Area of PRS: $\frac{1}{2}(SR\times SP)=\frac{1}{2}\times\left(\frac{125}{16}-5\right)\times\frac{9}{4}=\frac{405}{128}\ \left(=3\frac{21}{128}\right)$	M1 A1	(3)
		14	

Notes

(a)1M1 Differentitating
1A1 c.a.o.
2M1 $\frac{dy}{dx}$ in terms of t.
2A1 c.a.o.
3M1 Substituting gradient of **normal** into straight line equation.
3A1 c.s.o.

(b)1M1 Use of $b^2=a^2(e^2-1)$
1A1 c.a.o. for ae or for e
2M1 Using x coordinate of focus= x coordinate of P, to get single term $f(t)$= constant. (**Allow recovery in (c)**)
3M1 Substituting into P coordinates to a number for x and for y.
2A1 c.a.o.

(c)1M1 Attempt to find x coordinate of R.
2M1 Substituting into correct template i.e. ½ x |their R_x - their H_x| x their P_y
1A1 c.a.o. 3 s.f. or better.

Question number	Scheme	Marks
8.	(a) $\dot{x} = 3 + 3\cos t \quad \dot{y} = 3\sin t$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1+\cos t} = \frac{2\sin {t/2} \cos {t/2}}{2\cos^2 {t/2}} = \tan\frac{t}{2}$ (*)	M1 A (3)
	(b) $s = \int \sqrt{\dot{x}^2 + \dot{y}^2}\, \mathrm{d}t = 3\sqrt{2}\int \sqrt{1+\cos t}\, \mathrm{d}t$	M1 A1ft
	$= 6\int_0^t \cos\frac{t}{2}\mathrm{d}t = 12\sin\frac{t}{2}$ (Limits or establish $C = 0$ for A1) (*)	M1 A (4)
	(c) $\tan\psi = \tan\frac{t}{2} \Rightarrow \psi = \frac{t}{2} \Rightarrow s = 12\sin\psi$	B1 (1)
	(d) Surface area = $\int_0^t 2\pi y\sqrt{\dot{x}^2 + \dot{y}^2}\, \mathrm{d}t = 18\sqrt{2}\pi\int (1-\cos t)\sqrt{1+\cos t}\, \mathrm{d}t$	M1 A1ft
	$= 72\pi\int \sin^2\frac{t}{2}\cos\frac{t}{2}\, \mathrm{d}t$	M1
	$= \ldots\ldots\ldots\left(\frac{2}{3}\sin^3\frac{t}{2}\right)$	M1 A1
	But $\sin\frac{t}{2} = \frac{s}{12} = \frac{L}{12}$, so surface area $= \frac{144\pi}{3} \times \frac{L^3}{12^3} = \frac{\pi L^3}{36}$ (*)	M1 A (7)

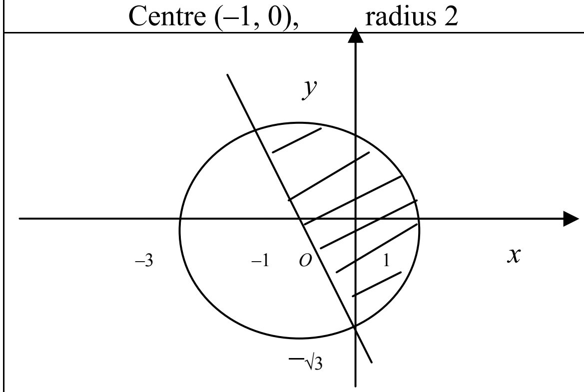
(a)1B1 both
1M1 Attempt at y'/x'
1A1 cso – on paper need to see half angles
(b)1M1 Attempt at arc length, integral formula
1A1 cao follow through on their x' and y' **one variable only**
2M1 Integrating
2A1 cso – on paper
(c) 1B1 cao
(d) 1M1 Attempt at Surface area, integral formula.Condone lack of 2π.
1A1 cao follow through on their x' and y' condone lack of 2π. **one variable only**
2DM1 Getting to integrable form condone lack of 2π. **Depends on previous M mark.**
3DM1 integrating condone lack of 2π. **Depends on previous M mark.**
2A1 cao
4DM1 Eliminating t to give expression in L only **Depends on previous M mark.**
3A1 cso – on paper.

6676 Further Pure Mathematics FP3
Mark Scheme

Question Number	Scheme	Marks
1.	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 0 + \cos 0.6 \;\; (= 0.825335\ldots)$ May be implicit	B1
	$y_1 \approx 0.05\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 + y_0$ $(= 0.05 \times 0.825335\ldots + 0.6)$	M1
	$y_1 \approx 0.641266\ldots$ $= 0.6413$ (4 d.p.) Allow awrt	A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 0.05 + \cos 0.641266...$ [or $0.05 + \cos(0.6 + 0.05\cos 0.6)$] $= 0.851338\ldots$	A1ft
	$y_2 \approx 0.05\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 + y_1$ $(= 0.05 \times 0.851338\ldots + 0.641266\ldots)$ Requires use of the differential equation to find $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1$	M1
	$y_2 \approx 0.683833\ldots$ $= 0.6838$ (4 d.p.)	A1 **(6)**
	<u>Degree mode in calculator</u>: Gives answers: 0.6500 (0.64999…) 0.7025 (0.70248…) This can score B1 M1 A0 A1ft M1 A0	

Question Number	Scheme	Marks
2. (a)	$\begin{pmatrix}1 & p & 2\\0 & 3 & q\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\lambda\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\begin{pmatrix}1+2p+2\\6+q\\2+2p+1\end{pmatrix}=\begin{pmatrix}\lambda\\2\lambda\\\lambda\end{pmatrix}$ is M1 A1 (2 eqns implied) $\begin{pmatrix}3+2p\\6+q\\3+2p\end{pmatrix}\Rightarrow 6+q=2(3+2p)$ is M1 A1 (2 eqns, use of parameter implied) $1+2p+2=\lambda$ $6+q=2\lambda$ M: Two equations, one in p, one in q $\therefore\ 6+q=6+4p\ \Rightarrow\ q=4p$ (*)	M1 A1 A1 (3)
(b)	$\begin{vmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{vmatrix}=0$ or $\begin{vmatrix}1-\lambda & p & 2\\0 & 3-\lambda & 4p\\2 & p & 1-\lambda\end{vmatrix}=0$ (or with q instead of $4p$) $[-4(8-4p^2)-p(0-8p)+2(0+4)=0]$ $p^2=1$ or $pq=4$ $p<0$ $p=-1$ $q=-4$ M: Use $q=4p$ to find value of p and of q A1: Positive values must be rejected	M1 A1 dM1 A1 (4)
(c)	$-4x-y+2z=0$, $-2y-4z=0$, $2x-y-4z=0$ Any 2 eqns, with value of p $2x=-y=2z$ (or 2 separate equations) E.vector is $k\begin{pmatrix}1\\-2\\1\end{pmatrix}$ (Any non-zero value of k)	M1 M1 A1 (3) **(10)**
	(a) Assuming a value for λ, e.g. $\lambda=1$, gives M1 A0 A0. (a) Assuming result and working 'backwards': $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}3+2p\\6+4p\\3+2p\end{pmatrix}=(3+2p)\begin{pmatrix}1\\2\\1\end{pmatrix}$, gives M1 A0 A0 (b) <u>Alternative</u>: $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=5\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $\begin{pmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ (or q instead of $4p$) $x+py+2z=5x$, $3y+4pz=5y$, $2x+py+z=5z$ $py+2z=4x$ (i), $2pz=y$ (ii), $2x+py=4z$ (iii) From (i) and (iii) $py=2z$ From (ii) $p^2=1$ (or equiv. in terms of p and/or q) $p<0$, $p=-1$, $q=-4$ A1: Positive values must be rejected (b) Using the eigenvector $\begin{pmatrix}1\\2\\1\end{pmatrix}$ scores <u>no marks</u> in this part.	 M1 A1 dM1 A1

Question Number	Scheme	Marks
3. (a)	$(x^2+1)\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}=4y\frac{dy}{dx}+(1-2x)\frac{d^2y}{dx^2}-2\frac{dy}{dx}$	M1 A1
	$(x^2+1)\frac{d^3y}{dx^3}=(1-4x)\frac{d^2y}{dx^2}+(4y-2)\frac{dy}{dx}$ (*)	A1 (3)
(b)	$\left(\frac{d^2y}{dx^2}\right)_0=3$	B1
	$\left(\frac{d^3y}{dx^3}\right)_0=5$ Follow through: $\frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}+2$	B1ft
	$y=1+x+\frac{3}{2}x^2+\frac{5}{6}x^3....$	M1 A1 (4)
(c)	$x=-0.5$, $y\approx 1-0.5+0.375-0.104166\ldots$	
	$=0.77$ (2 d.p.) [awrt 0.77]	B1 (1) **(8)**
	(a) M: Use of product rule (at least once) and implicit differentiation (at least once). (b) M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)	

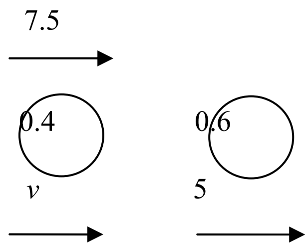
4.	(a)	$\lvert (x-3)+\mathrm{i}y \rvert = 2\lvert x+\mathrm{i}y \rvert \Rightarrow (x-3)^2 + y^2 = 4x^2 + 4y^2$ $\therefore\ x^2 + y^2 + 2x - 3 = 0$ $(x+1)^2 + y^2 = 4$ Centre $(-1, 0)$, radius 2	M1 A1 M1 A1, A1 (5)
	(b)	 Circle, centre on x-axis B1 $C\ (-1, 0)$, $r = 2$ dB1ft Follow through centre and radius, but dependent on first B1. There must be indication of their '–3', '–1' or '1' on the x-axis and no contradictory evidence for their radius. Straight line B1 Straight line through (–1, 0), or perp. bisector of (–3, 0) and $(0, \sqrt{3})$. B1 Straight line through point of int. of circle & –ve y-axis, or through $(0, -\sqrt{3})$ B1	B1 dB1 B1 B1 B1 (5)
	(c)	Shading (only) inside circle Inside correct circle and all of the correct side of the correct line… this mark is dependent on <u>all</u> the previous B marks in parts (b) and (c).	B1 dB1 (2) **(12)**
		(a) 1st M: Use $z = x + \mathrm{i}y$, and attempt square of modulus of each side. Not squaring the 2 on the RHS would be M1 A0. 2nd M: Attempting to express in the form $(x-a)^2 + (y-b)^2 = k$, or attempting centre and radius from the form $x^2 + y^2 + 2gx + 2fy + c = 0$	

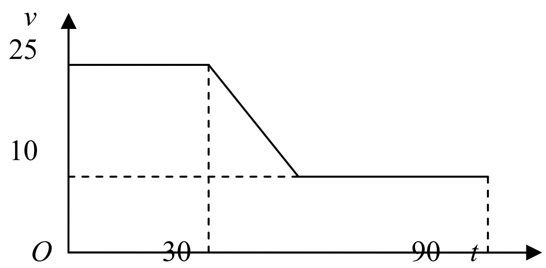
Question Number	Scheme	Marks
5. (a)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$	M1
	$t(1 + k) = 2t(k - 4)$	dM1
	$k = 9$	A1 (3)
(b)	det $\mathbf{A} = k^2 + 2(1 - k)$ (Must be seen in part (b))	M1
	$= (k - 1)^2 + 1$, which is always positive	M1
	A is non-singular	A1cso (3)
(c)	$\mathbf{A}^{-1} = \dfrac{1}{k^2 - 2k + 2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$	M1 A1 (2)
(d)	$k = 3,\quad \mathbf{A}^{-1} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$	B1
	$\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$	M1 A1 (3)
	Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4,\ -2x + 3y = -3$ B1	
	M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2$, $y = -0.2$ (o.e.)	**(11)**
	(b) 2nd M: Alternative is to use quadratic formula on the quadratic equation, or to use the discriminant, with a <u>comment</u> about 'no real roots', or 'can't equal zero', or a comment about the condition for singularity. $\left(x = \dfrac{2 \pm \sqrt{4-8}}{2}\right)$ A1 Conclusion. (c) M: Need $\dfrac{1}{\text{their det A}}$, k's unchanged and attempt to change sign for either -2 (leaving as top right) or $1-k$ (leaving as bottom left). (d) M: Requires an attempt to multiply the matrices.	

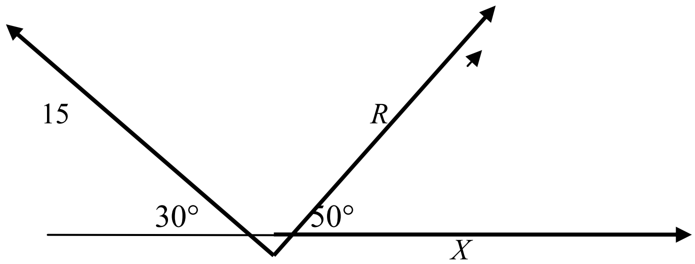
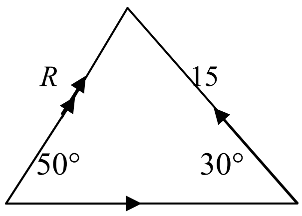
Question Number	Scheme	Marks
6. (a)	$(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta$ $\quad\therefore$ true for $n = 1$	B1
	Assume true for $n = k$, $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$	
	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1
	(Can be achieved either from the line above or the line below) $= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1
	Requires full justification of $(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	
	($\therefore$ true for $n = k + 1$ if true for $n = k$) $\quad\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1cso (5)
(b)	$\cos 5\theta = \mathrm{Re}\left[(\cos\theta + \mathrm{i}\sin\theta)^5\right]$	
	$= \cos^5\theta + 10\cos^3\theta\,\mathrm{i}^2\sin^2\theta + 5\cos\theta\,\mathrm{i}^4\sin^4\theta$	M1 A1
	$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
	$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	M1
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ $\quad$ (*)	A1cso (5)
(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \quad\Rightarrow\quad \cos 5\theta = 0$	M1
	$5\theta = \dfrac{\pi}{2} \qquad \theta = \dfrac{\pi}{10}$	A1
	$x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root $\quad$ (*)	A1 (3) **(13)**
	(a) <u>Alternative</u>: For the 2nd M mark: $\left(\mathrm{e}^{\mathrm{i}k\theta}\right)\left(\mathrm{e}^{\mathrm{i}\theta}\right) = \mathrm{e}^{\mathrm{i}\theta(k+1)}$ (b) <u>Alternative</u>: $\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 + 10z^2\left(\frac{1}{z}\right)^3 + 5z\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$ M1 $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ A1 $(2\cos\theta)^5 = \ldots\ldots$ and attempt to put $\cos 3\theta$ in powers of $\cos\theta$ M1 Correct method (or formula) for putting $\cos 3\theta$ in powers of $\cos\theta$ M1 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ A1cso (c) <u>Alternatives</u>: (i) Substitute given root into $x^4 - 5x^2 + 5$: $\left(2\cos\frac{\pi}{10}\right)^4 - 5\left(2\cos\frac{\pi}{10}\right)^2 + 5 = 2^4\left(\cos\frac{\pi}{10}\right)^4 - 5\times 2^2\left(\cos\frac{\pi}{10}\right)^2 + 5$ M1 'Multiply by $\cos\theta$' and use result from part (b): $\ldots = \cos\frac{5\pi}{10}$ A1 $= 0$ and conclusion A1 (ii) Use $5\theta = \frac{\pi}{2}$ in result from part (b) M1 $16\left(\cos\frac{\pi}{10}\right)^5 - 20\left(\cos\frac{\pi}{10}\right)^3 + 5\left(\cos\frac{\pi}{10}\right)$ and divide by $\cos\theta$ A1 $= 0$ and conclusion A1	

