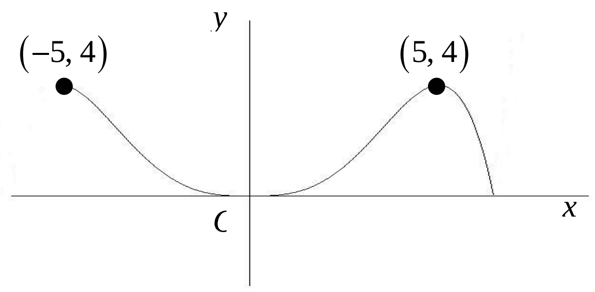
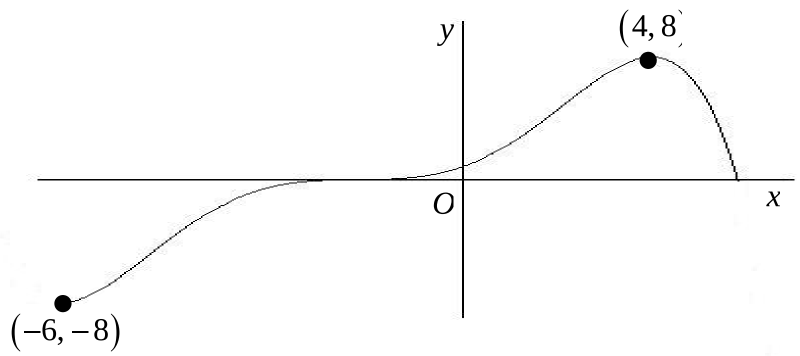
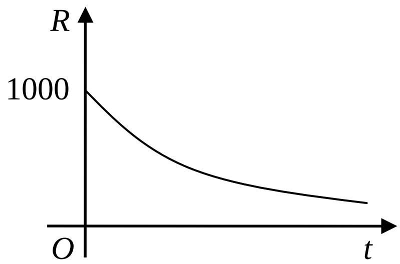
January 2008
6665 Core C3
Mark Scheme

Final Version for Standardisation

Question Number	Scheme	Marks
1.	$$\begin{array}{r\|l} & 2x^2 \quad\quad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \quad\quad -x^2+x+1 \\ & \quad\quad -x^2 \quad\quad +1 \\ \hline & \quad\quad\quad x \end{array}$$	M1
	$a=2$ stated or implied	A1
	$c=-1$ stated or implied	A1
	$2x^2-1+\dfrac{x}{x^2-1}$	
	$a=2, b=0, c=-1, d=1, e=0$	
	$d=1$ and $b=0, e=0$ stated or implied	A1
		[4]
2.	(a)	
	$\dfrac{dy}{dx}=2e^{2x}\tan x+e^{2x}\sec^2 x$	M1 A1+A1
	$\dfrac{dy}{dx}=0 \quad \Rightarrow \quad 2e^{2x}\tan x+e^{2x}\sec^2 x=0$	M1
	$2\tan x+1+\tan^2 x=0$	A1
	$(\tan x+1)^2=0$	
	$\tan x=-1$ * cso	A1 (6)
	(b) $\left(\dfrac{dy}{dx}\right)_0=1$	M1
	Equation of tangent at $(0, 0)$ is $y=x$	A1 (2)
		[8]

Question Number	Scheme		Marks	
3.	(a) $\mathrm{f}(2) = 0.38\ \ldots$			
	$\mathrm{f}(3) = -0.39\ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ $\ast$	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$\mathrm{f}(2.5045) \approx 6 \times 10^{-4}$			
	$\mathrm{f}(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp $\ast$	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

Question Number	Scheme	Marks
4.	(a)  Shape $A'(5,4)$ $B'(-5,4)$	B1 B1 B1 (3)
	(b) For the purpose of marking this paper, the graph is identical to (a) Shape $A'(5,4)$ $B'(-5,4)$	 B1 B1 B1 (3)
	(c)  General shape – unchanged Translation to left $A'(4,8)$ $B'(-6,-8)$	B1 B1 B1 B1 (4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.	**[10]**

Question Number	Scheme		Marks	
5.	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	 (2) **[9]**

