GCE

Edexcel GCE

Mathematics

Further Pure Mathematics 2 FP2 (6675)

June 2008

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Mark Scheme (Final)

Edexcel GCE

Mathematics

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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June 2008
Further Pure FP2
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Question number	Scheme	Marks
1.	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\tanh x)) = \frac{\operatorname{sech}^2 x}{\tanh x}$	M1 A1
	$= \frac{1}{\sinh x \cosh x} = \frac{2}{\sinh 2x} = 2\operatorname{cosech} 2x$ (*)	M1 A1 (4)
		4
	Notes **1M1** Any valid differentiation attempt including $\ln(e^x - e^{-x}) - \ln(e^x + e^{-x})$ **1A1** c.a.o. (o.e e.g. $\frac{\cosh x}{\sinh x} - \frac{\sinh x}{\cosh x}$) **2M1** Proceeding to a hyperbolic expression in $2x$ **2A1** c.s.o.	

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2.	$8\left(\frac{e^x + e^{-x}}{2}\right) - 4\left(\frac{e^x - e^{-x}}{2}\right) = 13$	B1
	$4e^x + 4e^{-x} - 2e^x + 2e^{-x} = 13$	
	$2e^{2x} - 13e^x + 6 = 0$ (or equiv.)	M1 A1
	$(2e^x - 1)(e^x - 6) = 0$	
	$e^x = \frac{1}{2}, \quad e^x = 6$	M1 A1ft
	$x = \ln\frac{1}{2}$ (or $-\ln 2$), $\quad x = \ln 6$	A1 (6)
		6
	Notes	
	B1 Correctly substituting exponentials for all hyperbolics	
	1M1 To a three term quadratic in e^x	
	1A1 c.a.o. (o.e.)	
	2M1 Solving their equation to $e^x =$	
	2A1ft f.t. their equation.	
	3A1 c.a.o.	

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June 2008
Further Pure FP2
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3.	$\int \frac{3}{\sqrt{x^2-9}}\,\mathrm{d}x + \int \frac{x}{\sqrt{x^2-9}}\,\mathrm{d}x$		B1	
	$= \left[3\operatorname{arcosh}\frac{x}{3} + \sqrt{x^2-9}\right]$		M1 A1 A1	
	$= \left[3\ln\left(\frac{x+\sqrt{x^2-9}}{(3)}\right) + \sqrt{x^2-9}\right]_5^6$			
	$= \left(3\ln(\frac{6+\sqrt{27}}{3}) + \sqrt{27}\right) - \left(3\ln(\frac{5+4}{3}) + 4\right)$		M1 A1	
	$= 3\ln\frac{6+\sqrt{27}}{9} + \sqrt{27} - 4 = 3\ln\frac{2+\sqrt{3}}{3} + 3\sqrt{3} - 4$	(*)	A1	(7)
	Notes			**7**
	B1 Correctly changing to an integrable form. **1M1** Complete attempt to integrate at least one bit. **1A1** One term correct **2A1** All correct **2DM1** Substituting limits in all. **Must have got first M1** **3A1** Correctly (no follow through) **4A1** c.s.o.			

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June 2008
Further Pure FP2
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Question number	Scheme	Marks
4.	(a) $\frac{dy}{dx} = \frac{3x^2}{\sqrt{1+x^6}}$, At $x = \sqrt{2}$ $\frac{dy}{dx} = \frac{6}{3} = 2$	M1 A1, A1
	$y - \text{arsinh}\,(2\sqrt{2}) = 2(x - \sqrt{2})$	M1
	$y = 2x - 2\sqrt{2} + \ln\left(3 + 2\sqrt{2}\right)$ (*)	A1 (5)
	(b) $\frac{3a^2}{\sqrt{1+a^6}} = 2$ $9a^4 = 4(1+a^6)$	M1 A1
	$4a^6 - 9a^4 + 4 = 0$ $(a^2-2)(4a^4 - a^2 - 2) = 0$	A1
	$a^2 = \frac{1 \pm \sqrt{1+32}}{8}$ $a = \sqrt{\frac{1+\sqrt{33}}{8}} \approx 0.92$	M1 A1 (5)
		10
	Notes **(a)1M1** Attempt to differentiate need $\left(1+x^6\right)^{-\frac{1}{2}}$ at least **1A1** correct **2A1** c.a.o. **2M1** Substituting into straight line equation (linear). Must use $x = \sqrt{2}$ **3A1** c.s.o. **(b)1M1** Their derivative = their gradient (condone x throughout) **2M1= A mark cao, any form** **1A1** quartic cao **3M1** Solving their quartic to 'a' = **2A1** c.a.o. (a.w.r.t. 0.92 to 2dp)	

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June 2008
Further Pure FP2
Mark Scheme