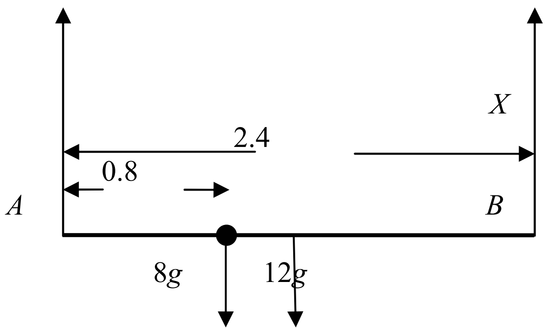
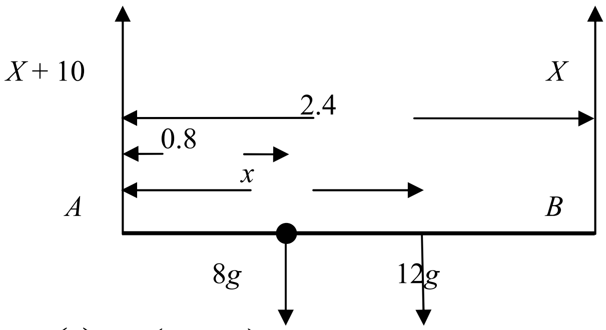
Question Number	Scheme	Marks
7. (a)	$\overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{PR} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	B1 M1 A1 (3)
(b)	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} - \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ [may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$] $\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$ o.e. ft from (a)	M1 A1ft (2)
(c)	$3x + y - z = 4$ (i), $x - 2y - 5z = 6$ (ii) (i) × 2 + (ii) $7x - 7z = 14$, $x = z + 2$ (M: Eliminate one variable) In (ii) $z + 2 - 2y - 5z = 6$, $y + 2 = -2z$ (M: Substitute back) $\therefore x = z + 2$ and $y + 2 = -2z$ o.e. ($y = 2 - 2x$) (Two correct '3-term' equations) $\dfrac{x-2}{(1)} = \dfrac{y+2}{-2} = \dfrac{z}{(1)}$ o.e. (M: Form cartesian equations)	 M1 M1 A1 M1 A1 (5)
(d)	Writing down direction vector of $\overrightarrow{PS}$ from part (c). $\overrightarrow{QR} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} = \overrightarrow{PS}$ $\therefore PS$ // QR (or cross-product = 0)	M1 A1 (2)
(e)	$\overrightarrow{PT} = 4\mathbf{i} + 2\mathbf{j}$ (or $\overrightarrow{QT} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{RT} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$) Volume = $\dfrac{1}{3}\left\|\overrightarrow{PQ} \times \overrightarrow{PR} . \overrightarrow{PT}\right\| = \dfrac{1}{3}\|(3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j})\|$ ft from (a) (Instead of $\overrightarrow{PQ} \times \overrightarrow{PR}$, it could be $\overrightarrow{PQ} \times \overrightarrow{QR}$ or $\overrightarrow{PR} \times \overrightarrow{QR}$) $= \dfrac{1}{3}(12 + 2)$ $= 4\frac{2}{3}$ o.e.	 M1 A1ft A1 (3) **(15)**
	(a) If both vectors are 'reversed', B0 M1 A1 is possible (c) <u>Alternative</u>: Direction of line: $\begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 7\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ M2 A1 Through P $(1, 0, -1)$: $\dfrac{x-1}{1} = \dfrac{y}{-2} = \dfrac{z+1}{1}$ M1 A1 (e) <u>Alternative</u>: $\dfrac{1}{3}\begin{vmatrix} 4 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix}$ gives M1 A1 directly. Here ft from 1st line of part (a). <u>Special case</u>: $\dfrac{1}{6}$ or $\dfrac{1}{2}$ instead of $\dfrac{1}{3}$, but method otherwise correct: M1 A0 A0	

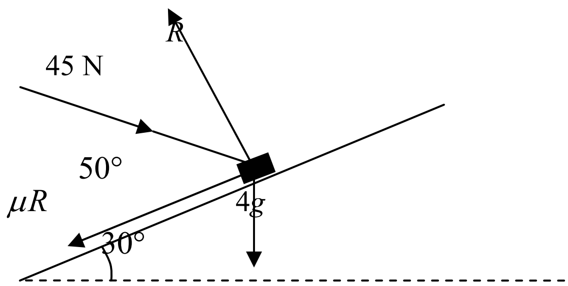
June 2008
6677 Mechanics M1
Mark Scheme

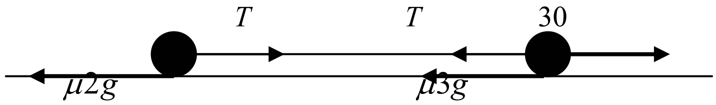
Question Number	Scheme	Marks
1.	(a) $I = mv \quad \Rightarrow \quad 3 = 0.4 \times v$	M1 A1
	$v = 7.5 \ \left(\mathrm{ms}^{-1}\right)$	A1 **(3)**
	(b) 	
	LM $\quad 0.4 \times 7.5 = 0.4v + 0.6 \times 5$	M1 A1
	$0 = 0.4v \quad \Rightarrow \quad v = 0$ * cso	A1 **(3)**
		[6]
2.	(a) $v^2 = u^2 + 2as \quad \Rightarrow \quad 17.5^2 = u^2 + 2 \times 9.8 \times 10$	M1 A1
	Leading to $u = 10.5$	A1 **(3)**
	(b) $v = u + at \quad \Rightarrow \quad 17.5 = -10.5 + 9.8T$	M1 A1 f.t.
	$T = 2\frac{6}{7}$ (s)	DM1 A1 **(4)**
	Alternatives for (b)	**[7]**
	$s = (\frac{u+v}{2})T \Rightarrow 10 = (\frac{17.5 + -10.5}{2})T$	M1A1 f.t.
	$\frac{20}{7} = T$	DM1A1 (4)
	OR $\quad s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad -10 = 10.5t - 4.9t^2$	M1 A1 f.t.
	Leading to $T = 2\frac{6}{7}, \left(-\frac{5}{7}\right)$ Rejecting negative	DM1 A1 **(4)**
	(b) can be done independently of (a)	
	$s = vt - \frac{1}{2}at^2 \quad \Rightarrow \quad -10 = -17.5t + 4.9t^2$	M1 A1
	Leading to $T = 2\frac{6}{7}, \frac{5}{7}$	DM1
	For final A1, second solution has to be rejected. $\frac{5}{7}$ leads to a negative u.	A1 **(4)**

Question Number	Scheme	Marks
3.	(a) $\tan\theta = \frac{8}{6}$	M1
	$\theta \approx 53°$	A1 **(2)**
	(b) $\mathbf{F} = 0.4(6\mathbf{i} + 8\mathbf{j})\ (= 2.4\mathbf{i} + 3.2\mathbf{j})$	M1
	$\lvert\mathbf{F}\rvert = \sqrt{(2.4^2 + 3.2^2)} = 4$	M1 A1 **(3)**
	The method marks can be gained in either order.	
	(c) $\mathbf{v} = 9\mathbf{i} - 10\mathbf{j} + 5(6\mathbf{i} + 8\mathbf{j})$	M1 A1
	$= 39\mathbf{i} + 30\mathbf{j}\ (\mathrm{ms}^{-1})$	A1 **(3)**
		[8]
4.	(a) 	
	shape	B1
	25, 10, 30, 90	B1 **(2)**
	(b) $30 \times 25 + \underline{\frac{1}{2}(25 + 10)}t + 10(60 - t) = 1410$	M1 <u>A1</u> A1
	$7.5t = 60$	
	$t = 8\ (\mathrm{s})$	DM1 A1
	$a = \frac{25 - 10}{8} = 1.875\ (\mathrm{ms}^{-2})$ $\quad 1\frac{7}{8}$	M1 A1 **(7)**
		[9]

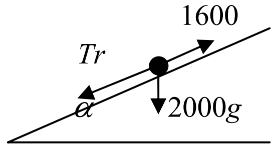
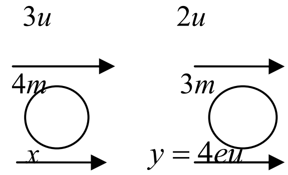
Question Number	Scheme	Marks
5.	(a) 	
	$(\uparrow)\quad 15\sin 30° = R\sin 50°$	M1 A1
	$R \approx 9.79\ (\mathrm{N})$	DM1 A1 **(4)**
	(b) $(\rightarrow)\ X - 15\cos 30° = R\cos 50°$ ft their R	M1 A2 ft
	$X \approx 19.3\ (\mathrm{N})$	DM1 A1 **(5)** **[9]**
	Alternatives using sine rule in (a) or (b); cosine rule in (b)	
	 X	
	(a) $\dfrac{R}{\sin 30°} = \dfrac{15}{\sin 50°}$	M1 A1
	$R \approx 9.79\ (\mathrm{N})$	DM1 A1 (4)
	(b) $\dfrac{X}{\sin 100°} = \dfrac{15}{\sin 50°} = \dfrac{R}{\sin 30°}$	M1 A2 ft on R
	$X \approx 19.3\ (\mathrm{N})$	DM1 A1 (5)
	$X^2 = R^2 + 15^2 - 2 \text{ x } 15 \text{ x } R\cos 100^o$ **OR**: cosine rule; any of $R^2 = X^2 + 15^2 - 2 \text{ x } 15 \text{ x } X\cos 30^o$ $15^2 = R^2 + X^2 - 2 \text{ x } X \text{ x } R\cos 50^o$	M1 A2 ft on R
	$X \approx 19.3\ (\mathrm{N})$	DM1 A1 (5)

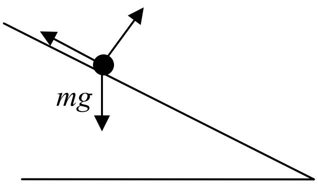
Question Number	Scheme	Marks
6.	(a) 	
	$M(A)$ $\quad 8g \times 0.8 + 12g \times 1.2 = X \times 2.4$	M1 A1
	$X \approx 85\ (\text{N})$ $\quad$ accept 84.9, $\frac{26g}{3}$	DM1 A1 **(4)**
	(b) 	
	$R(\uparrow)$ $\quad \underline{(X+10)} + \underline{X} = 8g + 12g$	M1 <u>B1</u> A1
	$(X = 93)$	
	$M(A)$ $\quad 8g \times 0.8 + 12g \times x = X \times 2.4$	M1 A1
	$x = 1.4\ (\text{m})$ $\quad$ accept 1.36	A1 **(6)**
		[10]

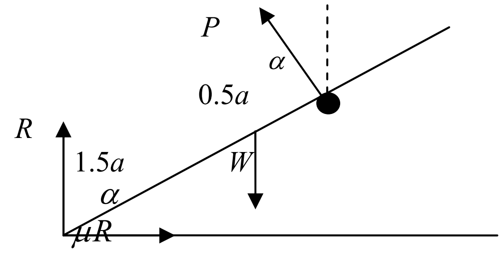
Question Number	Scheme	Marks
7.	(a) 	
	$R = 45\cos 40° + 4g\cos 30°$	M1 A2 (1, 0)
	$R \approx 68$ accept 68.4	DM1 A1 (5)
	(b) Use of $F = \mu R$	M1
	$F + 4g\sin 30 = 45\cos 50°$	M1 A2 (1, 0)
	Leading to $\mu \approx 0.14$ accept 0.136	DM1 A1 (6) **[11]**

Question Number	Scheme	Marks
8.	(a) 	
	$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad 6 = \frac{1}{2}a \times 9$	M1
	$a = 1\frac{1}{3}\ \left(\text{ms}^{-2}\right)$	A1 **(2)**
	(b) N2L for system $\quad 30 - \mu 5g = 5a \quad$ ft their a, accept symbol	M1 A1ft
	$\mu = \dfrac{14}{3g} = \dfrac{10}{21} \quad$ or $\quad$ awrt 0.48	DM1 A1 **(4)**
	(c) N2L for P $\quad T - \mu 2g = 2a \quad$ ft their μ, their a, accept symbols	M1 A1 ft
	$T - \dfrac{14}{3g} \times 2g = 2 \times \dfrac{4}{3}$	
	Leading to $T = 12\ (\text{N})$ $\quad$ awrt 12	DM1 A1 **(4)**
	Alternatively $\quad$ N2L for Q $30 - T - \mu 3g = 3a$	M1 A1
	Leading to $T = 12\ (\text{N})$ $\quad$ awrt 12	DM1 A1
	(d) The acceleration of P and Q (or the whole of the system) is the same.	B1 **(1)**
	(e) $v = u + at \quad \Rightarrow \quad v = \dfrac{4}{3} \times 3 = 4$	B1 ft on a
	N2L (for system or either particle) $-5\mu g = 5a \quad$ or equivalent	M1
	$a = -\mu g$	
	$v = u + at \quad \Rightarrow \quad 0 = 4 - \mu g t$	DM1
	Leading to $t = \dfrac{6}{7}\ (\text{s})$ $\quad$ accept 0.86, 0.857	A1 **(4)**
		[15]

June 2008
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
1.		
	Resolve ↗ : $T_r + \underline{2000g \times \sin\alpha} = 1600$	M1 <u>A1</u> A1
	$(T_r = 816)$	
	$P = 816 \times 14\,(\text{W})$ ft their T_r	M1 A1ft
	$\approx 11\,(\text{kW})$ accept 11.4	A1 cso **(6)**
		[6]
2.	(a) 	
	LM $12mu + 6mu = 4mx + 12meu$	B1
	NEL $4eu - x = eu$	M1 A1
	Eliminating x to obtain equation in e	DM1
	Leading to $e = \frac{3}{4}$ ∗ cso	A1 **(5)**
	(b) $x = 3eu$ or $\frac{9}{4}u$ or 4.5u – 3eu seen or implied in (b)	B1
	Loss in KE $= \frac{1}{2}4m(3u)^2 + \frac{1}{2}3m(2u)^2 - \frac{1}{2}4m\left(\frac{9}{4}u\right)^2 - \frac{1}{2}3m(3u)^2$	M1 A1ft
	ft their x	
	$= 24mu^2 - 23\frac{5}{8}mu^2 = \frac{3}{8}mu^2 = 0.375mu^2$	A1 **(4)**
		[9]

Question Number	Scheme	Marks
3.	(a $\Delta \text{KE} = \frac{1}{2}\times 3.5\left(12^2 - 8^2\right)$ $(=140)$ or KE at A, B correct separately	B1
	$\Delta \text{PE} = 3.5\times 9.8\times 14\sin 20°$ (≈ 164.238) or PE at A, B correct separately	M1 A1
	$\Delta \text{E} = \Delta \text{KE} + \Delta \text{PE} \approx 304$, 300	DM1 A1 **(5)**
	(b) Using Work-Energy	
	$F_r = \mu \times 3.5g\cos 20°$	M1 A1
	$304.238... = F_r \times 14$ ft their (a), F_r	M1 A1 ft
	$304.238... = \mu 3.5g\cos 20° \times 14$	
	$\mu \approx 0.674$, 0.67	A1 **(5)** **[10]**
	Alternative using N2L	
		
	$F_r = \mu \times 3.5g\cos 20°$	M1 A1
	$v^2 = u^2 + 2as \Rightarrow 8^2 = 12^2 - 2a\times 14$	
	$\left(a = \frac{20}{7}\right)$ (2.857 ….)	
	N2L R ↖ : {their F_r}- $mg\sin 20° = ma$ ft their F_r.	M1 A1ft
	Leading to $\mu \approx 0.674$ or 0.67	A1 **(5)**
4.	(a) N2L $(6t-5)\mathbf{i} + (t^2 - 2t)\mathbf{j} = 0.5\mathbf{a}$	M1
	$\mathbf{a} = (12t - 10)\mathbf{i} + (2t^2 - 4t)\mathbf{j}$	A1
	$\mathbf{v} = (6t^2 - 10t)\mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2\right)\mathbf{j}$ $(+\mathbf{C})$ ft their $\mathbf{a}$	M1 A1ft+A1ft
	$\mathbf{v} = (6t^2 - 10t + 1)\mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2 - 4\right)\mathbf{j}$	A1 **(6)**
	(b) When $t = 3$, $\mathbf{v}_3 = 25\mathbf{i} - 4\mathbf{j}$	M1
	$-5\mathbf{i} + 12\mathbf{j} = 0.5\left(\mathbf{v} - (25\mathbf{i} - 4\mathbf{j})\right)$ ft their $\mathbf{v}_3$	M1 A1ft
	$\mathbf{v} = 15\mathbf{i} + 20\mathbf{j}$	A1
	$\lvert\mathbf{v}\rvert = \sqrt{(15^2 + 20^2)} = 25$ (ms^{-1}) cso	M1 A1 **(6)** **[12]**

