GCE

Edexcel GCE

Mathematics

Core Mathematics C4 (6666)

June 2008

advancing learning, changing lives

Mark Scheme (Final)

Edexcel GCE

Mathematics

edexcel

June 2008
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	<table><tr><td>x</td><td>0</td><td>0.4</td><td>0.8</td><td>1.2</td><td>1.6</td><td>2</td></tr><tr><td>y</td><td>e^0</td><td>$e^{0.08}$</td><td>$e^{0.32}$</td><td>$e^{0.72}$</td><td>$e^{1.28}$</td><td>e^2</td></tr><tr><td>or y</td><td>1</td><td>1.08329…</td><td>1.37713…</td><td>2.05443…</td><td>3.59664…</td><td>7.38906…</td></tr></table>		
		Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)	B1
			[1]
(b) **Way 1**	$\text{Area} \approx \frac{1}{2}\times 0.4 ; \times\underline{\left[e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2 \right]}$	Outside brackets $\frac{1}{2}\times 0.4$ or 0.2	B1;
		<u>For structure of trapezium rule</u> $\underline{[\ \ldots\ldots\ldots\ldots\ ]}$;	<u>M1</u> √
	$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)	<u>4.922</u>	A1 **cao**
			[3]
Aliter (b) **Way 2**	$\text{Area} \approx 0.4\times\left[\frac{e^0+e^{0.08}}{2} + \frac{e^{0.08}+e^{0.32}}{2} + \frac{e^{0.32}+e^{0.72}}{2} + \frac{e^{0.72}+e^{1.28}}{2} + \frac{e^{1.28}+e^2}{2}\right]$	0.4 and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	<u>M1</u> √
	$\text{Area} \approx \frac{1}{2}\times 0.4 ; \times\underline{\left[e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2 \right]}$		
	$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)	<u>4.922</u>	A1 **cao**
			[3]
			4 marks

Note an expression like $\text{Area} \approx \frac{1}{2}\times 0.4 + e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the [ ] brackets for M1.

The M1 mark for structure is for the material found in the curly brackets ie
$\underline{\left[\text{ first } y \text{ ordinate} + 2(\text{intermediate ft } y \text{ ordinate}) + \text{final } y \text{ ordinate }\right]}$

Question Number	Scheme		Marks
2. (a)	$\left\{\begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$		
	$\int xe^x \, dx = xe^x - \int e^x.1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.) Correct expression. (Ignore dx)	M1 A1
	$= xe^x - \int e^x \, dx$		
	$= xe^x - e^x \; (+c)$	Correct integration with/without + c	A1
			[3]
(b) **Way 1**	$\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$		
	$\int x^2 e^x \, dx = x^2 e^x - \int e^x.2x \, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore dx)	M1 A1
	$= x^2 e^x - 2\int xe^x \, dx$		
	$= x^2 e^x - 2\left(xe^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW**
			[3]
	$\left\{\begin{array}{l} = x^2 e^x - 2xe^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{array}\right\}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example.

+ c is not required in part (a).
+ c is required in part (b).

Question Number	Scheme		Marks
Aliter **2.** (b) **Way 2**	$\left\{\begin{array}{lll} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = xe^x & \Rightarrow & v = xe^x - e^x \end{array}\right\}$		
	$\int x^2 e^x \, dx = x(xe^x - e^x) - \int (xe^x - e^x)\, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore dx)	M1 A1
	$= x(xe^x - e^x) + \int e^x \, dx - \int xe^x \, dx$		
	$= x(xe^x - e^x) + e^x - \int xe^x \, dx$		
	$= x(xe^x - e^x) + e^x - (xe^x - e^x) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW** **[3]**
	$\left\{\begin{array}{l} = x^2 e^x - xe^x + e^x - xe^x + e^x + c \\ = x^2 e^x - 2xe^x + 2e^x + c \end{array}\right\}$	*Ignore subsequent working*	

Question Number	Scheme		Marks
3. (a) **Way 1**	From question, $\frac{dA}{dt} = 0.032$	$\frac{dA}{dt} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}$; $\left\{= \frac{0.016}{\pi x}\right\}$	$0.032 \div$ Candidate's $\frac{dA}{dx}$;	M1;
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$		
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b) **Way 1**	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	$\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 . \left(\frac{0.016}{\pi x}\right)$; $\{= 0.24x\}$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$;	M1 $\sqrt{}$
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\text{s}^{-1})$	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Part (b): Remember to give this mark for correct differentiation of V with respect to x. The first B1 in part (b) can be implied by a candidate writing down $\frac{dV}{dx} = 15\pi x^2$.