Question Number	Scheme	Marks
6.	(a) $\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x$	M1
	$=\left(2\cos^2 x-1\right)\cos x-\left(2\sin x\cos x\right)\sin x$	M1
	$=\left(2\cos^2 x-1\right)\cos x-2(1-\cos^2 x)\cos x$ any correct expression	A1
	$=4\cos^3 x-3\cos x$	A1 (4)
	(b)(i) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}=\dfrac{\cos^2 x+\left(1+\sin x\right)^2}{\left(1+\sin x\right)\cos x}$	M1
	$=\dfrac{\cos^2 x+1+2\sin x+\sin^2 x}{\left(1+\sin x\right)\cos x}$	A1
	$=\dfrac{2\left(1+\sin x\right)}{\left(1+\sin x\right)\cos x}$	M1
	$=\dfrac{2}{\cos x}=2\sec x$ * cso	A1 (4)
	(c) $\sec x=2$ *or* $\cos x=\dfrac{1}{2}$	M1
	$x=\dfrac{\pi}{3},\dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1 (3)
		[11]
7.	(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=6\cos 2x-8\sin 2x$	M1 A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0=6$	B1
	$y-4=-\dfrac{1}{6}x$ or equivalent	M1 A1 (5)
	(b) $R=\sqrt{\left(3^2+4^2\right)}=5$	M1 A1
	$\tan\alpha=\dfrac{4}{3},\ \alpha\approx 0.927$ awrt 0.927	M1 A1 (4)
	(c) $\sin\left(2x+\text{ their }\alpha\right)=0$	M1
	$x=-2.03,-0.46,1.11,2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.	**[13]**

Question Number	Scheme	Marks
8.	(a) $x=1-2y^3 \quad\Rightarrow\quad y=\left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$\mathrm{f}: x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $\mathrm{gf}(x)=\frac{3}{1-2x^3}-4$	M1 A1
	$=\frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$=\frac{8x^3-1}{1-2x^3}$ $*$ cso	A1 (4)
	$\mathrm{gf}: x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3-1=0$ Attempting solution of numerator = 0	M1
	$x=\frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(1-2x^3)\times 24x^2+(8x^3-1)\times 6x^2}{(1-2x^3)^2}$	M1 A1
	$=\frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find y.	M1
	$x=0,\ y=-1$	A1 (5) **[13]**

January 2008 — Final Version for Standardisation

6665 Core C3
Marking Guidance
January 2008

The following abbreviations are used in Mark Schemes and in Guidance notes. Not all abbreviations are used in every scheme.

M	A method mark. These are awarded for ‘knowing a method and attempting to apply it’.
A	An accuracy mark. Can only be awarded if the relevant method mark(s) have been earned.
B	These marks are independent of method marks.
cso	correct solution only. There must be no errors in this part of the question to obtain this mark.
cao	correct answer only.
ft	follow through. The scheme or marking guidance will specify what is to be followed through.
oe	or equivalent.
awrt	answers which round to
	The second mark is dependent on gaining the first mark.
*	The answer is printed on the paper.
bod	benefit of doubt.
isw	ignore subsequent working. When a candidate has reached an acceptable answer they are awarded the appropriate marks. Subsequent working, even if incorrect, is ignored.
dp	decimal places
sf	significant figures
LHS	The left hand side of an equation
RHS	The right hand side of an equation

1. M1 A complete method.

If long division by x^2-1 is used, the process must get as far as obtaining a linear remainder $ax+b$, where a and/or b could be zero. If they get this far, be generous with the details. If they take the process further, ignore for the M1. For division by $x-1$ and $x+1$ see **Alternative solutions.**

If equating coefficients, substituting values of x, or some combination of these, is used there should be at least 5 equations or relations are solved to obtain all 5 values. See below for an example. As two of the values are zero, there may be some doubt about this and the benefit of any doubt should be given to the candidate.

A1 $a=2$ stated implicitly or implied. The coefficient of x^2 appearing as 2 is sufficient as long as the M has been gained.

A1 $c=-1$ stated implicitly or implied. The constant coefficient appearing as -1 is sufficient as long as the M has been gained.

A1 $d=1, b=0$ and $e=0$ stated or implied. The zeros may not always be clear and again benefit of any doubt should be given. Correct division with a correct remainder should be given full marks even if it is clear that the candidate is not sure how to interpret this. Any correct form of the answer should gain full marks, as long as there is some supporting working.