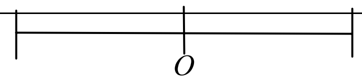
Question Number	Scheme		Marks
5.	(a) 		
	$R(\uparrow)\quad R + P\cos\alpha = W$		M1 A1
	$M(A)\quad P\times 2a = W\times 1.5a\cos\alpha$		M1 A1
	$\left(P = \frac{3}{4}W\cos\alpha\right)$		
	$R = W - P\cos\alpha = W - \frac{3}{4}W\cos^2\alpha$		DM1
	$= \frac{1}{4}\left(4 - 3\cos^2\alpha\right)W$ *	cso	A1 **(6)**
	(b) Using $\cos\alpha = \frac{2}{3}$, $\quad R = \frac{2}{3}W$		B1
	$R(\rightarrow)\quad \mu R = P\sin\alpha$		M1 A1
	Leading to $\mu = \frac{3}{4}\sin\alpha$		
	$\left(\sin\alpha = \sqrt{\left(1 - \frac{4}{9}\right)} = \frac{\sqrt{5}}{3}\right)$		
	$\mu = \frac{\sqrt{5}}{4}$	awrt 0.56	DM1 A1 **(5)**
			[11]

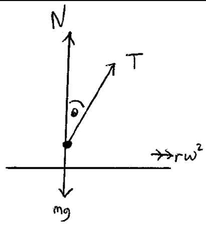
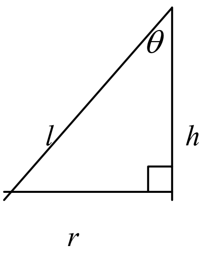
Question Number	Scheme	Marks
6.	(a) $M(Oy)$ $(8+k)m\times 6.4 = 5m\times 8 + km\times 8$	M1 A1
	$1.6k = 11.2 \quad \Rightarrow \quad k = 7$ ∗ cso	DM1 A1 **(4)**
	(b) $M(Oy)$ $27m\bar{x} = 12m\times 4 + 5m\times 8 + 7m\times 8$	M1 A1
	$\bar{x} = \dfrac{16}{3}$ 5.3 or better	A1
	$M(Ox)$ $27m\bar{y} = 12m\times 2.5 + 8m\times 5$	M1 A1
	$\bar{y} = \dfrac{70}{27}$ 2.6 or better	A1 **(6)**
	(c) $\tan\theta = \dfrac{\bar{y}}{\bar{x}} = \dfrac{35}{72}$	M1 A1ft
	$\theta \approx 26°$ awrt 25.9 °	A1 **(3)**
		[13]

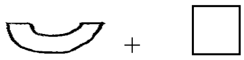
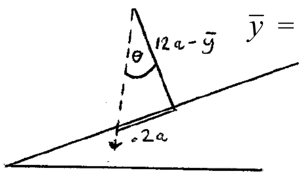
Question Number	Scheme	Marks
7.	(a) $(\downarrow)$ $u_y = 25\sin 30° \; (=12.5)$	B1
	$12 = 12.5t + 4.9t^2$ $\quad -1$ each error	M1 A2 (1, 0)
	Leading to $t = 0.743$, 0.74	A1 **(5)**
	(b) $(\rightarrow)$ $u_x = 25\cos 30° \left(= \dfrac{25\sqrt{3}}{2} \approx 21.65\right)$	B1
	$OB = 25\cos 30° \times t \; (\approx 16.094\,58)$ $\quad$ ft their (a)	M1 A1ft
	$TB \approx 1.1$ (m) $\quad$ awrt 1.09	A1 **(4)**
	(c) $(\rightarrow)$ $15 = u_x \times t \Rightarrow t = \dfrac{15}{u_x} (= \dfrac{2\sqrt{3}}{5} \approx 0.693$ or $0.69)$	M1 A1
	either $(\downarrow)$ $v_y = 12.5 + 9.8t \; (\approx 19.2896)$	M1
	$V^2 = u_x{}^2 + v_y{}^2 \; (\approx 840.840)$	
	$V \approx 29 \; (\text{ms}^{-1})$, 29.0	M1 A1 **(5)**
		[14]
	or $(\downarrow)$ $s_y = 12.5t + 4.9t^2 \; (\approx 11.0)$	M1
	$\dfrac{1}{2}m \times 25^2 + mg \times s_y = \dfrac{1}{2}mv^2$	
	$V \approx 29 \; (\text{ms}^{-1})$, 29.0	M1A1

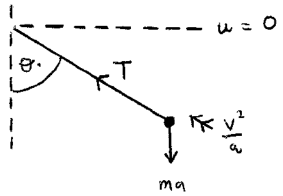
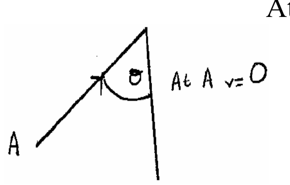
June 2008
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
Q1(a)	 EPE stored $= \frac{1}{2}\frac{\lambda}{L}\left(\frac{1}{2}L\right)^2 \quad \left(=\frac{\lambda L}{8}\right)$	B1
	KE gained $= \frac{1}{2}m\,2gL \quad (= mgL)$	B1
	EPE = KE $\Rightarrow \frac{\lambda L}{8} = \text{mg}\,L$ i.e. $\lambda = 8mg$*	M1A1cso
		(4)
		M1
(b)	 EPE = GPE + KE	
	$\frac{1}{2}\frac{8mg}{L}\left(\frac{1}{2}L\right)^2 = \frac{8mgL}{8} = mg\frac{L}{2} + \frac{1}{2}mu^2$	A1A1
	$\frac{mgL}{2} = \frac{m}{2}u^2 \quad \therefore u = \sqrt{gL}$	M1A1 **(5)**
		9 Marks

Question Number	Scheme	Marks
Q2 (a)		
	$T = 3 = \dfrac{2\pi}{\omega} \quad \therefore \; \omega = \dfrac{2\pi}{3}$	M1A1
	$u^2 = \omega^2 \left(a^2 - x^2\right)$; $a = 0.12$, $u^2 = a^2 \omega^2$, $u = 0.12 \times \omega$	M1
	$= 0.251 \text{ ms}^{-1}$ (0.25 m s^{-1})	A1 **(4)**
(b)	Time from $O \to A \to O = 1.5\text{s}$ $\quad \therefore t = 0.5$	B1
	$x = a \sin \omega t \quad \Rightarrow OP = 0.12 \sin\left(\dfrac{\pi}{3}\right)$	M1A1
	Distance from B is $0.12 - OP = 0.12 - 0.104\ldots = 0.016\text{m}$	M1A1 **(5)**
(c)	$v^2 = \omega^2 \left(a^2 - x^2\right)$	M1
	$v = \dfrac{2\pi}{3} \sqrt{0.12^2 - 0.104\ldots.^2} = \dfrac{2\pi}{3} \times 0.0598 = 0.13 \text{ ms}^{-1}$	A1 **(2)**
		11 Marks

Question Number	Scheme	Marks
Q3 (a)		
	$\uparrow \quad T\cos\theta + N = Mg \qquad$ **(1)**	M1A1
	$\rightarrow \quad T\sin\theta = mr\omega^2 \qquad$ **(2)**	M1A1
		
	$\sin\theta = \frac{r}{l}$ from **(2)** $T = ml\omega^2$	
	sub into **(1)** $ml\cos\theta\,\omega^2 + N = mg$	M1
	$N = mg - mh\omega^2$	A1
	Since in contact with table $N \geq 0 \quad \therefore \omega^2 \leq \frac{g}{h}$ *	M1A1 cso
		(8)
(b)		
	$r:h:l = 3:4:5 \quad \therefore$ extension $= \frac{h}{4}$	B1
	$T = \frac{2mg}{h} \times \frac{h}{4} \quad = \frac{mg}{2}$	M1A1
	$T = ml\omega^2 = \frac{5mh}{4}\omega^2 \quad \omega = \sqrt{\frac{2g}{5h}}$	M1A1
		(5)
		13 marks

Question Number	Scheme	Marks
Q4 (a)		
	Mass $a^3 \frac{2}{3}\pi \times$: 216 8 208	M1A1
	27 1 26	
	C of M from O: $\frac{3}{8}\times 6a$ $\frac{3}{8}\times 2a$ $\bar{x}$ Use of $\frac{3}{8}r$	M1
	Moment : $216\times\frac{6a\times 3}{8} = 8\times\frac{2a\times 3}{8}+208\bar{x}$	M1
	$\bar{x} = \frac{480a}{208} = \frac{30a}{13}$ *	A1 cso
		(5)
(b)	 = S	B1
		B1
	Mass $\pi a^3 \times$: $\frac{416}{3}$ + 24 = $\frac{488}{3}$	
	C of M: $\frac{30}{13}a$ + $9a$ = $\bar{y}$	M1
	Moments : $320a + 216a = \frac{488}{3}\bar{y}$	A1 cso
	$\bar{y} = \frac{201}{61}a$ *	**(4)**
(c)		
	$12a - \frac{201}{61}a$	M1
	$\tan\theta = \frac{2a}{12a-\frac{201}{61}a}$ $\tan\theta = \frac{2a}{....}$	M1
	$\theta = 12.93.....$	A1
	so critical angle = 12.93… $\therefore$ if $\theta = 12°$ it will <u>NOT</u> topple.	A1
		(4) **13 marks**

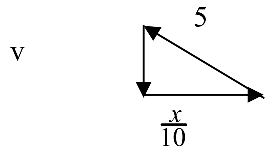
Question Number	Scheme	Marks
Q5(a)		
	ergy $\frac{1}{2}mv^2 = mga\cos\theta$	M1A1
	$v^2 = 2ga\cos\theta$	
	$F = ma$ $\nwarrow$ $T - mg\cos\theta = \frac{mv^2}{a}$	M1A1
	Sub for $\frac{v^2}{a}$: $T = mg\cos\theta + 2mg\cos\theta$: $\theta = 60$ $\therefore T = \frac{3}{2}mg$	M1A1
		(6)
(b)	Speed of P before impact = $\sqrt{2ga}$	B1
	$\rightarrow\sqrt{2ga}$ $\rightarrow 0$ $\rightarrow u$	
	PCLM : • • $\rightarrow$ • $\therefore u = \frac{\sqrt{2ga}}{4} = \sqrt{\frac{ga}{8}}$ *	M1A1cso
	m 3m 4m	
		(3)
(c) (i)		
	At A $v = 0$ so conservation of energy gives:	
	$\frac{1}{2}\cancel{4m}u^2 = \cancel{4m}ga(1-\cos\theta)$	M1A1
	$\frac{ga}{16} = ga(1-\cos\theta)$	M1
	$\cos\theta = \frac{15}{16}$, $\theta = 20^{\circ}$	A1
(ii)	$\nearrow$ At A $T = 4mg\cos\theta = \frac{15mg}{4}$ (accept 3.75mg)	M1A1 **(6)** **15 Marks**

Question Number	Scheme	Marks
Q6 (a)	$F = ma\ (\rightarrow)\quad \dfrac{3}{(x+1)^3} = 0.5\text{a} = 0.5\ v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1A1
	$\displaystyle\int \frac{3}{(x+1)^3}\,\mathrm{d}x = 0.5\int v\ \mathrm{d}v$ Separate and $\int$	M1
	$-\dfrac{3}{2(x+1)^2} = \dfrac{1}{4}v^2\,(+\ c)$	A1
	$x = 0,\ v = 0 \Rightarrow c' = -\dfrac{3}{2}\quad\therefore\quad v^2 = 6\left(1 - \dfrac{1}{(x+1)^2}\right)$ *	M1A1 cso **(6)**
(b)	$\forall x\quad v^2 < 6\quad\therefore\quad v < \sqrt{6}\quad (\because (x+1)^2 \text{ always} > 0)$	B1 **(1)**
(c)	$v = \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\sqrt{6}\sqrt{(x+1)^2 - 1}}{x+1}$	M1
	$\displaystyle\int \frac{x+1}{\sqrt{(x+1)^2 - 1}}\,\mathrm{d}x = \sqrt{6}\int \mathrm{d}t$	M1
	$\sqrt{(x+1)^2 - 1} = \sqrt{6}\,t + c'$	M1 A1
	$t = 0,\ x = 0 \Rightarrow c' = 0$	M1
	$t = 2 \Rightarrow (x+1)^2 - 1 = (2\sqrt{6})^2$	M1
	$(x+1)^2 = 25 \quad\Rightarrow x = 4\quad$ (c' need not have been found)	A1 cao
		(7)
		14 Marks

June 2008
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
1.	${}_Q\mathbf{v}_P = \mathbf{v}_Q - \mathbf{v}_P = (3\mathbf{i} + 7\mathbf{j}) - (5\mathbf{i} - 4\mathbf{j})$ $= (-2\mathbf{i} + 11\mathbf{j})$ $\tan\theta = \frac{11}{2} \Rightarrow \theta = 79.69^0...$ Bearing is 350^0	M1 A1 M1 A1 A1 **5**
2.	$2m(2\mathbf{i} - 2\mathbf{j}) + m(-3\mathbf{i} - \mathbf{j}) = 2m(\mathbf{i} - 3\mathbf{j}) + m\mathbf{v}$ $(\mathbf{i} - 5\mathbf{j}) = (2\mathbf{i} - 6\mathbf{j}) + \mathbf{v}$ $(-\mathbf{i} + \mathbf{j}) = \mathbf{v}$ $\lvert\mathbf{v}\rvert = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ m s^{-1} cwo	M1 A1 A1 DM1 A1 **5**
3.	$mg - mkv = m\frac{\mathrm{d}v}{\mathrm{d}t}$ $\int \mathrm{d}t = \int \frac{\mathrm{d}v}{g - kv}$ $t = -\frac{1}{k}\ln(g - kv) + c$ $t = 0, v = u \Rightarrow c = \frac{1}{k}\ln(g - ku)$ $T = \frac{1}{k}\ln(g - ku) - \frac{1}{k}\ln(g - 2ku)$ $= \frac{1}{k}\ln\left(\frac{g - ku}{g - 2ku}\right)$	M1* A1 A1 DM1* A1cao M1† DM1† A1 **8**

Question Number	Scheme	Marks
4.	$u\cos 2\theta = v\cos\theta$	M1 A1
	$\frac{3}{8}u\sin 2\theta = v\sin\theta$	M1 A1
	$3\tan 2\theta = 8\tan\theta$	M1
	$\dfrac{6\tan\theta}{1-\tan^2\theta} = 8\tan\theta$	M1
	$\tan^2\theta = \frac{1}{4} \quad (\tan\theta \neq 0)$	
	$\tan\theta = \frac{1}{2}$	M1 A1 **8**
5.(a)	$-T - \dfrac{1}{2}mg - 2mv\sqrt{\dfrac{g}{l}} = m\ddot{x}$	M1 A3,2,1,0
	$\dfrac{-mgx}{l} - \dfrac{1}{2}mg - 2m\dot{x}\sqrt{\dfrac{g}{l}} = m\ddot{x}$	M1
	$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\omega\dfrac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = -0.5g \quad \text{(AG)}$	A1 (6)
(b)	$u^2 + 2\omega u + \omega^2 = 0 \Rightarrow u = \omega$ (twice)	
	CF is $x = e^{-\omega t}(At + B)$	B1
	PI is $x = -\dfrac{1}{2}l \quad (-\dfrac{g}{2\omega^2})$	
	GS is $x = e^{-\omega t}(At + B) - \dfrac{1}{2}l$	M1
	$t = 0, x = 0 \Rightarrow B = \dfrac{1}{2}l \quad (\dfrac{g}{2\omega^2})$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\omega e^{-\omega t}(At + B) + Ae^{-\omega t}$	M1
	$t = 0, \dfrac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{gl} = \omega l \Rightarrow A = \dfrac{3}{2}\omega l (= \dfrac{3\sqrt{gl}}{2})(= \sqrt{gl} + \dfrac{0.5g}{\omega})$	M1
	so $x = e^{-\omega t}(\dfrac{3}{2}\omega l t + \dfrac{1}{2}l) - \dfrac{1}{2}l = \dfrac{1}{2}le^{-\omega t}(3\omega t + 1) - \dfrac{1}{2}l$	A1 (6)
(c)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 0 \Rightarrow -\omega e^{-\omega t}(At + B) + Ae^{-\omega t} = 0$	M1
	$\Rightarrow t = \dfrac{2}{3\omega}$	M1 A1 (3)
		15