Part (a): $0.032 \div$ Candidate's $\frac{dA}{dx}$ can imply the first B1.

Part (b): FOR THIS QUESTION ONLY: It is possible to award any or both of the B1 B1 marks **in part (b)** for working also seen in part (a), BUT if you do this it must be clear **in (a)** that V is assigned to $\underline{\pi x^2(5x)}$ or or $\underline{5\pi x^3}$.

Allow $x \equiv r$, but a mixture of variables like $V = \pi x^2(5r)$ is not appropriate. However, $V = \pi r^2(5r)$ is okay.

Question Number	Scheme		Marks
***Aliter* 3. (a) Way 2**	From question, $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.032$	$\frac{\mathrm{d}A}{\mathrm{d}t} = 0.032$ seen or implied from working.	B1
	Integrating gives, $A = 0.032t\ (+c)$		
	$A = \pi x^2 \Rightarrow \pi x^2 = 0.032t\ (+c)$		
	Differentiating gives, $2\pi x \frac{\mathrm{d}x}{\mathrm{d}t} = 0.032$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = (0.032)\frac{1}{2\pi x};\ = \frac{0.016}{\pi x}$	Candidate's $\frac{\mathrm{d}A}{\mathrm{d}t} \div \frac{\mathrm{d}A}{\mathrm{d}x}$;	M1
	When $x = 2\,\text{cm}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{0.016}{2\pi}$		
	Hence, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
***Aliter* 3. (b) Way 2**	$V = \pi x^2 h \Rightarrow V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$ or $\underline{V = A.5\sqrt{\frac{A}{\pi}}}$	B1
	$\underline{V = A.5\sqrt{\frac{A}{\pi}}} \Rightarrow \underline{V = \frac{5}{\sqrt{\pi}}A^{\frac{3}{2}}}$		
	$\frac{\mathrm{d}V}{\mathrm{d}A} = \frac{15}{2\sqrt{\pi}}A^{\frac{1}{2}}$	$\frac{\mathrm{d}V}{\mathrm{d}A} = \frac{15}{2\sqrt{\pi}}A^{\frac{1}{2}}$ or ft from candidate's V	B1 $\sqrt{}$
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{15}{2\sqrt{\pi}}A^{\frac{1}{2}}.(0.032)\ ;\ = \left\{\frac{0.24}{\sqrt{\pi}}A^{\frac{1}{2}}\right\}$	Candidate's $\frac{\mathrm{d}V}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t}$;	M1 $\sqrt{}$
	When $x = 2\,\text{cm}$, $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{0.24}{\sqrt{\pi}}\sqrt{\pi}\,(2) = \underline{0.48}$ (cm^3 s^{-1})	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]

In this question there are some other valid ways to arrive at the answer. If you are unsure of how to apply the mark scheme for these ways then send these items up to review for your team leader to look at.

Question Number	Example	
3. (a) **EG 1**	**WARNING:** 0.00255 does not necessarily mean 4 marks!! a) $\frac{dA}{dt} = 0.032.$ ~~$A = \pi x^2 5x$~~ ~~$A = 5x^2\pi$~~ ~~$\frac{dA}{dx} = 1$~~ ~~$\frac{dx}{dA}$~~ $\frac{dx}{dt} = \frac{dx}{dA} \times \frac{dA}{dt}$ ~~$= \frac{1}{10x\pi} \times 0.032$~~ ~~$\frac{dx}{dt} = \left(\frac{1}{20\pi}\right)(0.032)$~~ ~~$\frac{dx}{dt} =$~~ $A = 2\pi r^2 h$ $A = 2\pi x^2 5x$ $= 10x^2\pi.$ $\frac{dA}{dx} = 20x\pi.$ ~~$\frac{dx}{dA}$~~ $\frac{dx}{dt} = \frac{dx}{dA} \times \frac{dA}{dt}$ $= \frac{1}{20x\pi} \times 0.032$ $x = r = 2.$ $\frac{dx}{dt} = \left(\frac{1}{40\pi}\right)(0.032)$ $\frac{dx}{dt} = 0.0002546479$ $\frac{dx}{dt} = 0.000255$ (3.s.f)	
	Comment: EG 1 scores B1B0M1A0	
EG 2	(a) $\frac{dA}{dt} = 0.032$ $\frac{dx}{dt} = (0.032)\frac{1}{\pi x^2} = \frac{0.032}{\pi x^2}$ When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.032}{4\pi} = 0.00255$	
	Comment: EG 2 scores B1B0M0A0	