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5.	(a) $I_n = \int_0^{\pi} e^x \sin^n x\,dx = \left[e^x \sin^n x\right] - \int e^x n \sin^{n-1} x \cos x\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right] + n\int e^x(-\sin^n x + (n-1)\cos x \sin^{n-2} x \cos x)\,dx$	M1 A1
	$\left[e^x \sin^n x - ne^x \sin^{n-1} x \cos x\right]_0^{\pi} = 0$	B1
	$I_n = -n\int e^x \sin^n x\,dx + n(n-1)\int \sin^{n-2} x(1-\sin^2 x)\,dx$	M1
	$I_n = -nI_n + n(n-1)I_{n-2} - n(n-1)I_n \qquad I_n = \frac{n(n-1)}{n^2+1} I_{n-2}$ (*)	M1 A1 (8)
	(b) $I_4 = \frac{4 \times 3}{17} I_2, \qquad = \frac{12}{17} \times \frac{2}{5} I_0$	M1, A1
	$I_0 = \int_0^{\pi} e^x\,dx = \left[e^x\right]_0^{\pi} = \ldots. \quad , \qquad I_4 = \frac{24}{85}\left(e^{\pi} - 1\right)$	M1, A1 (4)
		12

(a)1M1 Complete attempt to use parts once in the right direction need $\sin^{n-1} x$

1A1 cao

2M1 Attempt to use parts again with sensible choice of parts, not reversing. Need to be differentiating a product.

2A1 cao

1B1 both = 0 at some point. (doesn't need to be correct, must must =0)

3DM1 I_n = expressions in $\int e^x \sin^k x\,dx$ **Depends on 2nd M**

4DM1 Expresssion in I_n and I_{n-2} to $I_n =$. **Depends on 3rd M**

3A1 c.s.o.

(b)1M1 I_4 in terms of I_2

1A1 I_4 correctly in terms of I_0 [o.e.]

2M1 $\int e^x dx$

2A1 c.a.o for I_4 .

Question number	Scheme	Marks
6.	(a) $\int \cosh x \arctan(\sinh x)\,dx = \sinh x \arctan(\sinh x) - \int \sinh x \frac{\cosh x}{1+\sinh^2 x}\,dx$	M1 A1 A1
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)\ \ (+C)$	M1 A1 (5)
	Or: $\ldots\ldots\ldots\ldots - \int \tanh x\,dx$	
	$= \sinh x \arctan(\sinh x) - \ln(\cosh x)\ \ (+C)$ M1 A1	
	<u>Alternative:</u>	
	Let $t = \sinh x$, $\frac{dt}{dx} = \cosh x$, $\int \arctan t\,dt = t\arctan t - \int \frac{t}{1+t^2}\,dt$ M1 A1 A1	
	$= \ldots\ldots - \frac{1}{2}\ln(1+t^2)$ M1	
	$= \sinh x \arctan(\sinh x) - \frac{1}{2}\ln(1+\sinh^2 x)\ \ (+C)$ (or equiv.) A1	
	(b) $\frac{1}{10}\left[\sinh x \arctan(\sinh x) - \ln(\cosh x)\right]_0^2 = \ldots\ldots,$ 0.34 (*)	M1, A1 (2)
		7
	(a) <u>Alternative:</u>	
	Let $\tan t = \sinh x$, $\sec^2 t\frac{dt}{dx} = \cosh x$, $\int t\sec^2 t\,dt = t\tan t - \int \tan t\,dt$ M1 A1 A1	
	$= \ldots\ldots - \ln(\sec t)$ M1	
	$= \sinh x \arctan(\sinh x) - \ln\sqrt{1+\sinh^2 x}\ \ (+C)$ (or equiv.) A1	
	Notes **(a)1M1** Complete attempt to use parts **1A1** One term correct. **2A1** All correct. **2M1** All integration completed. Need a ln term. **3A1** c.a.o. (in x) o.e, any correct form, simplified or not **(b)1M1** Use of limits 0 and 2 and 1/10. **1A1** c.s.o.	