6.(a)	 vector triangle	M1
	$v^2 + (\frac{x}{10})^2 = 5^2$	M1
	$\Rightarrow 100v^2 = 2500 - x^2$	A1 (3)
(b)	$200v\frac{\mathrm{d}v}{\mathrm{d}x} = -2x$	M1 A1
	$200\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2x = 0$	D M1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{x}{100} = 0$ *	A1 (4)
(c)	Aux equn: $m^2 + \frac{1}{100} = 0$	M1
	$\Rightarrow m = \pm\frac{i}{10}$	A1
	$x = A\sin\frac{t}{10} + B\cos\frac{t}{10}$	A1
	$t = 0, x = 0 \Rightarrow B = 0$	B1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{A}{10}\cos\frac{t}{10}$	M1
	$t = 0, x = 0 \Rightarrow v = \frac{\mathrm{d}x}{\mathrm{d}t} = 5$	
	$\Rightarrow 5 = \frac{A}{10} \Rightarrow A = 50$	M1
	$\Rightarrow x = 50\sin\frac{t}{10}$	A1
	$x = 30$: $30 = 50\sin\frac{t}{10}$	
	$\Rightarrow t = 10\sin^{-1}(\frac{3}{5}) = 6.44$ s	M1A1 (9)
		16

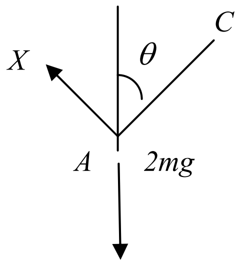
7.(a)	PE of rod $= -kMga\sin 2\theta$ $BP = 2\text{x}2a\sin\theta = 4a\sin\theta$ PE of mass $= -Mg(6a - 4a\sin\theta)$ $V = -Mg(6a - 4a\sin\theta) - kMga\sin 2\theta$ $= Mga(4\sin\theta - k\sin 2\theta) +$ constant *	B1 M1 A1 M1 A1 (5)
(b)	$\frac{dV}{d\theta} = Mga(4\cos\theta - 2k\cos 2\theta)$ so, $4\text{x}\frac{3}{4} - 2k(2(\frac{3}{4})^2 - 1) = 0$ $\Rightarrow k = 12$	M1 A1 M1 M1 A1 (5)
(c)	$4\cos\theta - 24(2\cos^2\theta - 1) = 0$ $12\cos^2\theta - \cos\theta - 6 = 0$ $(4\cos\theta - 3)(3\cos\theta + 2) = 0$ $\cos\theta = -\frac{2}{3}$	M1 D M1 A1 (3)
(d)	$\frac{d^2V}{d\theta^2} = (Mga)(-4\sin\theta + 4k\sin 2\theta)$ when $\cos\theta = \frac{3}{4}, \frac{d^2V}{d\theta^2} = (Mga)\times 44.97.. \Rightarrow$ stable when $\cos\theta = \frac{-2}{3}, \frac{d^2V}{d\theta^2} = (Mga)\times -50.68.. \Rightarrow$ unstable	M1 A1 M1 A1 A1 (5) **18**

June 2008
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	**d**= (7**i** - 14**j**) - (**i** - 6**j**) = (6**i** - 8**j**)	B1
	$(6k\mathbf{i} + k\mathbf{j}).(6\mathbf{i} - 8\mathbf{j}) = \frac{1}{2} \text{x} \frac{1}{2} \text{x} (2\sqrt{7})^2$	M1 A2 ft
	$28k = 7 \Rightarrow k = \frac{1}{4}$	D M1
	$\Rightarrow \mathbf{P} = \frac{\mathbf{3}}{\mathbf{2}}\mathbf{i} + \frac{1}{4}\mathbf{j}$	A1 **6**
2.	Aux Equn: $m^2 + 4m = 0 \Rightarrow m = 0$ or -4	M1
	$\mathbf{r} = \mathbf{A} + \mathbf{B}\mathrm{e}^{-4t}$	
	$t = 0, \mathbf{r} = \mathbf{i} - \mathbf{j}$: $\quad \mathbf{A} + \mathbf{B} = \mathbf{i} - \mathbf{j}$	A1 M1
	$\mathbf{v} = -\mathbf{4B}\mathrm{e}^{-4t}$	
	$t = 0, v = -8\mathbf{i} + 4\mathbf{j}$: $\quad -\mathbf{4B} = -8\mathbf{i} + 4\mathbf{j}$	M1
	$\mathbf{B} = 2\mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = -\mathbf{i}$	
	so, $\mathbf{r} = -\mathbf{i} + (2\mathbf{i} - \mathbf{j})\mathrm{e}^{-4t}$	A1 A1
	$= (2\mathrm{e}^{-4t} - 1)\mathbf{i} - \mathrm{e}^{-4t}\mathbf{j}$	A1 **7**
3.(a)	**R** = (-2**i** + **j** - **k**) + (3**i** - **j** + 2**k**)	M1
	= (**i** + **k**)	A1 (2)
(b)	**G** + (5**i** + **j** - **k**) x (**i** + **k**) = (**i** - **j** + **k**) x (-2**i** + **j** - **k**) + (4**i** - **j** - 2**k**) x (3**i** - **j** + 2**k**)	M1 A2 ft
	G + (**i** - 6**j** – **k**) = (- **j** – **k**) + (-4**i** - 14**j** - **k**)	A3 ft
	G = (-5**i** - 9**j** – **k**)	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-5^2 + (-9)^2 + (-1)^2} = \sqrt{107}$ Nm	M1 A1 (9)
		11

4. (a)	$-mg\,\delta t = (m+\delta m)(v+\delta v) + \delta m(U-v) - mv$ $-mg\,\delta t = mv + m\,\delta v + v\,\delta m + U\,\delta m - v\,\delta m - mv$ $-mg = m\dfrac{\mathrm{d}v}{\mathrm{d}t} + U\dfrac{\mathrm{d}m}{\mathrm{d}t}$ $m = M_0(1-\tfrac{1}{2}t) \Rightarrow \dfrac{\mathrm{d}m}{\mathrm{d}t} = -\tfrac{1}{2}M_0$ $-M_0 g(1-\tfrac{1}{2}t) = M_0(1-\tfrac{1}{2}t)\dfrac{\mathrm{d}v}{\mathrm{d}t} - \tfrac{1}{2}M_0 U$ $U - g(2-t) = (2-t)\dfrac{\mathrm{d}v}{\mathrm{d}t}$ $\dfrac{U}{(2-t)} - 9.8 = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ *	M1 A2 A1 B1 M1 A1 (7)
(b)	$\dfrac{\mathrm{d}v}{\mathrm{d}t} > 0$ when $t = 0 \Rightarrow \dfrac{U}{2} - 9.8 > 0$ $\Rightarrow \quad U > 19.6$ *	M1 A1 (2)
(c)	$v = \displaystyle\int \frac{U}{(2-t)} - 9.8\ \mathrm{d}t$ $= -U\ln(2-t) - 9.8t + C$ $t = 0, v = 0:\ 0 = -U\ln 2 + C \Rightarrow C = U\ln 2$ so, $v = U\ln\dfrac{2}{(2-t)} - 9.8t$ $t = 1:\ v = U\ln 2 - 9.8$	M1 A1 M1 M1 A1 (5) **14**

Question Number	Scheme	Marks
5.(a)	$I = \frac{1}{3}m(9a)^2 + \frac{1}{2}2ma^2 + 2m(9a)^2$	M1 A1 A1
	$= 27ma^2 + ma^2 + 162ma^2$	
	$= 190ma^2$	A1* (4)
(b)	M(L),	
	$mg\dfrac{9a}{2}\sin\theta + mg9a\sin\theta = -190ma^2\ddot{\theta}$	M1 A2
	$\ddot{\theta} = -\dfrac{9g}{76a}\sin\theta$	
	For small θ, $\sin\theta \approx \theta$,	M1
	$\Rightarrow \ddot{\theta} = -\dfrac{9g}{76a}\theta$ so S.H.M.	A1
	Period = $2\pi\sqrt{\dfrac{76a}{9g}} = \dfrac{4\pi}{3}\sqrt{\dfrac{19a}{g}}$	DM1 A1
		(7)
		11
6.	$\delta m = \pi a^2 \delta x . \dfrac{M}{\pi a^2 h} = \dfrac{M\delta x}{h}$	M1 A1
	$\delta I = \dfrac{1}{4}\delta m . a^2 + \delta m . x^2$	M1 A1
	$= \dfrac{M}{4h}(a^2 + 4x^2)\delta x$	M1 A1
	$I = \int_0^h \dfrac{M}{4h}(a^2 + 4x^2)\mathrm{d}x$	M1 A1
	$= \dfrac{M}{4h}\left[a^2 x + \dfrac{4}{3}x^3\right]_0^h$	M1
	$= \dfrac{M}{4}(a^2 + \dfrac{4}{3}h^2)$	
	$= \dfrac{M}{12}(3a^2 + 4h^2)$	A1
		10

7.(a)		
	$\frac{1}{2}24ma^2\dot{\theta}^2 = 2mg.3a(1-\cos\theta)$	M1 A1 A1
	$2a\dot{\theta}^2 = g(1-\cos\theta)$	A1 (4)
(b)	$(\searrow)$	
	$2mg\sin\theta - X = 2m3a\cos\theta$	M1 A2
	M(L), $2mg.3a\sin\theta = 24ma^2\ddot{\theta}$	M1 A1
	$\Rightarrow X = 2mg\sin\theta - 6ma\left(\frac{g\sin\theta}{4a}\right)$	DM1
	$= \frac{mg\sin\theta}{2}$ *	A1 (7)
(c)	$\theta = \pi : 2a\dot{\theta}^2 = g(1-\cos\pi)$	M1
	$\dot{\theta} = \sqrt{\frac{g}{a}}$	A1
	$6a.I = 24ma^2\sqrt{\frac{g}{a}}$	M1 A1
	$\Rightarrow I = 4m\sqrt{ag}$	A1
		(5)
		16

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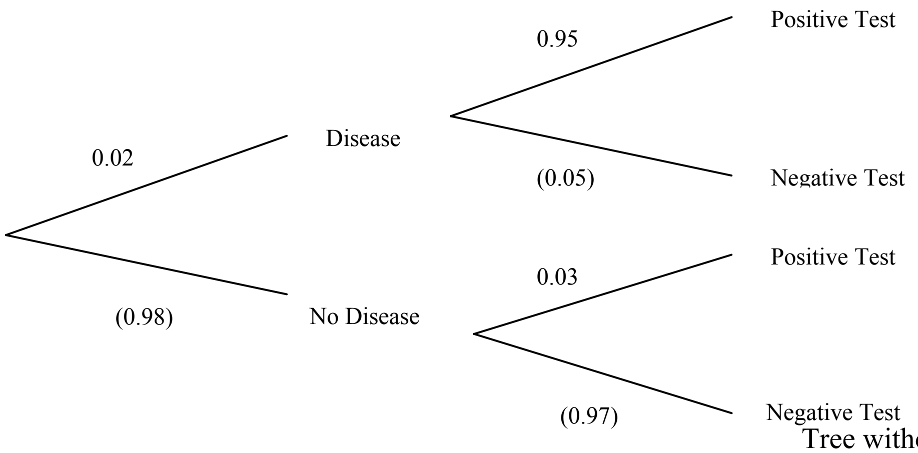
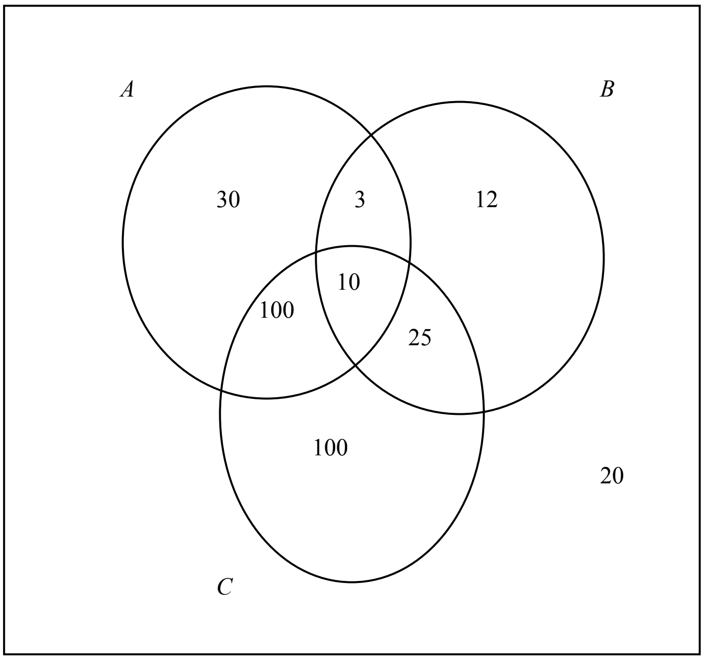
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)		
	Tree without probabilities or labels	**M1**
	0.02(Disease), 0.95(Positive) on correct branche	**A1**
	0.03(Positive) on correct branch.	**A1** **[3]**
(b)	P(Positive Test) $=0.02\times0.95+0.98\times0.03$	**M1A1ft**
	$=0.0484$	**A1** **[3]**
(c)	P(Do not have disease$\mid$Postive test) $=\dfrac{0.98\times0.03}{0.0484}$	**M1**
	$=0.607438..$ awrt 0.607	**A1** **[2]**
(d)	Test not very useful OR High probability of not having the disease for a person with a positive test	**B1** **[1]**
		Total 9
	<u>Notes</u>: (a) M1:All 6 branches. Bracketed probabilities not required. (b) M1 for sum of two products, at least one correct from their diagram A1ft follows from the probabilities on their tree A1 for correct answer only or $\dfrac{121}{2500}$ (c) M1 for conditional probability with numerator following from their tree and denominator their answer to part (b). A1 also for $\dfrac{147}{242}$.	