Example of substituting values and equating coefficients.

$$2x^4-3x^2+x+1=\left(ax^2+bx+c\right)\left(x^2-1\right)+dx+e$$

$x=1 \quad \Rightarrow \quad 1=d+e$

$x=-1 \quad \Rightarrow \quad -1=-d+e$

Solving $d=1, e=0$

Coefficient of x^4

$2=a$

Coefficient of x^3

$0=b$

Constant coefficient

$1=-c+e$ Only here, when the fifth relation is given, is the M1 gained. The As can then be awarded wherever gained.

$c=-1$

If **decomposition of the numerator** is used the solution can be reduced to

$$\frac{2x^4-3x^2+x+1}{x^2-1}=\frac{2x^4-2x^2-x^2+1+x}{x^2-1}$$ M1

$$=2x^2-1+\frac{x}{x^2-1}$$ A1+A1+A1 This is an example where the zero values can be taken as implied.

2.(a) M1 Use of $\frac{\mathrm{d}}{\mathrm{d}x}(uv) = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$. Must be applied to the question.

A1 $2\mathrm{e}^{2x}\tan x$. Even if this is the candidate's second term.

A1 $\mathrm{e}^{2x}\sec^2 x$ or equivalent. Even if this is the candidates first term. Other expressions than $\sec^2 x$ are possible an exact equivalent is needed for the M1.

M1 Putting their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Not formally dependent on the first M1 but there must be the sum of at least two terms put equal to zero. See below for attempts at verification of the result.

A1 Obtaining any correct equation in $\tan x$ alone, i.e. e^{2x} should have been removed. Usually this will use the identity $\sec^2 x = 1 + \tan^2 x$ but equivalent formulae are possible. Ignore any additional incorrect possibilities arising from $\mathrm{e}^{2x} = 0$.

A1 Obtaining the correct answer. cao and cso. All previous marks in this part must have been gained.

(b) The grid has B1 B1 but please mark as M1 A1. The second mark depends on the first and the second should follow from correct or, at least, not incorrect work.

M1 Substituting $x = 0$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and evaluating to find the gradient.

A1 $y = x$ or any equivalent form. The work may be very abbreviated as most of the numbers are 0s and 1s. If $y = x$ is given and there is no incorrect working, give both marks bod
If $y = x$ comes from incorrect working mark A0.

Attempts at verification in part (a)

Taking a particular value of x which satisfies $\tan x = -1$, usually $-\frac{\pi}{4}$, substituting it into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempting to show that this is 0 is worth M1 A0 A0. It is equivalent working to putting $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ but does not establish a general result.

Substituting $\tan x = -1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempting to show that this is 0 also gains the M1. If their $\frac{\mathrm{d}y}{\mathrm{d}x}$ is correct and they show correctly that it is 0, then the next A1 is gained but not the final A1.
They have used $\sec^2 x = 1 + \tan^2 x$, or its equivalent, correctly and established that $\tan x = -1$ is a general solution but not that it is the only one possible, so their maximum is M1 A1 A1 M1 A1 A0.

3.(a) M1 Calculation of $\mathrm{f}(2)$ and $\mathrm{f}(3)$. There must be some evidence of evaluation to at least one figure, either rounded or truncated is sufficient. The calculation at any two values between 2 and 3 enclosing the accurate root can lead to the conclusion that the root is in $(2, 3)$ but to gain the M1 there must be some reference to $(2, 3)$. For example showing that the root lies between 2.4 and 2.8 shows that it lies between 2 and 3 but there must be some evidence that they are attempting to prove the result the question were asked for.

A1 Calculating $\mathrm{f}(2)$ and $\mathrm{f}(3)$ correctly, to at least one figure, and a reason and conclusion. Reference to change of sign, $\mathrm{f}(2)\times\mathrm{f}(3)<0$ or a diagram are sufficient reasons. There must be a (minimal) conclusion, e.g. root in interval, QED or W is acceptable. Ignore the presence or absence of any reference to continuity.

(b) M1 Evaluating x_1 with $x = 2.5$ to at least 3 decimal places. Obtaining 2.504 … is sufficient evidence for this but incorrect evaluation can also gain this if a clear attempt has been made.

A1 **Both** x_1 and x_2 evaluated correctly and expressed correct to 5 decimal places. See the Mark Scheme.