Question Number	Scheme		Marks
4. (a) **Way 1**	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$		
	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \times \right\} \quad \underline{6x - 2y\frac{dy}{dx}} + \underline{\underline{\left(y + x\frac{dy}{dx}\right)}} = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $\underline{\underline{(\ )}}$ of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	$\underline{\text{A1}}$
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b) **Way 1**	At P & Q, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the y terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	$\underline{\text{A1}}$
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	$\underline{\text{A1}}$
			[3]
			9 marks

To award the final A1 mark you need to be convinced that the candidate has both coordinates. There must be link (albeit implied) between $x = 2$ and $y = 4$; and between $x = -2$ and $y = -4$. If you see extra points stated in addition to these two then award A0.

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 2**	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\} \quad \underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ *or* $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $(\;)$ of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	$\underline{\text{A1}}$
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow 6x - 2y\left(\frac{8}{3}\right) + y + x\left(\frac{8}{3}\right) = 0$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1 $*$
	giving $\frac{26}{3}x - \frac{13}{3}y = 0$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1 $*$
	giving $26x - 13y = 0$		
	Hence, $13y = 26x \Rightarrow y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
Aliter (b) **Way 2**	At P & Q, $x = \frac{y}{2}$. Substituting into eqn $*$		
	gives $3\left(\frac{y}{2}\right)^2 - y^2 + \left(\frac{y}{2}\right)y = 4$	Attempt replacing x by $\frac{y}{2}$ **in at least one** of the y terms in eqn $*$	M1
	Gives $\frac{3}{4}y^2 - y^2 + \frac{1}{2}y^2 = 4$		
	Simplifying gives, $y^2 = 16 \Rightarrow \underline{y = \pm 4}$	Either $y = 4$ or $y = -4$	$\underline{\text{A1}}$
	$x = \frac{y}{2} \Rightarrow x = \pm 2$		
	Hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$	Both $\underline{(2, 4)}$ and $\underline{(-2, -4)}$	$\underline{\text{A1}}$
			[3]

To award the final A1 mark you need to be convinced that the candidate has both coordinates. There must be link (albeit implied) between $x = 2$ and $y = 4$; and between $x = -2$ and $y = -4$. If you see extra points stated in addition to these two then award A0.

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 3**	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\} \quad 6x - 2y\frac{dy}{dx} + \left(y + x\frac{dy}{dx}\right) = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $(\ )$ of product rule	B1
		$(3x^2 - y^2) \to \left(6x - 2y\frac{dy}{dx}\right)$ and $(4 \to 0)$	A1
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$y = 2x \Rightarrow \frac{-6x - 2x}{x - 2(2x)} = \frac{dy}{dx}$	Substituting $y = 2x$ into their equation.	M1 *
	giving $\frac{dy}{dx} = \frac{-8x}{-3x}$	Attempt to combine x terms together.	dM1 *
	giving $\frac{dy}{dx} = \frac{8}{3}$	simplifying to give $\frac{dy}{dx} = \frac{8}{3}$ **AG**	A1 **cso**
			[6]

Very very few candidates may attempt ***partial*** differentiation. Please send these items to your team leader via review.