Question Number	Scheme	Marks
Q2		
(a)	50	B1 [1]
(b)	$Q_1 = 45$	B1
	$Q_2 = 50.5$ ONLY	B1
	$Q_3 = 63$	B1 [3]
(c)	Mean $= \frac{1469}{28} = 52.464286..$ awrt 52.5	M1A1
	Sd $= \sqrt{\frac{81213}{28} - \left(\frac{1469}{28}\right)^2}$	M1
	=12.164…. or 12.387216…for divisor n-1 awrt 12.2 or 12.4	A1 [4]
(d)	$\frac{52.46..-50}{sd} =$ awrt 0.20 or 0.21	M1A1 [2]
(e)	1. mode/median/mean Balmoral>mode/median/mean Abbey 2. Balmoral sd < Abbey sd or similar sd or correct comment from their values, Balmoral range<Abbey range, Balmoral IQR>Abbey IQR or similar IQR 3. Balmoral positive skew or almost symmetrical AND Abbey negative skew, Balmoral is less skew than Abbey or correct comment from their value in (d) 4. Balmoral residents generally older than Abbey residents or equivalent. Only one comment of each type max 3 marks	B1B1B1 [3] **Total 13**
	<u>Notes</u>: (c) M1for their 1469 between 1300 and 1600, divided by 28, A1 for awrt 52.5 .. Please note this is B1B1 on Epen M1 use of correct formula including sq root A1 awrt 12.2 or 12.4 Correct answers with no working award full marks. (d) M1 for their values correctly substituted A1 Accept 0.2 as a special case of awrt 0.20 with 0 missing (e) Technical terms required in correct context in lines 1 to 3 e.g. 'average' and 'spread' B0 1 correct comment B1B0B0 2 correct comments B1B1B0 3 correct comments B1B1B1	

Question Number	Scheme	Marks
Q3 (a)	$-1\times p+1\times 0.2+2\times 0.15+3\times 0.15 \quad = 0.55$	**M1dM1**
	$p = 0.4$	**A1**
	$p+q+0.2+0.15+0.15 \quad = 1$	**M1**
	$q = 0.1$	**A1**
		[5]
(b)	$\text{Var}(X) = (-1)^2\times p+1^2\times 0.2+2^2\times 0.15+3^2\times 0.15, -0.55^2$	**M1A1,M1**
	$= 2.55-0.3025 = 2.2475$ awrt 2.25	**A1**
		[4]
(c)	E(2X-4) =2E(X)-4	**M1**
	$= -2.9$	**A1**
		[2]
		Total 11
	Notes: (a) M1 for at least 2 correct terms on LHS Division by constant e.g. 5 then M0 dM1 dependent on first M1 for equate to 0.55 and attempt to solve. Award M1M1A1 for p=0.4 with no working M1 for adding probabilities and equating to 1. All terms or equivalent required e.g. $p+q$=0.5 Award M1A1 for q=0.1 with no working (b) M1 attempting E(X^2) with at least 2 correct terms A1 for fully correct expression or 2.55 Division by constant at any point e.g. 5 then M0 M1 for subtracting their mean squared A1 for awrt 2.25 Award awrt 2.25 only with no working then 4 marks (c) M1 for 2x(their mean) -4 Award 2 marks for -2.9 with no working	

Question Number	Scheme	Marks
Q4 (a)	$S_{tt} = 10922.81 - \frac{401.3^2}{15} = 186.6973$ awrt 187	**M1A1**
	$S_{vv} = 42.3356 - \frac{25.08^2}{15} = 0.40184$ awrt 0.402	**A1**
	$S_{tv} = 677.971 - \frac{401.3 \times 25.08}{15} = 6.9974$ awrt 7.00	**A1** **[4]**
(b)	$r = \frac{6.9974}{\sqrt{186.6973 \times 0.40184}}$	**M1A1ft**
	$= 0.807869$ awrt 0.808	**A1** **[3]**
(c)	*t* is the explanatory variable as we	**B1**
	can control temperature but not frequency of noise or equivalent comment	**B1** **[2]**
(d)	High value of *r* or *r* close to 1 or Strong correlation	**B1** **[1]**
(e)	$b = \frac{6.9974}{186.6973} = 0.03748$ awrt 0.0375	**M1A1**
	$a = \frac{25.08}{15} - b \times \frac{401.3}{15} = 0.6692874$ awrt 0.669	**M1A1** **[4]**
(f)	*t*= 19, *v*=0.6692874+0.03748x19=1.381406 awrt 1.4	**B1** **[1]** **Total 15**
	Notes: (a) M1 any one attempt at a correct use of a formula. Award full marks for correct answers with no working. Epen order of awarding marks as above. (b) M1 for correct formula and attempt to use A1ft for their values from part (a) NB Special Case for $\frac{677.971}{\sqrt{10922.81 \times 42.3356}}$ M1A0 A1 awrt 0.808 Award 3 marks for awrt 0.808 with no working (c) Marks are independent. Second mark requires some interpretation in context and can be statements such as 'temperature effects / influences pitch or noise' B1 'temperature is being changed' BUT B0 for 'temperature is changing' (e) M1 their values the right way up A1 for awrt 0.0375 M1 attempt to use correct formula with their value of *b* A1 awrt 0.669 (f) awrt 1.4	

Question Number	Scheme	Marks
Q5 (a)	 3 closed intersecting curves with labels 100 100,30 12,10,3,25 Box	M1 A1 A1 B1 **[4]**
(b)	P(Substance C) $=\dfrac{100+100+10+25}{300}=\dfrac{235}{300}=\dfrac{47}{60}$ or exact equivalent	**M1A1ft** **[2]**
(c)	$\text{P(All 3 } \mid A)=\dfrac{10}{30+3+10+100}=\dfrac{10}{143}$ or exact equivalent	**M1A1ft** **[2]**
(d)	$\text{P(Universal donor)}=\dfrac{20}{300}=\dfrac{1}{15}$ or exact equivalent	**M1A1 cao** **[2]** **Total 10**
	<u>Notes:</u> (a) 20 not required. Fractions and exact equivalent decimals or percentages. (b) M1 For adding their positive values in C and finding a probability A1ft for correct answer or answer from their working (c) M1 their 10 divided by their sum of values in A A1ft for correct answer or answer from their working (d) M1 for 'their 20' divided by 300 A1 correct answer only	

Question Number	Scheme	Marks
Q6 (a)	F(4)=1	
	$(4+k)^2 = 25$	**M1**
	$k = 1$ as $k > 0$	**A1**
		[2]
(b)	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X=x)</td><td>$\frac{9}{25}$</td><td>$\frac{7}{25}$</td><td>$\frac{9}{25}$</td></tr></table>	**B1ftB1B1**
		[3]
		Total 5
	<u>Notes</u>: (a) M1 for use of F(4) = 1 only If F(2)=1 and / or F(3)=1 seen then M0. F(2)+F(3)+F(4)=1 M0 A1 for k=1 and ignore k= -9 (b) B1ft follow through their k for P(X=2) either exact or 3sf between 0 and 1 inclusive. B1 correct answer only or exact equivalent B1 correct answer only or exact equivalent	

Question Number	Scheme		Marks
Q7 (a)	$z = \dfrac{53-50}{2}$	Attempt to standardise	M1
	P(X>53)=1-P(Z<1.5)	1-probability required can be implied	B1
	=1-0.9332		
	=0.0668		A1 [3]
(b)	$\mathrm{P}(X \le x_0) = 0.01$		M1
	$\dfrac{x_0 - 50}{2} = -2.3263$		M1B1
	$x_0 = 45.3474$	awrt 45.3 or 45.4	M1A1 [5]
(c)	P(2 weigh more than 53kg and 1 less) $= 3 \times 0.0668^2 (1 - 0.0668)$		B1M1A1ft
	$= 0.012492487..$	awrt 0.012	A1 [4]
			Total 12
	Notes: (a) M1 for using 53,50 and 2, either way around on numerator B1 1- any probability for mark A1 0.0668 cao (b) M1 can be implied or seen in a diagram or equivalent with correct use of 0.01 or 0.99 M1 for attempt to standardise with 50 and 2 numerator either way around B1 for ± 2.3263 M1 Equate expression with 50 and 2 to a z value to form an equation with consistent signs and attempt to solve A1 awrt 45.3 or 45.4 (c) B1 for 3, M1 $p^2(1-p)$ for any value of p A1ft for p is their answer to part (a) without 3 A1 awrt 0.012 or 0.0125		

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Mark Scheme

Question Number	Scheme	Marks
1(a)	$\mathrm{E}(X) = 5$	B1
	$\mathrm{Var}(X) = \frac{1}{12}(10-0)^2$ or attempt to use $\int \frac{x^2}{10}dx - \mu^2$	M1
	$= \frac{100}{12} = \frac{25}{3} = 8\frac{1}{3} = 8.\dot{3}$ awrt 8.33	A1
		(3)
(b)	$\mathrm{P}(X \le 2) = (2-0)\times\frac{1}{10} = \frac{1}{5}$ or $\frac{2}{10}$ or 0.2	M1 A1 (2)
(c)	$\left(\frac{1}{5}\right)^5 = 0.00032$ or $\frac{1}{3125}$ or 3.2×10^{-4} o.e.	M1 A1 (2)
(d)	$\mathrm{P}(X \ge 8)$ or $\mathrm{P}(X > 8)$	M1
	$\mathrm{P}(X \ge 8 \mid X \ge 5) = \frac{\mathrm{P}(X \ge 8)}{\mathrm{P}(X \ge 5)}$	M1
	$= \frac{2/10}{5/10}$	
	$= \frac{2}{5}$	A1
		(3)
	alternative	
	remaining time ~ U[0,5] or U[5,10] $\mathrm{P}(X \ge 3 \text{ or } 8) = \frac{2}{5}$	M1 M1 A1
		(Total 10)

	Notes (a) B1 cao M1 using the correct formula $\frac{(a-b)^2}{12}$ and subst in 10 or 0 **or** for an attempt at the integration they must increase the power of x by 1 and subtract their E(X) squared. A1 cao (b) M1 for P($X \leq 2$) or P($X < 2$) A1 cao (c) M1 (their b)^5. If the answer is incorrect we must see this. No need to check with your calculator A1 cao (d) writing P($X \geq 8$) (may use > sign). If they do not write P($X \geq 8$) then it must be clear from their working that they are finding it. 0.2 on its own with no working gets M0 M1 For attempting to use a correct conditional probability. NB this is an A mark on EPEN A1 2/5 Full marks for 2/5 on its own with no incorrect working Alternative M1 for P($X \geq 3$) or P($X \geq 8$) may use > sign M1 using either U[0,5] or U[5,10] A1 2/5	

Question Number	Scheme	Marks
2	X ~B(100,0.58) Y ~ N (58, 24.36)	B1 B1 B1
	[P(X > 50) = P($X \geq 51$)] using 50.5 or 51.5 or 49.5 or 48.5	M1
	$= \mathrm{P}\left(z \geq \pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their μ and σ for M1	M1
		A1
	= P(z ≥ -1.52…)	A1
	= 0.9357	(7)
	alternative X ~B(100,0.58) Y ~ N (42, 24.36)	B1 B1 B1
	[P(X < 50) = P($X \leq 49$)] using 50.5 or 51.5 or 49.5 or 48.5	M1
	$= \mathrm{P}\left(z \leq \pm\left(\frac{49.5-42}{\sqrt{24.36}}\right)\right)$ standardising 50.5, 51, 51.5 ,48.5,49, 49.5 and their μ and σ for M1	M1 A1
	= P(z ≤ 1.52…)	
	= 0.9357	A1 (Total 7)

Notes

The first 3 marks may be given if the following figures are seen in the standardisation formula :- 58 or 42,

24.36 or √24.36 or √24.4 or awrt 4.94.

Otherwise

B1 normal

B1 58 or 42

B1 24.36

M1 using 50.5 or 51.5 or 49.5 or 48.5. ignore the direction of the inequality.

M1 standardising 50.5, 51, 51.5 ,48.5, 49, 49.5 and their μ and σ . They may use √ 24 or √24.36 or √24.4 or awrt 4.94 for σ or the √of their variance.

A1 $\pm$ 1.52. may be awarded for $\pm\left(\frac{50.5-58}{\sqrt{24.36}}\right)$ or $\pm\left(\frac{49.5-42}{\sqrt{24.36}}\right)$ o.e.

A1 awrt 0.936

Question Number	Scheme	Marks
3(a)	$X \sim \text{Po}(9)$ may be implied by calculations in part a or b	M1
	$P(X \leq 3) = 0.0212$ $P(X \geq 16) = 0.0220$	
	CR $X \leq 3$; $\cup$ $X \geq 16$	A1; A1 (3)
(b)	P(rejecting Ho) = 0.0212 + 0.0220	M1
	= 0.0432 or 0.0433	A1 cao
		(2)
		Total 5

Notes

(a) M1 for using Po (9) – other values you might see which imply Po (9) are 0.0550, 0.0415, 0.9780, 0.9585, 0.9889,0.0111,0.0062 or may be assumed by at least one correct region.

A1 for $X \leq 3$ or $X < 4$ condone c1 or CR instead of X

A1 for $X \geq 16$ or $X > 15$

They must identify the critical regions at the end and not just have them as part of their working. Do not accept $P(X \leq 3)$ etc gets A0

(b) if they use 0.0212 and 0.0220 they can gain these marks regardless of the critical regions in part a. If they have not got the correct numbers they must be adding the values for their critical regions.(both smaller than 0.05) You may need to look these up. The most common table values for lambda = 9 are in this table

x	2	3	4	5	14	15	16	17	18
	0.0062	0.0212	0.0550	0.1157	0.9585	0.9780	0.9889	0.9947	0.9976

A1 awrt 0.0432 or 0.0433

Special case

If you see 0.0432 / 0.0433 and then they go and do something else with it eg 1 – 0.0432 award M1 A0

Question Number	Scheme	Marks
4(a)	$X \sim B(11000, 0.0005)$	M1 A1 (2)
(b)	$E(X) = 11000 \times 0.0005 = 5.5$	B1
	$\text{Var}(X) = 11000 \times 0.0005 \times (1 - 0.0005)$ $= 5.49725$	B1 (2)
(c)	$X \sim Po(5.5)$	M1 A1
	$P(X \leq 2) = 0.0884$	dM1 A1 (4)
		Total 8
	<u>Notes</u> (a) M1 for Binomial, A1 fully correct These cannot be awarded unless seen in part a (b)B1 cao B1 also allow 5.50, 5.497, 5.4973, do **not** allow 5.5 (c) M1 for Poisson A1 for **using** Po (5.5) M1 this is dependent on the previous M mark. It is for attempting to find $P(X \leq 2)$ A1 awrt 0.0884 Correct answer with no working gets full marks <u>Special case</u> If they use normal approximation they could get M0 A0 M1 A0 if they use 2.5 in their standardisation. NB exact binomial is 0.0883	

Question Number	Scheme	Marks
5(a)	$X \sim$ B(15, 0.5)	B1 B1 (2)
(b)	P (X = 8) = P ($X \le$ 8) – P($X \le$ 7) or $\left(\frac{15!}{8!7!}(p)^8(1-p)^7\right)$	M1
	= 0.6964 – 0.5	
	= 0.1964 awrt 0.196	A1 (2)
(c)	P ($X \ge$ 4) = 1 – P($X \le$ 3)	M1
	= 1 – 0.0176	
	= 0.9824	A1 (2)
(d)	$H_o : p = 0.5$	B1
	$H_1 : p > 0.5$	B1
	$X \sim$ B(15, 0.5)	
	P($X \ge$ 13) = 1 − P($X \le$ 12) | [P($X \ge$ 12) = 1− 0.9824 = 0.0176] att P($X \ge$ 13)	M1
	= 1 − 0.9963 | P($X \ge$ 13) = 1− 0.9963 = 0.0037	
	= 0.0037 | CR $X \ge$ 13 awrt 0.0037/ CR $X \ge$ 13	A1
	0.0037 < 0.01 | 13 ≥ 13	
	Reject H_0 or it is significant or a correct statement in context from their values	M1
	There is sufficient evidence at the 1% significance level that the coin is <u>biased in favour of heads</u> Or There is evidence that Sues belief is correct	A1 (6)

Notes

(a) B1 for Binomial
B1 for 15 and 0.5 must be in part a
This need not be in the form written

(b) M1 attempt to find P ($X = 8$) any method. Any value of p
A1 awrt 0.196
Answer only full marks

(c) M1 for 1 - P ($X \leq 3$).
A1 awrt 0.982

(d) B1 for correct H_0. must use p or π
B1 for correct H_1 must be one tail must use p or π
M1 attempt to find $P(X \geq 13)$ correctly. E.g. $1 - P(X \leq 12)$
A1 correct probability or CR

To get the next 2 marks the null hypothesis must state or imply that $(p) = 0.5$

M1 for correct statement based on their probability or critical region or a correct contextualised statement that implies that. not just 13 is in the critical region.

A1 This depends on their M1 being awarded for rejecting H_0. Conclusion in context. Must use the words biased in favour of heads or biased against tails or sues belief is correct .
NB this is a B mark on EPEN.

