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Mark Scheme (Pre-Standardisation)

January 2008

GCE

GCE Mathematics (6666/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2008
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π y: 0, 1.844321332…, 4.810477381…, 8.87207, 0		
		awrt 1.84432	B1
		awrt 4.81048	B1
			[2]
(b) **Way 1**	0 can be implied Area $\approx \frac{1}{2}\times\frac{\pi}{4}\,;\times\left\{\underline{0+2(1.84432+4.81048+8.87207)+0}\right\}$	Outside brackets $\frac{1}{2}\times\frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
		For structure of trapezium rule $\{............\}$;	M1 √
		Correct expression inside brackets which all must be multiplied by their "outside constant".	A1 √
	$=\frac{\pi}{8}\times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)	12.1948	A1 **cao**
			[4]
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{4}\times\left\{\frac{0+1.84432}{2}+\frac{1.84432+4.81048}{2}+\frac{4.81048+8.87207}{2}+\frac{8.87207+0}{2}\right\}$	$\frac{\pi}{4}$ and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	M1 √
	Area $\approx \frac{1}{2}\times\frac{\pi}{4}\,;\times\left\{\underline{0+2(1.84432+4.81048+8.87207)+0}\right\}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	A1 √
	$=\frac{\pi}{4}\times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)	12.1948	A1 **cao**
			[4]
			6 marks

Question Number	Scheme		Marks
	** represents a constant		
2. (a)	$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$	Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.	B1
	$= 2\left\{1+(\tfrac{1}{3})(**x)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3+...\right\}$ **with** $** \neq 1$	Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$; A correct simplified or an un-simplified $\{.........\}$ expansion with candidate's followed through $(**x)$	M1; A1√
		Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$	
	$= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3+...\right\}$		
	$= 2\{1-\tfrac{1}{8}x-\tfrac{1}{64}x^2-\tfrac{5}{1536}x^3-...\}$		
	$= 2-\frac{1}{4}x; -\frac{1}{32}x^2-\frac{5}{768}x^3-...$	Anything that cancels to $2-\frac{1}{4}x$; Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$	A1; A1 **[5]**
(b)	$(7.7)^{\frac{1}{3}} \approx 2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^2-\frac{5}{768}(0.1)^3-...$	Attempt to substitute $x=0.1$ into a candidate's binomial expansion.	M1
	$= 2 - 0.025 - 0.0003125 - 0.0000065104...$		
	$= 1.97468099...$	awrt 1.9746810	A1 **[2]**
			7 marks

You would award B1M1A0 for

$$=2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3+...\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	$(8-3x)^{\frac{1}{3}}$		
		2 or $(8)^{\frac{1}{3}}$ **(See note ↓)**	B1
	$= \left\{ \underline{(8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(-\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + \ldots} \right\}$	Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x)$;	M1
	with $** \neq 1$	A correct un-simplified or simplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= \left\{ \underline{(8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 + \frac{(-\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + \ldots} \right\}$		
	$= \left\{2 + (\tfrac{1}{3})(\tfrac{1}{4})(-3x); + (-\tfrac{1}{9})(\tfrac{1}{32})(9x^2) + (-\tfrac{5}{81})(\tfrac{1}{256})(27x^3) + \ldots\right\}$		
	$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \ldots$	Anything that cancels to $2 - \frac{1}{4}x$;	A1;
		Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$	A1
			[5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

> If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Question Number	Scheme		Marks
3.	Volume $= \underline{\pi \int_a^b \left(\frac{1}{2x+1}\right)^2}\, dx = \pi \int_a^b \frac{1}{(2x+1)^2}\, dx$	Use of $V = \underline{\pi \int y^2}\, dx$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2}\, dx$		
	$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$		
	$= (\pi)\left[\underline{-\tfrac{1}{2}(2x+1)^{-1}}\right]_a^b$	Integrating to give $\underline{\pm p(2x+1)^{-1}}$ $\underline{-\tfrac{1}{2}(2x+1)^{-1}}$	M1 A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of b and a and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$	A1 aef **[5]**
			5 marks

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme		Marks
4. (i)	$\int \ln\left(\frac{x}{2}\right) dx = \int 1.\ln\left(\frac{x}{2}\right) dx \Rightarrow \begin{Bmatrix} u = \ln\left(\frac{x}{2}\right) & \Rightarrow & \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow & v = x \end{Bmatrix}$		
	$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\, dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\, dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	dM1
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef **[4]**
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$	Consideration of double angle formula for $\sin^2 x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\, dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\underline{1 - \cos 2x}\right) dx$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	Integrating to give $\pm ax \pm b\sin 2x$; Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	dM1 A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$ $= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)$ or $\frac{\pi}{8} + \frac{1}{4}$ Candidate must collect their π term and constant term together for A1	A1 **aef** **[5]**
			9 marks