Question Number	Scheme	Marks
5. (a) **Way 1**	** represents a constant (which must be consistent for first accuracy mark)	
	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ $\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	B1
	Expands $(1+{**}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({**}x)$;	M1;
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})({**}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2+\ldots}\right]$ **with** $** \neq 1$ A correct simplified or an un-simplified $[\ \underline{\ldots\ldots\ldots}\ ]$ expansion with candidate's followed through $({**}x)$	A1 $\sqrt{}$
	Award SC M1 if you see $(-\frac{1}{2})({**}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2$	
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+\ldots}\right]$	
	$= \frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+\ldots\right]$ $\frac{1}{2}\left[1+\frac{3}{8}x;\ldots\right]$	A1 isw
	SC: $K\left[1+\frac{3}{8}x+\frac{27}{128}x^2+\ldots\right]$ $\frac{1}{2}\left[\ldots\ldots\ldots;\ \frac{27}{128}x^2\right]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+\ldots\right\}$ *Ignore subsequent working*	
		[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\ldots\right)$ Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+\ldots\ldots$ $+4+\frac{3}{2}x+\frac{27}{32}x^2+\ldots\ldots$ Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$= 4+2x;+\frac{33}{32}x^2+\ldots$ Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
		[4]
		9 marks

(a) You would award B1M1A0 for $= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-3x)^2+\ldots}\right]$ because ** is not consistent.

(a) If you see the constant term "$\frac{1}{2}$" in a candidate's final binomial expansion, then you can award B1.

Question Number	Scheme		Marks
Aliter **5.** (a) **Way 2**	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}}$		
		$\frac{1}{2}$ or $(4)^{-\frac{1}{2}}$ **(See note ↓)**	B1
	$= \left[\underline{(4)^{-\frac{1}{2}} + (-\frac{1}{2})(4)^{-\frac{3}{2}}(**x); + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(4)^{-\frac{5}{2}}(**x)^2 +} \right]$	Expands $(4-3x)^{-\frac{1}{2}}$ to give an un-simplified or simplified $(4)^{-\frac{1}{2}} + (-\frac{1}{2})(4)^{-\frac{3}{2}}(**x)$;	M1;
	with $** \neq 1$	A correct un-simplified or simplified [..........] expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= \left[\underline{(4)^{-\frac{1}{2}} + (-\frac{1}{2})(4)^{-\frac{3}{2}}(-3x); + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(4)^{-\frac{5}{2}}(-3x)^2 +} \right]$	Award SC M1 if you see $(-\frac{1}{2})(4)^{-\frac{3}{2}}(**x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(4)^{-\frac{5}{2}}(**x)^2$	
	$= \left[\frac{1}{2} + (-\frac{1}{2})(\frac{1}{8})(-3x) + (\frac{3}{8})(\frac{1}{32})(9x^2) + ... \right]$		
	$= \frac{1}{2} + \frac{3}{16}x; + \frac{27}{256}x^2 + ...$	Anything that cancels to $\frac{1}{2} + \frac{3}{16}x$; Simplified $\frac{27}{256}x^2$	A1; A1
			[5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

> If you see the constant term "$\frac{1}{2}$" in a candidate's final binomial expansion, then you can award B1.

> **Note**: In part (b) it is possible to award M1M0A1A0.

edexcel

Question Number	Scheme		Marks
6. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\begin{array}{lll}\mathbf{i}: & -9+2\lambda=3+3\mu & (1)\\ \mathbf{j}: & \lambda=1-\mu & (2)\\ \mathbf{k}: & 10-\lambda=17+5\mu & (3)\end{array}$	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda=1--2=3$	Both $\underline{\lambda=3}$ & $\underline{\mu=-2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively**. This mark can be implied by any two correct components of $(-3, 3, 7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3, 3, 7)$	A1
	Either check **k:** $\lambda=3$: LHS $=10-\lambda=10-3=7$ $\mu=-2$: RHS $=17+5\mu=17-10=7$ (As LHS = RHS then the lines intersect.)	**Either** check that $\lambda=3$, $\mu=-2$ in a third equation **or** check that $\lambda=3$, $\mu=-2$ give the same coordinates on the other line. Conclusion not needed.	B1 **[6]**
(b) **Way 1**	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times3)+(1\times-1)+(-1\times5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1 **[2]**