They may also attempt to find $P(X < 13) = 0.9963$ and compare with 0.99

Question Number	Scheme	Marks
6(a)	Calls occur singly Calls occur at a constant rate Calls occur independently or randomly. any two of the 3 only need calls once	B1 B1 (2)
(b) (i)	X~ Po(4.5) used or seen in (i) or (ii) P (X = 5) = P (X $\leq$ 5) – P(X $\leq$ 4) = 0.7029 – 0.5321 = 0.1708	M1 M1 A1 (3)
(ii)	P (X > 8) = 1 – P (X $\leq$ 8) = 1 – 0.9597 = 0.0403	M1 A1 (2)
(c)	Ho : λ = 9 (λ = 18) may use λ or μ H1 : λ > 9 (λ > 18)	B1
	X ~ Po (9) may be implied	B1
	P(X $\geq$ 14) = 1 − P(X $\leq$ 13) | [P(X $\geq$ 14) = 1− 0.9261 = 0.0739] att P(X $\geq$ 14) | P(X $\geq$ 15)	M1
	= 1 − 0.9261 | P(X $\geq$ 15) = 1− 0.9780 = 0.0220 = 0.0739 | CR X $\geq$ 15 awrt 0.0739	A1
	0.0739 > 0.05 | 14 $\leq$ 15	
	Accept H0. or it is not significant or a correct statement in context from their values	M1
	There is insufficient evidence to say that the number of calls per hour handled by the agent has increased.	A1 (6)
	Notes (a) B1 B1 They must use calls at least once. Independently and randomly are the same reason. Award the first B1 if they only gain 1 mark. Special case if they don't put in the word calls but write two correct statements award B0B1 (b) correct answers only score full marks (i) M1 Po (4.5) may be implied by them using it in their calculations in (i) or (ii) M1 for P(X < 5) – P(X < 4) or $\dfrac{e^{-\lambda}\lambda^5}{5!}$	

	A1 only awrt 0.171 (ii) M1 for 1 – P ($X \leq 8$) A1 only awrt 0.0403 (c) B1 both . Must be one tail test. They may use λ or μ and either 9 or 18 and match H0 and H1 M1 Po (9) may be implied by them using it in their calculations. M1 attempt to find $P(X \geq 14)$ eg $1 - P(X < 13)$ or $1 - P(X < 14)$ A1 correct probability or CR To get the next2 marks the null hypothesis must state or imply that (λ) = 9 or 18 M1 for a correct statement based on their probability or critical region or a correct contextualised statement that implies that. A1. This depends on their M1 being awarded for accepting H0. Conclusion in context. Must have calls per hour has not increased. Or the rate of calls has not increased. Any statement that has the word calls in and implies the rate not increasing e.g. no evidence that the rate of calls handled has increased Saying the number of calls has not increased gains A0 as it does not imply rate NB this is an A mark on EPEN They may also attempt to find $P(X < 14) = 0.9261$ and compare with 0.95	

Question Number	Scheme		Marks
7(a)			
	$\int_0^1 \frac{1}{2}x \, dx = \left[\frac{1}{4}x^2\right]_0^1 = \frac{1}{4}$ oe	attempt to integrate both parts	M1
	$\int_1^2 kx^3 \, dx \left[\frac{1}{4}kx^4\right]_1^2 = 4k - \frac{1}{4}k$ oe	both answer correct	A1
	$\frac{1}{4} + 4k - \frac{1}{4}k = 1$	adding two answers and putting = 1	dM1dep on previous M
	$\frac{15k}{4} = \frac{3}{4}$		
	$k = \frac{1}{5}$ *		A1 (4)
(b)	$\int_0^1 \frac{1}{2}x^2 \, dx = \left[\frac{1}{6}x^3\right]_0^1 = \frac{1}{6}$	attempt to integrate $x\mathrm{f}(x)$ for one part	M1
		1/6	A1
	$\int_1^2 \frac{1}{5}x^4 \, dx = \left[\frac{1}{25}x^5\right]_1^2 = \frac{32}{25} - \frac{1}{25}$		
	$= \frac{31}{25}$ or 1.24		A1
	$\mathrm{E}(X) = \frac{1}{6} + \frac{31}{25}$		
	$= \frac{211}{150} = 1\frac{61}{150} = 1.40\dot{6}$		A1 (4)
(c)			
	$\mathrm{F}(x) = \int_0^x \frac{1}{2}t \, dt$ (for $0 \le x \le 1$)	ignore limits for M	M1
	$= \frac{1}{4}x^2$	must use limit of 0	A1
	$\mathrm{F}(x) = \int_1^x \frac{1}{5}t^3 \, dt; + \int_0^1 \frac{1}{2}t \, dt$ (for $1 < x \le 2$)	need limit of 1 and variable upper limit; need limit 0 and 1	M1; M1
	$= \frac{1}{20}x^4 + \frac{1}{5}$		A1

	$F(x)\begin{cases} 0 & x<0 \\ \frac{1}{4}x^2 & 0\le x\le 1 \\ \frac{1}{20}x^4+\frac{1}{5} & 1<x\le 2 \\ 1 & x>2 \end{cases}$ middle pair ends	B1 ft B1	(7)
(d)	F(m) = 0.5 either eq $\frac{1}{20}m^4+\frac{1}{5}=0.5$ eq for their $1\le x\le 2$ $m = \sqrt[4]{6}$ or 1.57 or awrt 1.57	M1 A1ft A1	(3)
(e)	negative skew This depends on the previous B1 being awarded. One of the following statements which must be compatible with negative skew and their figures. If they use mode then they must have found a value for it Mean < Median Mean < mode Mean < median (< mode) Median < mode Sketch of the pdf.	B1 dB1	(2)
	<u>Notes</u> (a) M1 for adding two integrals together =1, ignore limits A1 for correct integration, ignore limits M1 using correct limits A1 cso (b) M1 attempting to use integral of x f(x) A1 correct two integrals added with limits A1 correct integration ignore limits A1 awrt 1.41 (c) M1 Att to integrate $\frac{1}{2}$t (they need to increase the power by 1). Ignore limits for method mark A1 $\frac{1}{4}x^2$ allow use of t. must have used/implied use of limit of 0. This must be on its own without anything else added M1 att to integrate $\int_1^x \frac{1}{5}t^3$ dt and correct limits. M1 $\int_0^1 \frac{1}{2}t\ \mathrm{d}t +$ Att to integrate using limits 0 and 1. no need to see them put 0 in .		

they must add this to their $\int_1^x \frac{1}{5}t^3 \, dt$. may be given if they add 1/4

$$\left(\begin{array}{l}\text{Alternative method for these last two M marks} \\ \text{M1 for att to } \int \frac{1}{5}t^3 \text{ dt and putting } + \text{ C} \\ \text{M1 use of } \text{F}(2) = 1 \text{ to find C} \\ \end{array}\right)$$

A1 $\frac{1}{20}x^4 + \frac{1}{5}$ must be correct

B1 middle pair followed through from their answers. condone them using < or ≤ incorrectly they do not need to match up

B1 end pairs. condone them using < or ≤. They do not need to match up

NB if they show no working and just write down the distribution. If it is correct they get full marks. If it is incorrect then they cannot get marks for any incorrect part. So if $0<x<1$ is correct they can get M1 A1 otherwise M0 A0. if $3<x<4$ is correct they can get M1 A1A1 otherwise M0 A0A0. you cannot award B1ft if they show no working unless the middle parts are correct.

(d) M1 either of their $\frac{1}{4}x^2$ or $\frac{1}{20}x^4 + \frac{1}{5} = 0.5$

A1 for their F(X) $1<x<2 = 0.5$

A1 cao

If they add both their parts together and put = 0.5 they get M0

If they work out both parts separately and do not make the answer clear they can get M1 A1 A0

(e) B1 negative skew only

B1 Dependent on getting the previous B1. their reason must follow through from their figures.

June 2008
6691 Statistics S3
Mark Scheme

Question number	Scheme	Marks
1. (a)	$\bar{x} = \left(\frac{6046}{36} = \right)167.94...$ awrt **168**	B1
	$s^2 = \frac{1016338 - 36 \times \bar{x}^2}{35}$	M1
	$= 27.0253....$ awrt **27.0** (Accept 27)	A1 (3)
(b)	99% Confidence Interval is: $\bar{x} \pm 2.5758 \times \frac{5.1}{\sqrt{36}}$	M1A1ft
	2.5758	B1
	= (165.755…, 170.133…) awrt (**166,170**)	A1 A1 (5)
		8 marks
(a)	M1 for a correct expression for s^2, follow through their mean, beware it is very "sensitive" $167.94 \rightarrow \frac{999.63..}{35} \rightarrow 28.56...$ $167.9 \rightarrow \frac{1483.24..}{35} \rightarrow 42.37...$ $168 \rightarrow \frac{274}{35} \rightarrow 7.82$ These would all score M1A0 Use of 36 as the divisor (= 26.3…) is M0A0	
(b)	M1 for substituting their values in $\bar{x} \pm z \times \frac{5.1 \text{ or } s}{\sqrt{36}}$ where z is a recognizable value from tables 1st A1 follow through their mean and their z (to 2dp) in $\bar{x} \pm z \times \frac{5.1}{\sqrt{36}}$ Beware: $167.94 \pm 2.5758 \times \frac{5.1^2}{36} \rightarrow (166.07..,169.8..)$ but scoresB1M0A0A0A0 Correct answer only in (b) scores 0/5 2nd & 3rd A marks depend upon 2.5758 **and** M mark.	

Question	Scheme	Marks
2.	$\frac{115\times 70}{217}=37.0967...$ or $\frac{1150}{31}$ (etc) $\frac{1265}{31},\frac{1020}{31},\frac{1122}{31}$	M1

Expected (Obs)	A	S	H
Boy	37.1 (30)	37.1 (50)	40.8 (35)
Girl	32.9 (40)	32.9 (20)	36.2 (42)

A1A1

H_0 : There is no association between course and gender

H_1 : There is some association between course and gender (both) B1

$$\sum\frac{(O-E)^2}{E}=\frac{(37.1-30)^2}{37.1}+\frac{(32.9-40)^2}{32.9}+...+\frac{(36.2-42)^2}{36.2}$$ M1A1ft

= 1.358 + 4.485 + 0.824 + 1.532 + 5.058 + 0.929 = 14.189… awrt **14.2** A1

$\nu=(3-1)(2-1)=2,$ $\chi_2^{\,2}(1\%)$ critical value is 9.210 (condone 9.21) B1, B1ft

Significant result or reject null hypothesis M1

There is evidence of an association between course taken and gender A1ft (11)

[Correct answers only score full marks] **11 marks**

ALT	$\sum\frac{O^2}{E}-N=\frac{30^2}{37.1}+\frac{40^2}{32.9}+...+\frac{42^2}{36.2}-217$	M1A1ft

1st M1 for some use of the $\frac{\text{row total}\times\text{col total}}{\text{grand total}}$ formula

1st A1 for one correct row or one correct column of expected frequencies to nearest integer

2nd A1 for all expected frequencies correct to awrt 1 dp (Allow exact fractions)

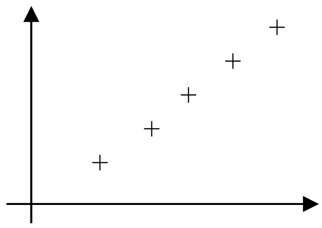
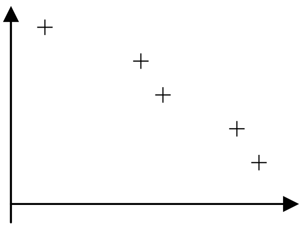
1st B1 for hypotheses. Independence is OK. Must mention courses and gender at least once.
Use of ρ or "correlation" is B0 but allow ISW.

2nd M1 for an attempt to calculate test statistic. At least one correct expression, ft expected freq.

3rd A1 follow through expected frequencies for at least 3 expressions

3rd M1 for a correct statement relating their test statistic and their cv (may be implied by comment)

5th A1 for a contextualised comment relating their test statistic and their cv. Ignore their H_0 or H_1 or assume that they were correct. Must mention courses and gender

Question number	Scheme	Marks
3. (a)	(i)  (ii) 	(i) B1 (ii) B1B1 (3)
(b)(i)		M1M1

	A	*B*	*C*	*D*	*E*	*F*	*G*
Rank (Judge 1)	1	4	2	3	5	6	7
Rank (Judge 2)	1	2	4	3	5	7	6
d^2	0	4	4	0	0	1	1

$\sum d^2 = 10$ — M1A1

$r_s = 1 - \dfrac{6\times 10}{7\times(49-1)} = 1 - \dfrac{5}{28} = \dfrac{23}{28}$ or awrt **0.821** — M1A1 (6)

(ii) $H_0 : \rho = 0$ $H_1 : \rho > 0$ (Allow ρ_S) ($H_1 : \rho \neq 0$ scores B0) — B1,B1

r_s 5% one tail critical value is **0.7143** B1

Significant result or reject null hypothesis — M1

There is evidence of a (positive) correlation between the judges <u>or</u> the judges agree A1ft — (5)

14 marks

(a) (i) 1st B1 for 5 or more points on a straight line of positive gradient

(ii) 2nd B1 for 4 or more points satisfying $-1<r<0$

3rd B1 for 5 or more points of decreasing ranks not on a straight line

(b)(i) 1st M1 for attempting to rank one of the judges (at least 2 correct rankings)

2nd M1 for ranking both (may be reversed) (at least 2 correct rankings)

3rd M1 for attempting d^2.

1st A1 for $\sum d^2 = 10$

4th M1 for correct use of the r_s formula

(ii) 3rd B1 for the correct critical value - depends upon their H_1: $\rho > 0$ needs 0.7143, $\rho \neq 0$, 0.7857

The H_1 may be in words so B0B1 is possible. If no H_1 award for 0.7143 only.

5th M1 for a correct statement relating their r_s and their cv (may be implied by correct comment)

3rd A1ft follow through their r_s and their cv. Comment in context. Must mention judges.

Don't insist on "positive" and condone it if they are using $\rho \neq 0$.

Question number	Scheme	Marks
4. (a)	$X = M_1 + M_2 + M_3 + M_4 \sim \mathrm{N}(336, 22^2)$ $\quad \mu =$ **336**	B1
	$\sigma^2 = 22^2$ or **484**	B1
	$\mathrm{P}(X < 350) = \mathrm{P}(Z < \frac{350-336}{22})$	M1
	$= \mathrm{P}(Z < 0.64)$ $\quad$ awrt **0.64**	A1
	$=$ $\quad$ awrt **0.738** or **0.739**	A1
	(5)	
(b)	M~N(84, 121) and W~N(62, 100) Let $Y = M - 1.5W$ $\quad$ M1	
	$\mathrm{E}(Y) = 84 - 1.5 \times 62 = -9$	A1
	$\mathrm{Var}(Y) = \mathrm{Var}(M) + 1.5^2\,\mathrm{Var}(W)$	M1
	$= 11^2 + 1.5^2 \times 10^2 = 346$	A1
	$\mathrm{P}(Y < 0)$, $\quad = \mathrm{P}(Z < 0.48\ldots) =$ $\quad$ awrt **0.684 ~ 0.686**	M1, A1
	(6)	
		11 marks
(a)	2nd B1 for □= 22 or $\sigma^2 = 22^2$ or 484 M1 for standardising with their mean and standard deviation (ignore direction of inequality)	
(b)	1st M1 for attempting to find Y. Need to see $\pm(M - 1.5W)$ or equiv. May be implied by Var(Y). 1st A1 for a correct value for their E(Y) i.e. usually $\pm$ 9. Do not give M1A1 for a "lucky" $\pm$ 9. 2nd M1 for attempting Var(Y) e.g. $\ldots + 1.5^2 \times 10^2$ or $11^2 + 1.5^2 \times \ldots$ 3rd M1 for attempt to calculate the correct probability. Must be attempting a probability > 0.5. Must attempt to standardise with a relevant mean and standard deviation Using ${\sigma^2}_M = 11$ or ${\sigma^2}_W = 10$ is not a misread.	