Question Number	Scheme		Marks
Aliter **4.** (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right)\,dx = \int \left(\ln x - \ln 2\right)\,dx = \int \ln x\,dx - \int \ln 2\,dx$		
	$\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \left\{\begin{matrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow & v = x \end{matrix}\right\}$		
	$\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without + c	A1
	$\int \ln 2\,dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without + c	M1
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$	Correct integration with + c	A1 aef
			[4]

Question Number	Scheme		Marks
Aliter **4.** (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x.\sin x\,dx$ and $I = \int \sin^2 x\,dx$		
	$\begin{Bmatrix} u = \sin x & \Rightarrow & \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow & v = -\cos x \end{Bmatrix}$		
	$\therefore I = \left\{-\sin x\cos x + \int \cos^2 x\,dx\right\}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{-\sin x\cos x + \int (1-\sin^2 x)\,dx\right\}$		
	$\int \sin x\,dx = \left\{-\sin x\cos x + \int 1\,dx - \int \sin^2 x\,dx\right\}$		
	$2\int \sin^2 x\,dx = \left\{-\sin x\cos x + \int 1\,dx\right\}$	For the LHS becoming $2I$	dM1
	$2\int \sin^2 x\,dx = \{-\sin x\cos x + x\}$		
	$\int \sin^2 x\,dx = \left\{-\frac{1}{2}\sin x\cos x + \frac{x}{2}\right\}$	Correct integration	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \left[\left(-\frac{1}{2}\sin(\frac{\pi}{2})\cos(\frac{\pi}{2}) + \frac{(\frac{\pi}{2})}{2}\right) - \left(-\frac{1}{2}\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) + \frac{(\frac{\pi}{4})}{2}\right)\right]$ $= \left[(0 + \frac{\pi}{4}) - (-\frac{1}{4} + \frac{\pi}{8})\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{\pi}{8} + \frac{1}{4}$	$\frac{1}{8}(\pi + 2)$ or $\frac{\pi}{8} + \frac{1}{4}$ Candidate must collect their pi term and constant term together for A1	A1 **aef** **[5]**

Question Number	Scheme		Marks
5. (a)	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$		
	$4y^2 - 96y + 512 ;= 0$ $y^2 - 24y + 128 ;= 0$	Substitutes $x = -8$ into $*$ to obtain a quadratic in y of the form $ay^2 + by + c$ on one side. Condone the loss of $= 0$.	M1;
	$(y-16)(y-8) = 0$	An attempt to solve the quadratic in y by either factorising or by the formula.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \cancel{=} \right\}$ $3x^2 - 8y\frac{dy}{dx} = \underline{\left(12y + 12x\frac{dy}{dx}\right)}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \ldots$ Correct application of product rule Correct equation	M1 B1 A1
	$\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient correctly found. Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	dM1 A1 A1
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 2**	$\left\{\frac{\cancel{d}x}{\cancel{d}y}\times\right\}$ $3x^2\frac{dx}{dy}-8y = \underline{\left(12y\frac{dx}{dy}+12x\right)}$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy}=...$	M1
		Correct application of product rule	B1
		Correct equation	A1
	$\left\{\frac{dy}{dx}=\frac{3x^2-12y}{12x+8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx}=\frac{3(64)-12(8)}{12(-8)+8(8)} = \frac{96}{-32} = \underline{-3}$,	Substitutes $x=-8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.	dM1
	@ $(-8, 16)$, $\frac{dy}{dx}=\frac{3(64)-12(16)}{12(-8)+8(16)} = \frac{0}{32} = \underline{0}$.	One gradient correctly found.	A1
		Both gradients of -3 and 0 correctly found.	A1
			[6]

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8 \quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3},\ \underline{0}.$	One gradient correctly found. Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1 A1
			[6]

Question Number	Scheme		Marks
6. (a)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.	M1
		Correct answer.	A1
			[2]
(b)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	Using $\mathbf{r} = \overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} + \lambda(\text{their } \overrightarrow{AB})$	M1
		Correct answer	A1 aef
			[2]
(c)	$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Applies dot product formula between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1+0+2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ}$.	$\underline{\theta = 45^\circ}$	A1 **cao**
			[3]

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
6. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	Any two of $\mathbf{i}:\ 2+\lambda=\mu$ (1); $\mathbf{j}:\ 6-2\lambda=0$ (2); $\mathbf{k}:\ -1+2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1
	(2) yields $\lambda=3$ Any two yields $\lambda=3,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	M1
		either one of λ or μ correct.	A1
	$l_1:\ \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter 6. (d) **Way 2**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	Any two of $\mathbf{i}:\ 3+\lambda=\mu$ (1); $\mathbf{j}:\ 4-2\lambda=0$ (2); $\mathbf{k}:\ 1+2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1
	(2) yields $\lambda=2$ Any two yields $\lambda=2,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	M1
		either one of λ or μ correct.	A1
	$l_1:\ \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+2\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