Question Number	Scheme		Marks
6. (c) **Way 1**	Equating $\mathbf{i}$; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$		
	$\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ $(= \overrightarrow{OA}$. Hence the point A lies on l_1.)	Substitutes candidate's $\lambda = 7$ into the line l_1 and finds $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$. The conclusion on this occasion is not needed.	B1
			[1]
Aliter (c) **Way 2**	At A; $\underline{-9 + 2\lambda = 5}$, $\underline{\lambda = 7}$ & $\underline{10 - \lambda = 3}$ gives $\lambda = 7$ for all three equations. (Hence the point A lies on l_1.)	Writing down all three <u>underlined equations</u> and finds $\lambda = 7$ for all three equations. The conclusion on this occasion is not needed.	B1
			[1]
(d) **Way 1**	Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \underline{\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}\right)$	M1 $\sqrt{}$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2\left(\text{their } \overrightarrow{AX}\right)$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	$\underline{\begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$	A1
			[3]
			12 marks

Question Number	Scheme		Marks
Aliter **6.** (d) **Way 2**	Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \underline{\begin{pmatrix}-3\\3\\7\end{pmatrix} - \begin{pmatrix}5\\7\\3\end{pmatrix}} = \underline{\begin{pmatrix}-8\\-4\\4\end{pmatrix}}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix}-3\\3\\7\end{pmatrix} - \begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1 $\sqrt{\ }$ ±
	$\overrightarrow{OB} = \overrightarrow{OX} + \overrightarrow{XB} = \overrightarrow{OX} + \overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix}-3\\3\\7\end{pmatrix} + \begin{pmatrix}-8\\-4\\4\end{pmatrix}$	$\begin{pmatrix}\text{their } \overrightarrow{OX}\end{pmatrix} + \begin{pmatrix}\text{their } \overrightarrow{AX}\end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$	A1
			[3]
Aliter (d) **Way 3**	At A, $\lambda = 7$. At X, $\lambda = 3$.		
	Hence at B, $\lambda = 3 - (7 - 3) = -1$	$\lambda_B = (\text{their } \lambda_X) - (\text{their } \lambda_A - \text{their } \lambda_X)$ $\lambda_B = 2(\text{their } \lambda_X) - (\text{their } \lambda_A)$	M1 $\sqrt{\ }$
	$\overrightarrow{OB} = \begin{pmatrix}-9\\0\\10\end{pmatrix} - 1\begin{pmatrix}2\\1\\-1\end{pmatrix}$	Substitutes their value of λ into the line l_1.	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$	A1
			[3]

Question Number	Scheme		Marks
Aliter **6.** (d) **Way 4**	$\overrightarrow{OA} = 5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$		
	$\begin{pmatrix}5\\7\\3\end{pmatrix} \rightarrow \begin{pmatrix}\text{Minus } 8\\ \text{Minus } 4\\ \text{Plus } 4\end{pmatrix} \rightarrow \begin{pmatrix}-3\\3\\7\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\left(\overrightarrow{AX} =\right) \pm \left(\begin{pmatrix}-3\\3\\7\end{pmatrix} - \begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1 $\sqrt{\ }$ ±
	$\begin{pmatrix}-3\\3\\7\end{pmatrix} \rightarrow \begin{pmatrix}\text{Minus } 8\\ \text{Minus } 4\\ \text{Plus } 4\end{pmatrix} \rightarrow \begin{pmatrix}-11\\-1\\11\end{pmatrix}$	$\begin{pmatrix}\text{their } \overrightarrow{OX}\end{pmatrix} + \begin{pmatrix}\text{their } \overrightarrow{AX}\end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$	A1 **[3]**
Aliter (d) **Way 5**	$\overrightarrow{OA} = 5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ As X is the midpoint of AB, then		
	$(-3, 3, 7) = \left(\frac{5+a}{2}, \frac{7+b}{2}, \frac{3+c}{2}\right)$	Writing down any two of these "equations" correctly.	M1 $\sqrt{\ }$
	$a = 2(-3) - 5 = -11$ $b = 2(3) - 7 = -1$ $c = 2(7) - 3 = 11$	An attempt to find at least two of a, b or c.	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}}$ or $\underline{(-11, -1, 11)}$ or $a = -11, b = -1, c = 11$	A1 **[3]**