Question number	Scheme	Marks
5. (a)	Only cleaners - no managers i.e. not all <u>types</u>. OR Not a random sample	B1g
	1^{st} 50 may be in same shift/group/share <u>same views</u>. OR Not a random sample	B1h (2)
	(Allow "not a representative sample" in place of "not a random sample")	
(b)(i)	Label employees (1-550) or obtain an ordered list	B1
	Select <u>first</u> using <u>random numbers</u> (from 1 - 11)	B1
	Then select every 11^{th} person from the list	B1
(ii)	Label managers (1-55) and cleaners (1-495)	M1
	Use random numbers to select…	M1
	…5 managers and 45 cleaners	A1 (6)
(c)	390, 372 (They must be in this order)	B1, B1 (2)
		10 marks
	After 1^{st} B1, comments should be in **context**, i.e. mention cleaners, managers, types of worker etc	
(a)	1^{st} B1g for one row	
	2^{nd} B1h for both rows. "Not a random sample" only counts once.	
	Score B1B0 or B1B1 or B0B0 on EPEN	
(b)(i)	1^{st} B1 for idea of labelling or getting an ordered list. No need to see 1-550.	
	2^{nd} B1 selecting first member of sample using random numbers (1-11 need not be mentioned)	
	3^{rd} B1 selecting every nth where $n = 11$.	
(ii)	1^{st} M1 for idea of <u>two</u> groups and labelling <u>both</u> groups. (Actual numbers used not required)	
	2^{nd} M1 for use of random numbers within each strata. Don't give for SRS from all 550.	
	"Assign random numbers to managers and cleaners" scores M0M1	
	A1 for 5 managers <u>and</u> 45 cleaners. (This mark is dependent upon scoring at least one M)	

Question	Scheme	Marks
6. (a)	$p = \dfrac{0\times 11 + 1\times 21 + ...}{10\times(11+21+...) \text{ or } 10\times 100}, = \dfrac{223}{1000} = 0.223$ **(*)** (Accept $\dfrac{223}{1000}$)	M1, A1cso (2)
(b)	$r = (0.8)^{10}\times 100 = 10.7374$ awrt **10.74**	M1A1
	$s = \binom{10}{2}(0.8)^8\times(0.2)^2\times 100 = 30.198\ldots$ awrt **30.2**	A1
	$t = 100 - [r + s + 26.84 + 20.13 + 8.81] =$ awrt **3.28** (4)	A1cao
(c)	H_0 : Binomial ([n =10], p = 0.2) is a suitable model for these data	B1
	H_1 : Binomial ([n =10], p = 0.2) is NOT a suitable model for these data (2)	B1
(d)	Since $t < 5$, the last two groups are combined	M1
	and $\nu = 4 = 5 - 1$	A1
	(2)	
(e)	Critical value $\chi_4^2(5\%) = 9.488$	B1
	Not significant or do not reject null hypothesis	M1
	The binomial distribution with p = 0.2 is a suitable model for the number of cuttings that do not grow	A1
	(3)	**13 marks**

	M1	Must show clearly how to get either 223 or 1000. As printed or better.
	A1cso	for showing how to get <u>both</u> 223 and 1000 and reaching p = 0.223
(b)	M1	for any correct method (a correct expression) seen for r or s.
	1st A1	for correct value for r awrt 10.74
	2nd A1	for s = awrt 30.2
	3rd A1	for t = 3.28 only
(c)		B1 for each. The value of p must be mentioned at least once. Accept B(10, 0.2)
		If hypotheses are correct but with no value of p then score B0B1
		Minimum is X~B(10, 0.2). If just B(10, 0.2) and not B(10, 0.2) award B1B0
(d)	M1	for combining groups (must be stated or implied by a new table with combined cell seen)
	A1	for the calculation 4 = 5 - 1
(e)	M1	for a correct statement based on 4.17 and their cv(context not required) (may be implied)
		Use of 4.17 as a critical value scores B0M0A0
	A1	for a correct interpretation in context and p =0.2 and cuttings mentioned.

Question number	Scheme	Marks
7. (a)	$H_0 : \mu_F = \mu_M$ $\quad$ $H_1 : \mu_F \neq \mu_M$ (Allow μ_1 and μ_2)	B1
	$z = \dfrac{6.86 - 5.48}{\sqrt{\dfrac{4.51^2}{200} + \dfrac{3.62^2}{100}}}$	M1 A1
	$= 2.860\ldots$ awrt ($\pm$)**2.86**	A1
	2 tail 5% critical value ($\pm$) 1.96 (or probability awrt 0.0021~0.0022) B1	
	Significant result or reject the null hypothesis (o.e.) M1	
	There is evidence of a difference in the (mean) amount spent on junk food by male and female teenagers	A1ft
	(7)	
(b)	CLT enables us to assume $\bar{F}$ and $\bar{M}$ are normally distributed B1	(1)
		8 marks
(a)	1st M1 for an attempt at $\dfrac{a-b}{\sqrt{\dfrac{c}{100 \text{ or } 200} + \dfrac{d}{100 \text{ or } 200}}}$ with 3 of *a, b, c* or *d* correct 1st A1 for a fully correct expression 2nd B1 for $\pm$ 1.96 <u>but</u> only if their H_1 is two-tail (it may be in words so B0B1 is OK) If H_1 is one-tail this is automatically B0 too. 2nd M1 for a correct statement based on comparison of their *z* with their cv. May be implied 3rd A1 for a correct conclusion in context based on their *z* and 1.96. Must mention <u>junk food</u> or <u>money</u> and <u>male vs female</u>.	
(b)	B1 for $\bar{F}$ or $\bar{M}$ mentioned. Allow "<u>mean</u> (amount spent on junk food) is <u>normally distributed</u>" Read the whole statement e.g. " original distribution is normal so mean is…" scores B0	

June 2008
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1 a	$\mathrm{E}(\theta_1) = \dfrac{\mathrm{E}(X_3)+\mathrm{E}(X_4)+\mathrm{E}(X_5)}{3}$	
	$= \dfrac{3\mu}{3}$	
	$= \mu$ Bias = 0 allow unbiased	B1
	$\mathrm{E}(\theta_2) = \dfrac{E(X_{10}) - E(X_1)}{3}$	
	$=$ 1/3([illegible]	
	$= 0$ Bias = $-\mu$ allow $\pm\mu$	B1,B1
	$\mathrm{E}(\theta_3) = \dfrac{3\mathrm{E}(X_1)+2\mathrm{E}(X_2)+\mathrm{E}(X_{10})}{6}$	
	$= \dfrac{3\mu+2\mu+\mu}{6}$	
	$= \mu$ Bias = 0 allow unbiased	B1 (4)
b	$\mathrm{Var}(\theta_1) = \dfrac{1}{9}\{(\mathrm{Var}\ X_2) + \mathrm{Var}(X_3) + \mathrm{Var}(X_4)\}$	M1
	$= \dfrac{1}{9}\{\sigma^2 + \sigma^2 + \sigma^2\}$	
	$= \dfrac{1}{3}\sigma^2$	A1
	$\mathrm{Var}(\theta_2) = \dfrac{2}{9}\sigma^2$	B1
	$\mathrm{Var}(\theta_3) = \dfrac{1}{36}\{9\sigma^2 + 4\sigma^2 + \sigma^2\}$	M1
	$= \dfrac{7}{18}\sigma^2$	A1 (5)
ci)	θ_1 is the better estimator. It has a lower var. and no bias	B1 depB1
ii)	θ_2 is the worst estimator. It is biased	B1 depB1 (4)

Question Number	Scheme	Marks
2 a	$H_1 : \sigma_A^2 = \sigma_B^2$ $H_0 : \sigma_A^2 \neq \sigma_B^2$	B1
	$S_A^2 = 22.5$ $s_B^2 = 21.6$ awrt	M1 A1A1
	$\frac{s_1^2}{s_2^2} = 1.04$	M1 A1
	$F_{(8, 6)} = 4.15$	B1
	$1.04 < 4.15$ do not reject H_0. The variances are the same.	B1 (8)
b	Assume the samples are selected at random, (independent)	B1 (1)
c	$s^2_p = \frac{8(22.5)+6(21.62)}{14} = 22.12$ awrt 22.1	M1 A1
	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A \neq \mu_B$	B1
	$t = \frac{40.667-39.57}{\sqrt{22.12}\sqrt{\frac{1}{9}+\frac{1}{7}}}$	M1
	$= 0.462$ 0.42 – 0.47	A1
	Critical value = $t_{14}(2.5\%) = 2.145$	B1
	$0.462 < 2.145$ No evidence to reject H_0. The means are the same	B1 (7)
d	Music has no effect on performance	B1 (1)

Question Number	Scheme	Marks
3	Differences 2.1 -0.7 2.6 -1.7 3.3 1.6 1.7 1.2 1.6 2.4	M1
	$\bar{d} = 1.41$	M1
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d > 0$	B1
	$s = \sqrt{\dfrac{40.65 - 10 \times 1.41^2}{9}} = 1.5191\ldots$	M1
	$t = \dfrac{1.41}{\left(\dfrac{1.519...}{\sqrt{10}}\right)} = 2.935\ldots$ awrt 2.94 /2.93	M1 A1
	$t_9\ (1\%) = 2.821$	B1
	2.935.. > 2.821 Evidence to reject H_0. There has been an increase in the mean weight of the mice.	B1ft
		(8)

2 sample test can score
M0 M0
B1 for $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$

M1 $\dfrac{9 \times 24.5 + 9 \times 17.16}{18}$

M0 A0
B1   2.552
B1 ft

ie 4/8

Question Number	Scheme	Marks
4a	$\bar{x} = 668.125 \quad s = 84.428$	M1 M1
	$T_7(5\%) = 1.895$	B1
	Confidence limits = $668.125 \pm \dfrac{1.895 \times 84.428}{\sqrt{8}}$	M1
	= 611.6 and 724.7	A1A1
	Confidence interval = (612, 725)	(6)
b	Normal distribution	B1 (1)
c	£650 is within the confidence interval. No need to worry.	B1 √ B1 √ (2)

Question Number	Scheme	Marks
5 a		
	Confidence interval = $\left(\frac{15\times 0.003}{27.488}, \frac{15\times 0.003}{6.262}\right)$	M1 B1B1
	= (0.00164, 0.00719)	A1 A1 (5)
b	0.07^2 =0.0049	M1
	0.0049 is within the 95% confidence interval.	A1
	There is no evidence to reject the idea that the standard deviation of the volumes is not 0.07 or The machine is working well.	A1 (3)

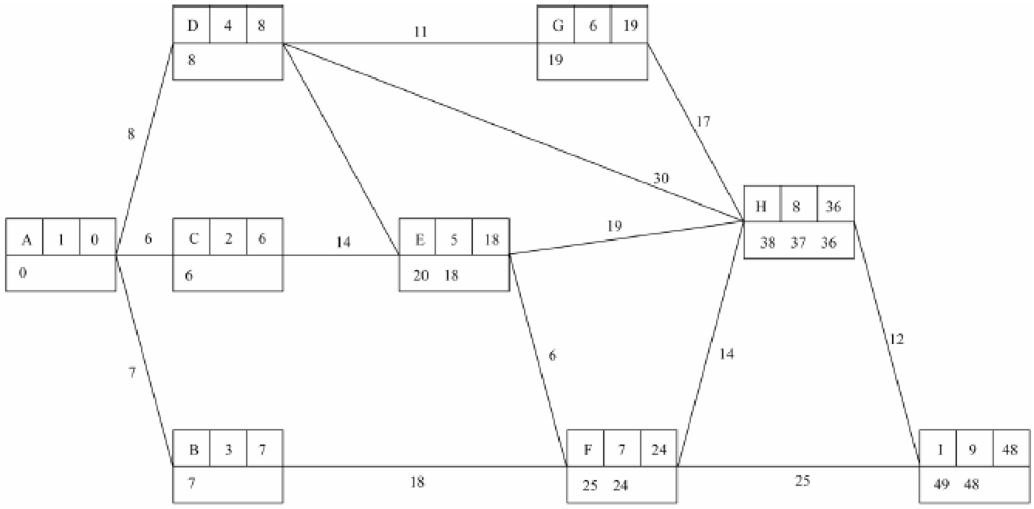
Question Number	Scheme Marks	
6 a	H_0 $p = 0.35$ $H_1 : p \neq 0.35$	B1 B1 (2)
b	Let X = Number cured then $X \sim B(20, 0.35)$	B1
	α = P(Type I error) = P($x \leq 3$) +P($x \geq 11$) given p= 0.35 = 0.0444 + 0.0532 = 0.0976	M1 A1 (3)
c	β = P(Type II error) = P($4 \leq x \leq 10$)	M1
	p 0.2 0.3 0.4 0.5 β 0.5880 0.8758 0.8565 0.5868	A1A1 (3)
d	Power = 1 – B 0.4120 0.1435	M1 A1 (2)
e	Not a good procedure. Better further away from 0.35 or This is not a very powerful test (power = 1 - β)	B1 B1dep (2)

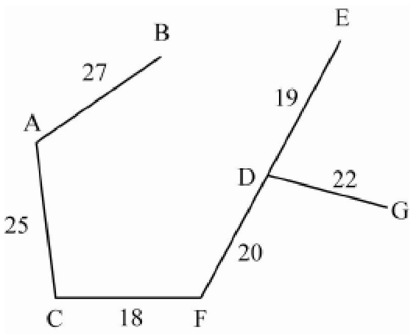
Question Number	Scheme	Marks
7 a	$H_0 : \mu = 230 \quad H_1 : \mu < 230$	B1
	$\nu = 9$	
	From table critical value = ± 1.833	B1√
	$\bar{x} = 228.3 \quad S = 17.858$	B1 B1
	$t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	M1
	$= \pm \dfrac{228.3 - 230}{\frac{17.858}{\sqrt{10}}} = \pm 0.301$	A1
	$\pm 0.301 > \pm 1.833$. No evidence to reject H_0. Mean is 230 N/mm^2	B1 (7)
b	Since the tensile strength is the same and the price is cheaper recommend use new supplier.	B1 (1)

June 2008
6689 Decision Mathematics D1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$\frac{502}{100} = 5.02$ so 6 tapes.	M1 A1 (2)
(b)	Bin 1: 29, 52 Bin 2: 73 Bin 3: 87 Bin 4: 74 Bin 5: 47, 38 Bin 6: 61 Bin 7: 41	M1 A1 A1 (3)
(c)	Bin 1: 87 Bin 2: 74 Bin 3: 73 Bin 4: 61, 38 Bin 5: 52, 47 Bin 6: 41, 29	M1 A1 A1 (3)
	Notes: (a) 1M1: $(502 \pm 40) \div 100$ (maybe implicit) 1A1: cao 6 tapes (b) 1M1: Bin 1 correct and at least 8 values put in bins 1A1: Condone one error, (e.g. extra, omission, 'balanced' swap). 2A1: All correct (c) 1M1: Bin 1 correct and at least 8 values put in bins 1A1: Condone one error, (e.g. extra, omission, 'balanced' swap). 2A1: All correct	**Total 8**

Question Number	Scheme	Marks
Q2		
(a)	G – 5 = W – 3 change status G = 5 – W = 3	M1 A1 (2)
(b)	A – no match E = 2 G = 5 R = 4 W = 3	A1 (1)
(c)	e.g. R is the only person who can do 1 and the only person who can do 4	B 2, 1, 0 (2)
(d)	A – 2 = E – 3 = W – 4 = R – 1 change status A = 2 – E = 3 – W = 4 – R = 1 A = 2 E = 3 G = 5 R = 1 W = 4	 M1 A1 A1 (3) **Total 8**
	Notes: (a) 1M1: Path from G to 3 1A1: CAO including change status (stated or shown), chosen path clear. (b) 2A1: CAO must ft from stated path (c) 1B1: Correct answer, may be imprecise or muddled (bod gets B1) but all nodes refered to must be correct. 2B1: Good, clear, correct answer. (d) 1M1: Path from A to 1 1A1: CAO including change status (stated or shown) but don't penalise twice. Chosen path clear. 1A1: CAO must ft from stated path **Misread** (remove last two A or B marks if earned.) A – 2 = E – 3 c.s. A=2 – E = 3 Matching A = 2, E = 3, R = 4 W = 5 Then G – 5 = W – 4 = R – 1 c.s. G = 5 – W = 4 – R =1 Matching A = 2, E = 3, G = 5, R = 1, W = 4	

Question Number	Scheme	Marks
Q3 (a)		M1 A1 A1ft
	Route: ADGHI Length: 48 (km)	A1 A1ft (5)
(b)	Odd vertices are A and H Attempt to find shortest route from A to H = ADGH New length: 197 + 36 = 233 Route: e.g. ADGHGDACEDHIFHEFBA (18)	B1 M1 A1ft A1 (4)
	Notes: (a) 1M1: Smaller number replacing larger number in the working values at E or F or H or I. (generous – give bod) 1A1: All values in boxes A to E and G correct 2A1ft: All values in boxes F, H and I correct (ft). Penalise order of labelling just once. 3A1: CAO (not ft) 4A1ft: Follow through from their I value, condone lack of units here. (b) 1B1: A and H identified in some way – allow recovery from M mark. 1M1: Accept, if correct, path, or its length. Accept attempt if finding shortest. 1A1ft: 197 + their shortest A to H (36) 2A1: A correct route.	**Total 9**

Question Number	Scheme	Marks
Q4 (a)	e.g. • Prims starts with any vertex, Kruskal starts with the shortest arc. • It is not necessary to check for cycles when using Prim. • Prims adds nodes to the growing tree, Kruskal adds arcs. • The tree 'grows' in a connected fashion when using Prim. • Prim can be used when data in a matrix form. Other correct statements also get credit.	B 2, 1, 0 (2)
(b)(i)	e.g. AC, CF, FD, DE, DG, AB.	M1, A1, A1 (3)
(ii)	CF, DE, DF, not CD, not EF, DG, not FG, not EG, AC, not AD, AB. [18, 19, 20, not 21, not 21 , 22 , not 23, not 24, 25, not 26, 27]	M1, A1, A1 (3) **Total 8**
	Notes: (a) 1B1: Generous one correct difference. If bod give B1 2B1: Generous two distinct, correct differences. (b) 1M1: Prim's algorithm – first three arcs chosen correctly, in order, or first four nodes chosen correctly, in order. 1A1: First five arcs chosen correctly; all 7 nodes chosen correctly, in order. 2A1: All correct and arcs chosen in correct order. 2M1: Kruskal's algorithm – first 4 arcs selected chosen correctly. 1A1: All six non-rejected arcs chosen correctly. 2A1: All rejections correct and in correct order and at correct time. 	