A1 x_3 evaluated correctly and expressed correct to 5 decimal places. If previous A1 has been lost for giving more than 5 decimal places for one or both answers do not penalise again here. Only penalise too great an accuracy once.

(c) The grid has B1 B1 but please mark as M1 A1. That is the second mark depends on the first.

M1 Choosing the interval $[2.5045, 2.5055]$,or any tighter interval which includes the accurate answer 2.50524, and evaluating $\mathrm{f}(x)$ at the end points Hence, for examples, evaluating at the end points of $(2.505, 2.5055)$ or $(2.5045, 2.5054)$, gains the mark.

A1 The calculation of $\mathrm{f}(x)$ at both end values correctly to at least one significant figure and giving a reason and conclusion. Accept if rounded or truncated to one significant figure. $\mathrm{f}(2.5045) \approx 5.768\,97\times10^{-4}$, $\mathrm{f}(2.5055) \approx -2.011\,273\times10^{-4}$, $\mathrm{f}(2.505) \approx 1.878\times10^{-4}$.
Acceptable reasons are a reference to the sign change, noting that $\mathrm{f}(x_1)\mathrm{f}(x_2) < 0$ or drawing a sketch. Ignore the presence or absence of any reference to continuity. Adopt a minimalist approach to the conclusion. QED or, its modern equivalent, W are acceptable. Explicit reference to 3 dp is not required, e.g. "Hence result is proved" is acceptable.

In part (c). we will accept repeated iteration but they must obtain at least x_6 and show all their intermediate results to at least 5 decimal places for the M1. For the A1 the working must be correct and, as above, there must be a conclusion. With this method there must be some reference to the third decimal place, or better, not changing.
To 6 decimal places, truncated $x_4 = 2.505\,228$, $x_5 = 2.505\,238$, $x_6 = 2.505\,240$

4.(a) Only mark the graph for the domain from $x = -5$ to the point where the graph meets the positive x-axis $(x \approx 6.455)$. Ignore anything outside this domain, whether right, wrong or debatable.

B1 The general shape unchanged for $x > 0$ and reflected above x-axis for $x < 0$. The graph should have a minimum at $x = 0$. Ignore gradient approaching $(-5, 4)$.

B1 $(5, 4)$ shown at the maximum point on the right hand side of the curve in the correct quadrant. Both the 5 and the 4 must be indicated, either as a coordinate or by reference to scales on the axes. If A is $(5, 4)$ is written somewhere near the diagram, accept this.

B1 $(-5, 4)$ shown on the left hand side of the curve in the correct quadrant. Do not worry about the gradient of the curve as it approaches $(-5, 4)$ and ignore if the curve is extended further to the left. Otherwise as for previous mark.

(b) For the purposes of marking this paper, this part will be marked identically to part (a). (There should probably be an additional part of the curve joining $(-5, 4)$ to the negative x-axis but ignore the presence or absence of this on this occasion.)

B1 As above. B1 As above. B1 As above.

(c) Again, mark only the domain indicated in the scheme. Anything outside this domain is to be ignored. Candidates are often rather bad at drawing points of inflection, which are not explicitly in the syllabus. Be generous about the shape of the curve at the point of inflection and do not insist that the gradient is zero there.

B1 The general shape of the curve is unchanged. Ignore changes in scale, or the lack of them, for this mark. This mark is gained if the curve is moved to the right, rather than the left, or is left with the point of inflection at the origin.

B1 The curve is moved to the left. The point of inflection on the negative x-axis is sufficient evidence of this. The point of inflection must not be moved up or down. The amount moved to the left is not considered for this mark.

B1 $(4, 8)$ indicated at the maximum point on the right hand side of the curve in the correct quadrant. Other conditions as the second B of part (a). See below for an **additional case**.

B1 $(-6, -8)$ indicated on the left hand side of the curve in the correct quadrant. Other conditions as the third B of part (a).

Additional case.
The third B1 should also be awarded if both *x*s are correct $(4, -6)$ but both *y*s are incorrect. It should also be awarded if both *y*s are correct $(8, -8)$ but both *x*s are incorrect.

5.(a) B1 1000 cao. Ignore any working.

(b) M1 Substituting $t = 5730$ and handling condition that half of the substance has decayed, usually by substituting in half of their value found in (a).