Question Number	Scheme		Marks
7. (a)	$x = \ln(t+2)$, $\quad y = \dfrac{1}{t+1}$		
	$\dfrac{dx}{dt} = \dfrac{1}{t+2}$	$\dfrac{dx}{dt} = \dfrac{1}{t+2}$	B1
	$\text{Area}(R) = \int_{\ln 2}^{\ln 4} y\,dx; = \int_0^2 \left(\dfrac{1}{t+1}\right)\left(\dfrac{1}{t+2}\right) dt$	Area $= \int y\,dx$. Ignore limits.	M1;
		$\int \left(\dfrac{1}{t+1}\right) \times \left(\dfrac{1}{t+1}\right) dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$ $x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$	changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$	B1
	Hence, $\text{Area}(R) = \int_0^2 \dfrac{1}{(t+1)(t+2)}\,dt$		
			[4]
(b)	$\dfrac{1}{(t+1)(t+2)} = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ $1 = A(t+2) + B(t+1)$	Expresses $\dfrac{1}{(t+1)(t+2)}$ as a partial fraction and forms this identity. Can be implied.	M1
	Let $t = -1$, $\quad 1 = A(1) \quad \Rightarrow \quad \underline{A = 1}$ Let $t = -2$, $\quad 1 = B(-1) \quad \Rightarrow \quad \underline{B = -1}$	Finds both A and B correctly. Can be implied.	A1
	$\int_0^2 \dfrac{1}{(t+1)(t+2)}\,dt = \int_0^2 \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}\,dt$		
	$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ Both ln terms correct	dM1 A1
	$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	Substitutes limits of 2 and 0 and subtracts the correct way round.	ddM1
	$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$	A1 aef isw
			[6]

Question Number	Scheme	Marks
	$x = \ln(t+2), \quad y = \dfrac{1}{t+1}$	
7. (c)	$e^x = t+2 \;\Rightarrow\; t = e^x - 2$ — Attempt to make t the subject; giving $t = e^x - 2$	M1 A1
	$y = \dfrac{1}{e^x - 2 + 1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$ — Eliminates t by substituting in y; giving $y = \dfrac{1}{e^x - 1}$	dM1 A1 **[4]**
Aliter **7.** (c) **Way 2**	$t+1 = \dfrac{1}{y} \;\Rightarrow\; t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$ — Attempt to make t the subject	M1
	$y(t+1) = 1 \;\Rightarrow\; yt + y = 1 \;\Rightarrow yt = 1-y \;\Rightarrow t = \dfrac{1-y}{y}$ — Giving either $t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\dfrac{1-y}{y} + 2\right)$ — Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$	
	$e^x = \dfrac{1}{y} + 1$	
	$e^x - 1 = \dfrac{1}{y}$	
	$y = \dfrac{1}{e^x - 1}$ — giving $y = \dfrac{1}{e^x - 1}$	A1 **[4]**
(c)	Domain: $\underline{x > 0}$ — $\underline{x > 0}$ or just > 0	B1 **[1]**
		15 marks

Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$V = 4000h \Rightarrow \frac{dV}{dh} = 4000$	Both of these statements required	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Convincing proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Convincing proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	Separates the variables with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Convincing proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h = (20-x)^2$		
	$\frac{\mathrm{d}h}{\mathrm{d}t} = 2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = -2(20-x)$	Correct $\frac{\mathrm{d}h}{\mathrm{d}t}$	B1 aef
	$h = (20-x)^2 \Rightarrow \sqrt{h} = 20-x \Rightarrow x = 20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$	$\pm\lambda\int\frac{20-x}{x}\,\mathrm{d}x$ where λ is a constant	M1*
	$= 100\int \frac{x-20}{x}\,\mathrm{d}x$		
	$= 100\int \left(1-\frac{20}{x}\right)\mathrm{d}x$		
	$= 100\left(x - 20\ln x\right) + c$	$\pm\alpha\left(x-20\ln x\right)$ $100x - 2000\ln x$	M1 A1
	change limits: when $h = 0$ then $u = 20$ and when $h = 100$ then $u = 10$		
	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100x - 2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right) - 2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$= \left(1000 - 2000\ln 10\right) - \left(2000 - 2000\ln 20\right)$	Correct use of limits Either $x = 10$ and $x = 20$ or $h = 100$ and $h = 0$	depM1*
	$= 2000\ln 20 - 2000\ln 10 - 1000$		
	$= 2000\ln 2 - 1000$	$2000\ln 2 - 1000$	A1
			[6]
(e)	Time required $= 2000\ln 2 - 1000 = 386.2943611...$ sec		
	= 386 seconds (nearest second)		
	= 6 minutes and 26 seconds (nearest second)	<u>6 minutes, 26 seconds</u>	B1
			[1]
			13 marks