Question Number	Scheme		Marks
Aliter **6.** (b) **Way 2**	$\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}\ ,\ \mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}\ \ \&\ \theta$ is angle		
	$\cos\theta = \dfrac{\mathbf{d}_1 \bullet \mathbf{d}_2}{\left(\lvert\mathbf{d}_1\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix}2\\1\\-1\end{pmatrix} \bullet \begin{pmatrix}3\\-1\\5\end{pmatrix}}{\left(\sqrt{(2)^2+(1)^2+(-1)^2}\,.\sqrt{(3)^2+(-1)^2+(5)^2}\right)}$		
	$\cos\theta = \dfrac{6-1-5}{\left(\sqrt{(2)^2+(1)^2+(-1)^2}\,.\sqrt{(3)^2+(-1)^2+(5)^2}\right)}$ ←	Dot product calculation between the ***two direction vectors***: $\underline{(2\times 3)+(1\times -1)+(-1\times 5)}$	M1
	$\cos\theta = 0 \Rightarrow \underline{\theta = 90^\circ}$ or <u>lines are perpendicular</u>	$\cos\theta = 0$ and $\underline{\theta = 90^\circ}$ or <u>lines are perpendicular</u>	A1 **cao**
			[2]

Question Number	Scheme		Marks
7. (a) **Way 1**	$\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}}$	$\underline{\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}}$, aef	<u>A1</u> **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)		**[3]**
Aliter **7.** (a) **Way 2**	$\frac{2}{4-y^2} \equiv \frac{-2}{y^2-4} \equiv \frac{-2}{(y-2)(y+2)} \equiv \frac{A}{(y-2)} + \frac{B}{(y+2)}$		
	$-2 \equiv A(y+2) + B(y-2)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $-2 = B(-4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $-2 = A(4) \Rightarrow A = -\frac{1}{2}$	Either one of $A=-\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\frac{-\frac{1}{2}}{(y-2)} + \frac{\frac{1}{2}}{(y+2)}}$	$\underline{\frac{-\frac{1}{2}}{(y-2)} + \frac{\frac{1}{2}}{(y+2)}}$, aef	<u>A1</u> **cao**
			[3]
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)		

Note also that: $2 \equiv A(y-2) + B(-y-2)$ gives $A = -\frac{1}{2}, B = -\frac{1}{2}$

Note: that the partial fraction needs to be correctly stated for the final A mark in part (a). This partial fraction must be stated in part (a) and cannot be recovered from part (b).

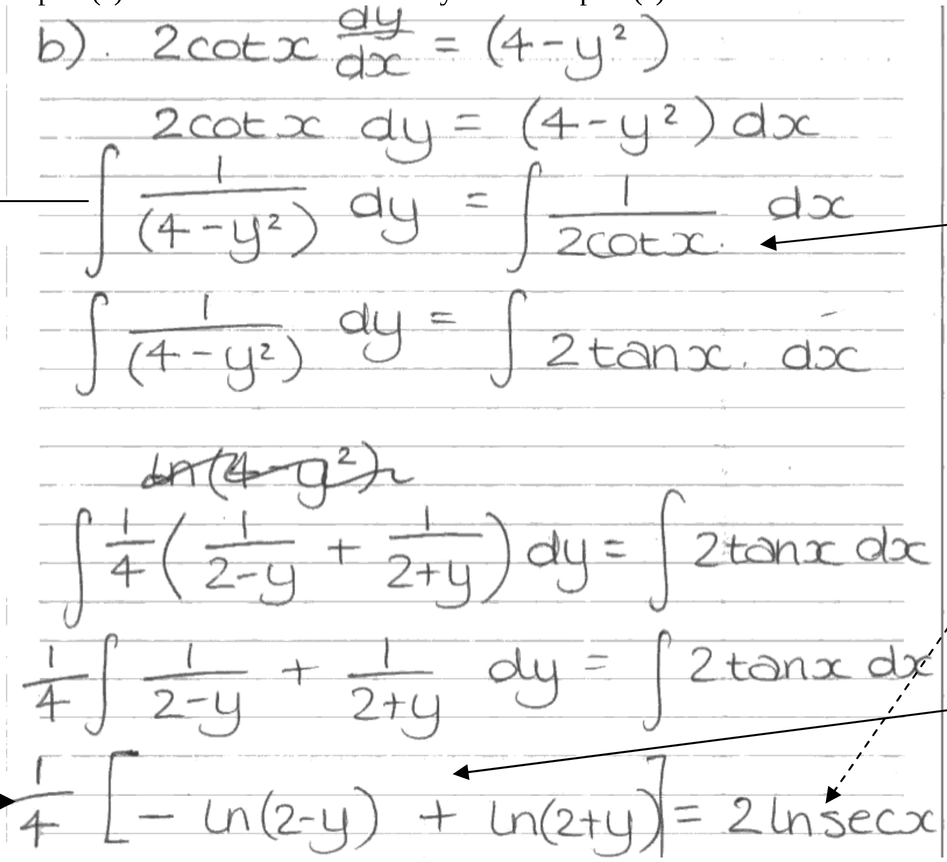
Question Number	Scheme		Marks
7. (b) **Way 1**	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
	$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) \quad + (c)$	their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y=0,\ x=\frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