A1 *Either*

Obtaining $e^{-5730c} = \frac{1}{2}$ or $e^{5730c} = 2$. If this is done, logarithms are not needed for this mark.

Here, and elsewhere on the paper, accept equivalent fractions, e.g. $\frac{500}{1000}$.

Or

Obtaining any correct linear equation in c after taking logarithms. If this is done, this A will be awarded after gaining the next M. It is still put in the same place on the grid.
For example, $\ln 1000 - 5730c = \ln 500$ gains this mark.

M1 Taking logarithms correctly to obtain a linear equation in c. Not formally dependent on the previous M1. It could, for example, be gained for correctly taking logarithms to solve any equation of the form $e^{lc} = m$, where l and m where any real numbers. In principle, logs could be taken to any base.

A1 $0.000\,121$ or 1.21×10^{-4}. The correct answer to the correct number of significant figures.

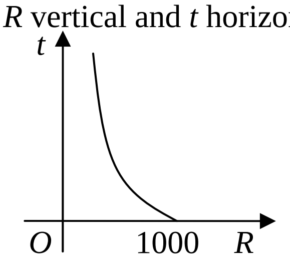
(c) M1 Substituting $t = 22\,920$ into $R = 1000\,e^{-ct}$, with their c, and evaluating R.

A1 Anything in the range 62 to 63 inclusive. (It is not clear in the context of modern physics that the answer has to be a whole number!) Accept any degree of accuracy in this range.

(d) B1 A monotonic decreasing function, wholly above, not touching, the t-axis, curving in the right direction; $\frac{d^2R}{dt^2}$ must be positive throughout. The curve must meet the positive R-axis and descend from there. Straight lines are not acceptable but be tolerant of a tendency to become a straight line for large values of t. Ignore the closeness of approach to the asymptote. Also ignore extension to left of the R-axis (technically wrong but there is only one mark here).

B1 1000 or $(0, 1000)$ marked in some way. The graph must go through this point but there are no other requirements on the shape of the graph for this B1.

In part (d), ignore the labelling of the axes. Assume that is R vertical and t horizontal, even if y and x are used. If R and t are clearly reversed accept for B1 B1:-



Give credit for their graph wherever it is drawn.

6.(a) M1 Using the printed formula and replacing *A* and *B* by $2x$ and x, either way round. If a plus sign is given in the middle of the formula, treat as a misread and the maximum the candidate can score is M1 M1 A0 A0.

M1 Replacing $\cos 2x$ by $2\cos^2 x - 1$ and $\sin 2x$ by $2\sin x \cos x$. Both double angle formulae used must be correct. Dependent on the previous M mark. If $\cos 2x$ is replaced by $\cos^2 x - \sin^2 x$, the $\sin^2 x$ must be correctly replaced by $1 - \cos^2 x$ before this mark is gained.

A1 Using $\cos^2 x + \sin^2 x = 1$ and obtaining a correct, not necessarily simplified, expression in $\cos x$.

A1 Collecting together terms and obtaining the correct answer. Apart from the order of the terms there is only one possible answer. If another letter is used instead of *x* throughout, only this mark is lost.

(b)(i) M1 Putting the fractions over a common denominator. A soft mark but at least one of the numerators, $\cos x$ and/or $1 + \sin x$, must be modified. As $x \neq (2n+1)\frac{\pi}{2}$ is given, it is legitimate to multiply both sides by $(1 + \sin x)\cos x$. See **Alternative solutions**.

A1 The bracket in the numerator multiplied out and a correct numerator obtained.

M1 Simplifying the numerator using $\cos^2 x + \sin^2 x = 1$. The numerator does not have to be factorised, e.g. both $2 + 2\sin x$ and $2(1 + \sin x)$ are acceptable. If the numerator is incorrect the numerator must be at least as complicated as the correct one, i.e. have at least 4 terms. If their numerator cannot be simplified, the mark cannot be gained; it must be possible to collect $\cos^2 x$ and $\sin^2 x$ together and replace them by 1.

A1 Completing the proof. cso – all previous marks in this part must have been gained.

(ii) M1 Using (b)(i) and reducing the equation to $\sec x = \text{constant}$ or $\cos x = \text{constant}$. The constant does not have to be correct but it does have to make sense, e.g. $\cos x = 2$ does not gain the mark. The question has "hence" so no alternative methods are acceptable.