Question Number	Scheme	Marks
Q5		
(a)	$x = 9$, $y = 11$	B1,B1 (2)
(b)	AC DC DT ET	B2,1,0 (2)
(c)	36	B1 (1)
(d)	$C_1 = 49$, $C_2 = 48$, $C_3 = 39$	B1,B1,B1 (3)
(e)	e.g. SAECT	B1 (1)
(f)	maximum flow = minimum cut cut through DT, DC, AC and AE	M1 A1 (2)
		Total 11
	Notes: (a) 1B1: cao (permit B1 if 2 correct answers, but transposed) 2B1: cao (b) 1B1: correct (condone one error – omission or extra) 2B1: all correct (no omissions or extras) (c) 1B1: cao (d) 1B1: cao 2B1: cao 3B1: cao (e) 1B1: A correct route (flow value of 1 given) (f) 1M1: Must have attempted (e) and made an attempt at a cut. 1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of four.	

Question Number	Scheme	Marks
Q6 (a)		

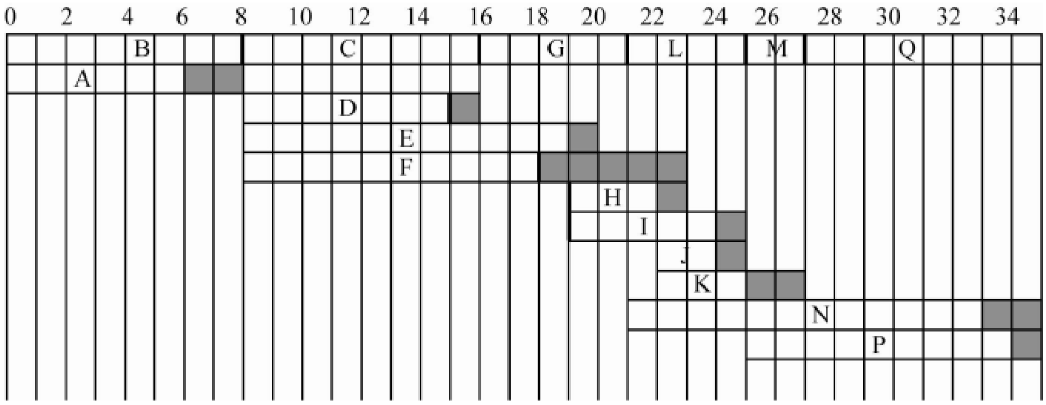
b.v	x	y	z	R	s	t	value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	-5	$-\frac{7}{2}$	-4	0	0	0	0

b.v	x	y	z	R	s	t	value	Row ops	Marks
r	0	$\frac{1}{3}$	$\frac{1}{2}$	1	0	-1	4	$R_1 - 4R_3$	M1 A1
s	0	$\frac{5}{2}$	$-\frac{1}{2}$	0	1	$-\frac{1}{4}$	1	$R_2 - R_3$	M1 A1ft A1
x	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{4}$	15	$R_3 \div 4$	
P	0	-1	$-\frac{3}{2}$	0	0	$\frac{5}{4}$	75	$R_4 + 5R_3$	

b.v	x	y	z	R	s	t	value	Row ops	Marks
z	0	$\frac{2}{3}$	1	2	0	-2	8	$R_1 \div \frac{1}{2}$	M1 A1ft
s	0	$\frac{17}{6}$	0	1	1	$-\frac{5}{4}$	5	$R_2 + \frac{1}{2}R_1$	M1 A1
x	1	$\frac{1}{6}$	0	-1	0	$\frac{5}{4}$	11	$R_3 - \frac{1}{2}R_1$	(9)
P	0	0	0	3	0	$-\frac{7}{4}$	87	$R_4 + \frac{3}{2}R_1$	

(b) There is still a negative number in the profit row. B1 (1)

Total 10

Question Number	Scheme	Marks
Q7		
(a)	v = 16 w = 25 x = 23 y = 20 z = 8	B3,2,1,0 (3)
(b)	B C G L M Q	B1 (1)
(c)	Float on H = 23ft – 19 – 3 = 1 Float on J = 25 – 22 – 2 = 1	B1 B1 (2)
(d)		M1 A1 A1 A1 (4)
(e)	E has one day of float, so project can still be completed on time.	B2,1,0 (2)
(f)	e.g • At time 23 ½ activities L, I, J and N must be taking place • At time 13 ½ or 14 ½ activities C, D, E and F must be taking place So 4 workers needed.	B2,1,0 (2)
		Total 14

Question Number	Scheme	Marks
Q8	Maximise (P=) $0.2a + 0.15b$ or $20a + 15b$ **o.e.**	B1 B1 (2)
	Subject to $a + b \leq 800$ $a \geq 2b$ $50 \leq b \leq 100$ $a \geq 0$	B1 B2,1,0 B1 B1 (5) **Total 7**
	Notes: 1B1: ‘Maximise’ 2B1: ratio of coefficients correct 3B1: cao 4B1: ratio of coefficients of a and b correct. 5B1: inequality correct way round i.e. □$a \geq$□b 6B1: cao accept < – accept two separate inequalities here 7B1: cao • Penalise < and > only once with last B mark earned • Be generous on letters a, b, A, B, x, y etc and mixed, but remove last B mark earned if inconsistent or 3 letters in the ones marked.	

June 2008
6690 Decision Mathematics D2
Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q1
(a)</td><td>A walk is a finite sequence of arcs such that the end vertex of one arc is the start vertex of the next.</td><td>B2,1,0</td></tr>
<tr><td>(b)</td><td>A tour is a walk that visits every vertex, returning to its stating vertex.</td><td>B2,1,0
(4)

Total 4</td></tr>
<tr><td></td><td>Notes:
(a) 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.
2B1: A good clear complete answer: End vertex=start vertex + finite.
(b) 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.
2B1: A good clear complete answer: Every vertex + return to start.

<u>From the D1 and D2 glossaries</u>
<u>D1</u>
A path is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, <u>and in which no vertex appears more than once</u>.

A cycle (circuit) is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

<u>D2</u>

A walk in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a tour.</td><td></td></tr>
</table>

Question Number	Scheme	Marks
Q2		
(a)	Total supply > total demand	B2,1,0 (2)
(b)	Adds 0, 0 and 5 to the dummy column	B2,1,0 (2)
(c)		B1 (1)

	L	E	D
A	35	20	
B		40	5

(d) M1 A1

		80	70	20
		L	E	D
0	A	35	20	
-20	B		40	5

$I_{AD} = 0 - 0 - 20 = -20$
$I_{BL} = 60 + 20 - 80 = 0$

A1 (3)

	L	E	D
A	35	20-θ	θ
B		40+θ	5-θ

M1

θ = 5 ; entering square is AD; exiting square is BD

		80	70	0
		L	E	D
0	A	35	15	5
-20	B		45	

A1ft
B1ft (2)

$I_{BL} = 60 + 20 - 80 = 0$
$I_{BD} = 0 + 20 - 0 = 20$

B1ft (2)

(e) Cost is (£) 6100 — B1 (1)

Total 13

Question Number	Scheme	Marks
Q3 (a)	Maximin : we seek a route where the shortest arc used is a great as possible. Minimax : we seek a route where the longest arc used is a small as possible.	B2,1,0 (2)
(b)		

Stage	State	Action	Dest.	Value
	G	GR	R	132*
1	H	HR	R	175*
	I	IR	R	139*
	D	DG	G	min (175,132) = 132
		DH	H	min (160,175) = 160*
2	E	EG	G	min (162,132) = 132
		EH	H	min (144,175) = 144*
		EI	I	min (102,139) = 102
	F	FH	H	min (145,175) = 145*
		FI	I	min (210,139) = 139
	A	AD	D	min (185,160) = 160*
		AE	E	min (279,144) = 144
3	B	BD	D	min (119,160) = 119
		BE	E	min (250,144) = 144*
		BF	F	min (123,145) = 123
	C	CE	E	min (240,144) = 144
		CF	F	min (170,145) = 145*
	L	LA	A	min (155,160) = 155*
4		LB	B	min (190,144) = 144
		LC	C	min (148,145) = 145

M1A1 (2)

M1A1

A1 (3)

M1A1ft

A1ft

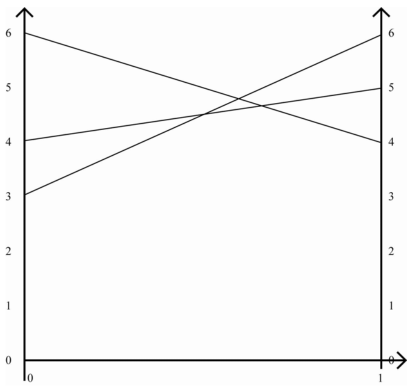
A1ft

Maximin route: LADHR

A1ft (5)

Total 12

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q4
(a)</td><td>For each row the element in column x must be less than the element in column y.</td><td>B2,1,0
(2)</td></tr>
<tr><td>
(b)</td><td>Row minimum {2,4,3} row maximin = 4
Column maximum {6,5,6} column minimax = 5
$4 \neq 5$ so not stable</td><td>
M1
A1
A1 (3)</td></tr>
<tr><td>(c)</td><td>Row 3 dominates row 1, so matrix reduces to
<table>
<tr><th></th><th>M1</th><th>M2</th><th>M3</th></tr>
<tr><td>L2</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>L3</td><td>6</td><td>4</td><td>3</td></tr>
</table></td><td>B1</td></tr>
<tr><td></td><td>Let Liz play 2 with probability p and 3 with probability (1- p)
If Mark plays 1: Liz’s gain is 4p + 6 (1- p) = 6 – 2p
If Mark plays 2: Liz’s gain is 5p + 4 (1- p) = 4 + p
If Mark plays 3: Liz’s gain is 6p + 3 (1- p) = 3 + 3p</td><td>M1

A1 (3)</td></tr>
<tr><td></td><td>
</td><td>B2, 1ft, 0
(2)</td></tr>
<tr><td></td><td>$4+p=6-2p$
$p=\frac{2}{3}$</td><td>M1
A1

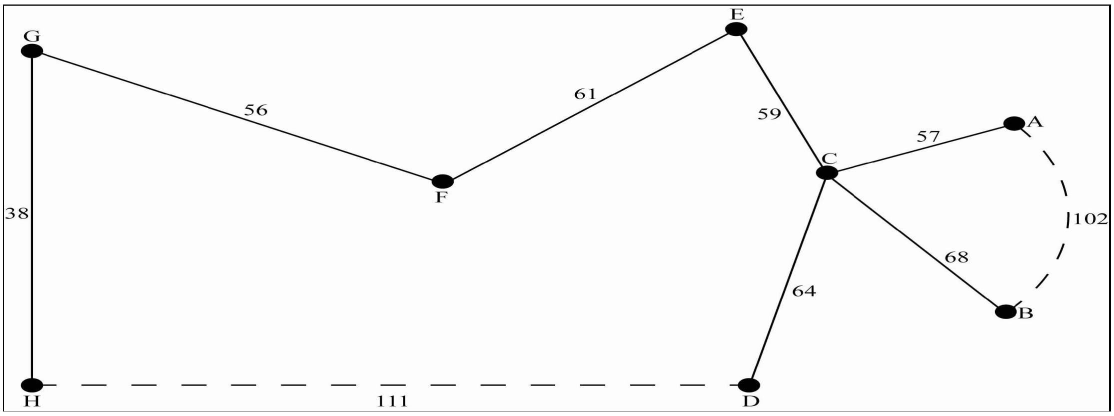
A1ft</td></tr>
<tr><td>
(d)</td><td>Liz should play row 1 – never, row 2 - $\frac{2}{3}$ of the time, row 3 - $\frac{1}{3}$ of the time
and the value of the game is $4\frac{2}{3}$ to her.
Row 3 no longer dominates row 1 and so row 1 can not be deleted.
Use Simplex (linear programming).</td><td>A1 (4)
B1
B1 (2)
Total 16</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Question	Scheme	
Q5 (a)	Since maximising, subtract all elements from some $n \geq 53$	
	$\begin{bmatrix} 5 & 4 & 11 & 11 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 6 & 3 & 7 & 10 \end{bmatrix}$	M1 A1 (2)
	Reduce rows $\begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 3 & 0 & 4 & 7 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 4 & 0 & 0 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 2 & 4 \end{bmatrix}$	M1 A1ft (2)
	Minimum element 1	M1
		A1ft
		A1ft (3)
	$\begin{bmatrix} 0 & 0 & 4 & 3 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$	M1
		A1ft
		A1ft (3)
(b)	$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 2 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$	M1 A1ft (2) M1A1(2) Total 14

Joe	A	A
Min-Seong	C	D
Olivia	D	B
Robert	B	C

Value £197 000

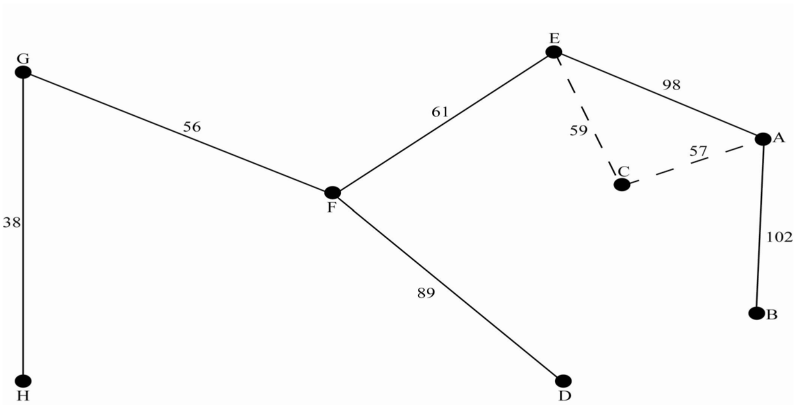
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>Q6
(a)</td><td>GH(38) GF(56) CA(57) EC(59) FE(61) CD(64) CB(68)</td><td>M1A1 (2)</td></tr>
<tr><td>(b)</td><td>2 x 403 = 806 (km)</td><td>B1 (1)</td></tr>
<tr><td>(c)</td><td>e.g. DH saves 167
AB saves 23
806 – 190 = 616 (km)



</td><td>M1 A1

A1

A1 (4)</td></tr>
<tr><td>(d)</td><td>eg A B C E F G H D C A</td><td>M1 A1
A1 (3)</td></tr>
<tr><td>(e)</td><td>B C A E F G H D B
68 +57 + 98 + 61 + 56 + 38 + 111 + 108 = 597 (km)</td><td></td></tr>
<tr><td>(f)</td><td>Delete C



RMST weight = 444
Lower bound = 444 + 59 + 57 = 560 (km)

560 < length ≤ 597</td><td>M1

A1

M1
A1ft (4)

B2,1,0 (2)
Total 16</td></tr>
</table>

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