Note: This M1 mark for finding c appears as B1 on ePEN.

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 2**	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) + c$	their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$	A1$\sqrt{}$
	$\Rightarrow -\ln(2-y) + \ln(2+y) = 2\ln(\sec x) + c$		
	See below for the award of M1	***decide to award M1 here!!***	M1
	$\Rightarrow\ \ln\left(\frac{2+y}{2-y}\right) = 2\ln(\sec x) + c$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\Rightarrow\ \ln\left(\frac{2+y}{2-y}\right) = \ln(\sec x)^2 + c$		
	$\Rightarrow\ \ln\left(\frac{2+y}{2-y}\right) = \ln(\sec x)^2 + \ln K$		
	$\Rightarrow\ \ln\left(\frac{2+y}{2-y}\right) = \ln\left(K(\sec x)^2\right)$	Using the log laws correctly to obtain a single log term on both sides of an equation which includes a constant of integration.	M1
	$\Rightarrow\ \left(\frac{2+y}{2-y}\right) = K\sec^2 x$		
	$y=0,\ x=\frac{\pi}{3}\ \Rightarrow\ 1 = \frac{K}{\cos^2\left(\frac{\pi}{3}\right)}\ \Rightarrow 1 = 4K$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c or K ;	**award above**
	$\left\{\Rightarrow K = \frac{1}{4}\right\}$		
	$\Rightarrow\ \left(\frac{2+y}{2-y}\right) = \frac{1}{4}\sec^2 x$		
	Hence, $\sec^2 x = \frac{8+4y}{2-y}$	$\sec^2 x = \frac{8+4y}{2-y}$	A1 aef
			[8]

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 3**	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{-\frac{1}{2}}{(y-2)} + \frac{\frac{1}{2}}{(y+2)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(y-\lambda)$ or $\pm b\ln(y+\lambda)$	M1;
	$\therefore\ -\frac{1}{2}\ln(y-2) + \frac{1}{2}\ln(y+2) = \ln(\sec x)\ \ + (c)$	their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y=0,\ x=\frac{\pi}{3} \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(y-2) + \frac{1}{2}\ln(y+2) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{y+2}{y-2}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{y+2}{y-2}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{y+2}{y-2}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$	Note taking out the logs results in $y-2 \rightarrow 2-y$	
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 4**	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{1}{(4-2y)} + \frac{1}{(4+2y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		$\pm a\ln(\lambda-\mu y)$ or $\pm b\ln(\lambda+\mu y)$	M1;
	$\therefore\ -\tfrac{1}{2}\ln(4-2y)+\tfrac{1}{2}\ln(4+2y)=\ln(\sec x)\ \ +(c)$	their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their A and B and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y=0,\ x=\tfrac{\pi}{3}\ \Rightarrow\ -\tfrac{1}{2}\ln 4+\tfrac{1}{2}\ln 4=\ln\left(\frac{1}{\cos(\frac{\pi}{3})}\right)+c$	Use of $y=0$ and $x=\tfrac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0=\ln 2+c\ \Rightarrow\ \underline{c=-\ln 2}\}$		
	$-\tfrac{1}{2}\ln(4-2y)+\tfrac{1}{2}\ln(4+2y)=\ln(\sec x)-\ln 2$		
	$\frac{1}{2}\ln\left(\frac{4+2y}{4-2y}\right)=\ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{4+2y}{4-2y}\right)=2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{4+2y}{4-2y}\right)=\ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{4+2y}{4-2y}=\frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x=\frac{16+8y}{4-2y}}$	$\underline{\sec^2 x=\frac{16+8y}{4-2y}}$ or $\underline{\sec^2 x=\frac{8+4y}{2-y}}$	A1 aef
			[8]