A1 $\frac{\pi}{3}$ or anything which rounds to 1.05. Degrees are not acceptable.

A1 $\frac{5\pi}{3}$ or anything which rounds to 5.24. Degrees are not acceptable but if both correct in degrees $(60°, 300°)$, allow A0 A1 as a special case. Accept equivalent fractions, e.g. $\frac{10\pi}{6}$ is acceptable.

Ignore any answers, correct or incorrect, outside the range $0 < x < 2\pi$. If there are incorrect answers within range withhold the final A1, if otherwise gained, only .

7.(a) M1 Differentiating both terms. There must be some modification of both terms and, at least, either $\sin 2x \mapsto \pm 2\cos 2x$ or $\cos 2x \mapsto \pm 2\sin 2x$ must be seen or implied. Differentiating a single term must give rise to just one term. The terms must be differentiated separately and not confused in a product.

A1 A completely correct expression for $\frac{dy}{dx}$.

B1 $\frac{dy}{dx}$ at $(0, 4)$ is 6. This can be implied by the gradient of the normal being $-\frac{1}{6}$. This is a B mark and could come from incorrect previous working or no working at all.

M1 Using $mm' = -1$ and substituting their value of m' into $y - 4 = m'x$, or $(0, 4)$ substituted into $y = m'x + c$ and c found. $(0, 4)$ is given and must be used correctly. Not dependent on the previous M1.

A1 Any correct form on the equation of the normal, e.g. $x + 6y = 24$. Require a completely correct solution or, at least, one with no errors – some working can be done mentally.

(b) M1 Using a Pythagoras relation to find R. Do not consider with the details of the working, e.g. using identities etc.

A1 $R = 5$. If this appears give M1 A1 – it can just be written down.

M1 $\tan\alpha = \pm\frac{4}{3}$ or $\pm\frac{3}{4}$, or $\cot\alpha = \pm\frac{4}{3}$ or $\pm\frac{3}{4}$. Do not consider the details of how these are obtained. Accept $\cos\alpha = \pm\frac{4}{5}$ or $\pm\frac{3}{5}$ or $\sin\alpha = \pm\frac{4}{5}$ or $\pm\frac{3}{5}$.

A1 A value of α correct to at least 3 significant figures. If they just give $\arctan\frac{4}{3}$, do not accept immediately but if a correct value is given in (c) allow retrospectively. Degrees, $53.1°$, is not acceptable.

(c) M1 If they use (b), reducing the equation to $\sin(2x + \text{ their } \alpha) = 0$. Their R must have been eliminated. There is no "hence" here, see **Alternative Solutions**.

A1 Any one correct answer in range to at least 2 decimal places. Accept degrees for this mark. $-116.6°, -26.6°, 63.4°, 153.4°$, accept answers in degrees to 1 d.p. Accept $63.45°$ and $153.45°$. Accept $-0.65\pi, -0.15\pi, 0.35\pi, 0.85\pi$ for all A marks in (c).

A1 Any second correct answer in range to at least 2 decimal places. Accept degrees for this mark as above.

A1 All four answers correct in radians, to 2 or more decimal places. There must be no incorrect answers within the range $-\pi \leqslant x \leqslant \pi$.

Ignore any answers, correct or incorrect, outside the range $-\pi \leqslant x \leqslant \pi$.
Ignore the absence, or otherwise, of reference to the y-coordinate, 0.

8.(a) M1 Changing the subject of the formula and "switching variable". These procedures can be carried out in either order. They need not use y but an expression in x must be obtained.

A1 A correct function of x. Accept equivalent forms. This may be called f, $\mathrm{f}(x)$, y or appear in function definition, e.g. $x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$. Ignore domain whether correct, incorrect or omitted.

(b) M1 Attempting to apply the functions in the correct order. First f, then g. $1-2\left(\frac{3}{x}-4\right)^3$ is definitely wrong but is, now, uncommon.

A1 Any correct function of x.

M1 Putting the terms over a common denominator. The 4 must be modified. Depends on previous M1.

A1 ∗ Simplifying to the printed expression. cao and cso.

(c) M1 Attempting and obtaining a, not necessarily correct, solution of $8x^3-1=0$. Obtaining $x=2$ from this equation is M1 A0. The presence of other solutions does not lose this mark.