EDEXCEL
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7.	(a) $\frac{2x}{16}-\frac{2y}{9}\frac{dy}{dx}=0$ $\left[\frac{dx}{dt}=4\sec t\tan t,\ \frac{dy}{dt}=3\sec^2 t\right]$	M1 A1	
	$\frac{dy}{dx}=\frac{9x}{16y}=\frac{36\sec t}{48\tan t}=\frac{3}{4\sin t}$	M1 A1	
	$y-3\tan t=\frac{-4\sin t}{3}(x-4\sec t)$	M1	
	$4x\sin t+3y=25\tan t$ (*)	A1	(6)
	(b) Using $b^2=a^2(e^2-1)$: $ae=\sqrt{a^2+b^2}=5$ or $e=\frac{5}{4}$	M1 A1	
	P: $4\sec t=5$ $\cos t=\frac{4}{5}$	M1	
	Coordinates of P: $(4\sec t, 3\tan t)=\left(5,\frac{9}{4}\right)$	M1 A1	(5)
	(c) R: $x=\frac{25\tan t}{4\sin t}=\frac{125}{16}$	M1	
	Area of PRS: $\frac{1}{2}(SR\times SP)=\frac{1}{2}\times\left(\frac{125}{16}-5\right)\times\frac{9}{4}=\frac{405}{128}\left(=3\frac{21}{128}\right)$	M1 A1	(3)
		14	

Notes

(a)1M1 Differentitating
1A1 c.a.o.
2M1 $\frac{dy}{dx}$ in terms of t.
2A1 c.a.o.
3M1 Substituting gradient of **normal** into straight line equation.
3A1 c.s.o.
(b)1M1 Use of $b^2=a^2(e^2-1)$
1A1 c.a.o. for ae or for e
2M1 Using x coordinate of focus= x coordinate of P, to get single term $f(t)$= constant. (**Allow recovery in (c)**)
3M1 Substituting into P coordinates to a number for x and for y.
2A1 c.a.o.
(c)1M1 Attempt to find x coordinate of R.
2M1 Substituting into correct template i.e. ½ x |their R_x - their H_x| x their P_y
1A1 c.a.o. 3 s.f. or better.

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8.	(a) $\dot{x} = 3 + 3\cos t \quad \dot{y} = 3\sin t$		B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}} = \frac{\sin t}{1+\cos t} = \frac{2\sin {}^{t}\!/_{2} \cos {}^{t}\!/_{2}}{2\cos^2 {}^{t}\!/_{2}} = \tan\frac{t}{2}$	(*)	M1 A1	(3)
	(b) $s = \int \sqrt{\dot{x}^2 + \dot{y}^2}\,\mathrm{d}t = 3\sqrt{2}\int\sqrt{1+\cos t}\,\mathrm{d}t$		M1 A1ft	
	$= 6\int_0^t \cos\frac{t}{2}\mathrm{d}t = 12\sin\frac{t}{2}$ (Limits or establish $C = 0$ for A1)	(*)	M1 A1	(4)
	(c) $\tan\psi = \tan\frac{t}{2} \Rightarrow \psi = \frac{t}{2} \Rightarrow s = 12\sin\psi$		B1	(1)
	(d) Surface area $= \int_0^t 2\pi y\sqrt{\dot{x}^2 + \dot{y}^2}\,\mathrm{d}t = 18\sqrt{2}\pi\int(1-\cos t)\sqrt{1+\cos t}\,\mathrm{d}t$		M1 A1ft	
	$= 72\pi\int\sin^2\frac{t}{2}\cos\frac{t}{2}\,\mathrm{d}t$		M1	
	$= \ldots\ldots\ldots\left(\frac{2}{3}\sin^3\frac{t}{2}\right)$		M1 A1	
	But $\sin\frac{t}{2} = \frac{s}{12} = \frac{L}{12}$, so surface area $= \frac{144\pi}{3}\times\frac{L^3}{12^3} = \frac{\pi L^3}{36}$	(*)	M1 A1	(7)

(a)1B1 both
1M1 Attempt at y'/x'
1A1 cso – on paper need to see half angles
(b)1M1 Attempt at arc length, integral formula
1A1 cao follow through on their x' and y' **one variable only**
2M1 Integrating
2A1 cso – on paper
(c) 1B1 cao
(d) 1M1 Attempt at Surface area, integral formula.Condone lack of 2π.
1A1 cao follow through on their x' and y' condone lack of 2π. **one variable only**
2DM1 Getting to integrable form condone lack of 2π. **Depends on previous M mark.**
3DM1integrating condone lack of 2π. **Depends on previous M mark.**
2A1 cao
4DM1 Eliminating t to give expression in L only **Depends on previous M mark.**
3A1 cso – on paper.

Alternative solution for 8d (from Charles)

$$S = 2\pi\int y ds$$
$$= 2\pi\int (3-3\cos 2\psi)(12\cos\psi)d\psi$$
$$= 2\pi\int (36\cos\psi - 36\cos\psi\cos 2\psi)d\psi$$
$$= 72\pi\int \cos\psi(1-\cos 2\psi)d\psi$$
$$= 72\pi\int \cos\psi.2\sin^2\psi d\psi$$
$$= 72\pi.\frac{2}{3}\sin^3\psi$$
$$= 48\sin^3\frac{t}{2}$$
$$= 48\pi\frac{L^3}{12^3}$$
$$= \frac{\pi L^3}{36}$$