Question Number	Example	
7. (b)	The first four marks in part (b). In part (a) this candidate had correctly answered part (a). 	B1 B1 M1 A0
	Comment 1: Even though the candidate has correctly substituted and then integrated the LHS, the constant 2 on the right hand side is incorrect. Therefore this expression is equivalent to $\therefore\ -\frac{1}{8}\ln(2-y)+\frac{1}{8}\ln(2+y)=\int \tan x \, dx$ which is incorrect from the candidate's working.	
	Comment 2: If the candidate had omitted line 3 of part (b), then the candidate will still score the first B (separating the variables) for $\int \frac{1}{4-y^2}\,dy = \int 2\tan x \, dx$, because the position of the "2" would be ignored.	

Question Number	Scheme		Marks
8. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leq t \leq \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leq t \leq \frac{\pi}{2}$	A1
			[2]
(b) **Way 1**	$x = 8\cos t, \quad y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence $m(\mathbf{N}) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses $m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}$.	dM1*
	$\mathbf{N}$: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their } m_N)x + \text{"}c\text{"}$.	dM1*
	$\mathbf{N}$: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so $\mathbf{N}$: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Note that "$(\text{their } m_N)$", means that the tangent gradient has to be changed. Note a change like $m(\mathbf{N}) = \frac{1}{\text{their } m(\mathbf{T})}$ is okay. This could score a maximum of M1 A1 M1* dM0* dM1* A0.

Note the final A1 is cso, meaning that the previous 5 marks must be awarded before the final mark can be awarded.

Note in (b) the marks are now M1A1M1M1M1A1. Apply the marks in this order on ePEN.

Question Number	Scheme		Marks
(c)	$A = \int_0^4 y\,dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	attempt at $A = \int y \frac{dx}{dt}\,dt$	M1
		correct expression (ignore limits and dt)	A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{du}{dt} = \cos t$ } {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$ }		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$	M1
		Correct integration ignoring limits.	A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = \frac{\sqrt{3}}{2} \text{ and } u = 1\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw**
			[4]
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks

t limits must be used in a t integrand and u limits must be used in a u integrand.

(d) To get the second M1 mark the candidates need to have gained the first M1 mark.

In (c), $\int 4\sin 2t(8\cos t)dt$, would be given the first M0.

Question Number	Scheme		Marks
Aliter **8.** (b) **Way 2**	$x = 8\cos t\,,\quad y = 4\sin 2t = 8\sin t\cos t$		
	$\left\{\begin{matrix} u = 8\sin t & v = \cos t \\ \frac{du}{dt} = 8\cos t & \frac{dv}{dt} = -\sin t \end{matrix}\right\}$		
	$\frac{dx}{dt} = -8\sin t\,,\quad \frac{dy}{dt} = 8\cos^2 t - 8\sin^2 t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and attempts to apply $vu' + uv'$ for their terms.	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \frac{8\cos^2\left(\frac{2\pi}{3}\right) - 8\sin^2\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{ = \frac{8\left(\frac{1}{2}\right) - 8\left(\frac{3}{4}\right)}{(-1)\left(\frac{\sqrt{3}}{2}\right)} = \frac{-2}{-\frac{\sqrt{3}}{2}} = \text{awrt } 0.58 \right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence m(**N**) = $-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses m(N) $= -\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x-4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x-4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (m_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso** **AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Note that "$(\text{their } m_N)$", means that the tangent gradient has to be changed. Note a change like m(**N**) $= \frac{1}{\text{their m}(\mathbf{T})}$ is okay. This could score a maximum of M1 A1 M1* dM0* dM1* A0.

Note the final A1 is cso, meaning that the previous 5 marks must be awarded before the final mark can be awarded.

Note in (b) the marks are now M1A1M1M1M1A1. Apply the marks in this order on ePEN.

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  dM1$*$ denotes a method mark which is dependent upon the award of the previous method M1$*$ mark.