A1 $x=\frac{1}{2}$ and no other solutions which have not been rejected. Additional solutions from the numerator equals zero or denominator equals zero, lose this.

(d) M1 Differentiating using a quotient rule or using the product rule with the correct modified expression $\left(8x^3-1\right)\left(1-2x^3\right)^{-1}$. If the quotient rule, the correct rule, with the negative sign in the right place, must be quoted or implied. $\mathrm{gf}(x)$ can also be differentiated when expressed in other forms, e.g. $3(1-2x^3)^{-1}+4$

A1 Any correct, not necessarily simplified, form of $\frac{\mathrm{d}y}{\mathrm{d}x}$. Among possible alternatives are $24x^2\left(1-2x^3\right)^{-1}+6x^2\left(8x^3-1\right)\left(1-2x^3\right)^{-2}$ and $8x^2\left(1-2x^3\right)^{-2}$.

A1 Obtaining a numerator of $18x^2$. Also award for equating to zero first and manipulating correctly to obtain a multiple (usually 18) of $x^2=0$. Differentiating $3(1-2x^3)^{-1}+4$ correctly to $18x^2\left(1-2x^3\right)^{-2}$ would gain both this and the previous A1 together.

M1 Solving their numerator equal to zero and substituting their value into $\mathrm{gf}(x)$ to find y.
If the product rule is used, the candidate must manipulate their expression to a point where there is a numerator , solve that = 0, and obtain a value of y. This M is not dependent on the previous M, e.g. it could come from the quotient rule "the wrong way round" .

A1 The correct values of x and y. The values must come from correct working. Do not accept $x=0$ from incorrect working, e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{30x^2}{\left(1-2x^3\right)^2}$. In this part, ignore additional solutions.

ALTERNATIVE SOLUTIONS

1. It is possible to solve this by dividing successively by $(x-1)$ and $(x+1)$. The M1 can only be gained here if the candidate shows how to handle the remainders and this is very unlikely. Most will get 0/4 using this. Working is:

$$
\begin{array}{r|l}
 & \quad\; 2x^3 + 2x^2 - x \\
\hline
x-1 & 2x^4 \qquad\quad -3x^2 + x + 1 \\
 & \underline{2x^4 - 2x^3} \\
 & \qquad 2x^3 - 3x^2 \\
 & \qquad \underline{2x^3 - 2x^2} \\
 & \qquad\qquad -x^2 + x \\
 & \qquad\qquad \underline{-x^2 + x} \\
 & \qquad\qquad\qquad\quad +1
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad 2x^2 \qquad -1 \\
\hline
x+1 & 2x^3 + 2x^2 - x \\
 & \underline{2x^3 + 2x^2} \\
 & \qquad\qquad -x \\
 & \qquad\qquad \underline{-x-1} \\
 & \qquad\qquad\quad\;\; 1
\end{array}
$$

Hence $\dfrac{2x^4 - 3x^2 + x + 1}{x^2 - 1} = 2x^2 - 1 + \dfrac{1}{x+1} + \dfrac{1}{x^2-1}$

$$= 2x^2 - 1 + \frac{x-1+1}{x^2-1}$$

The M cannot be awarded until here.
Only then can any As be awarded.

$$= 2x^2 - 1 + \frac{x}{x^2-1}$$

The order of division can be reversed.

6.(b)(i)Multiplying <u>both</u> sides by $\cos x(1+\sin x)$

$\cos^2 x+(1+\sin x)^2 = 2\sec x\cos x(1+\sin x)$ M1

By analogy with the main scheme, the RHS must be correct and at least one term on the LHS modified.

LHS $= \cos^2 x+1+2\sin x+\sin^2 x$ A1

$= 2+2\sin x$ M1

RHS $= 2+2\sin x$

LHS = RHS

Hence $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x} = 2\sec x$ A1

With this method there must be a minimal conclusion, e.g. qed, W or a reference to the identity which has to be proved. Here LHS=RHS is insufficient – that amounts to $2+2\sin x = 2+2\sin x$, which is an insufficient conclusion, as is 0 = 0 etc.

7.(c) $3\sin 2x+4\cos 2x=0$

$\tan 2x = -\dfrac{4}{3}$ M1

Allow this M1 for putting $y=0$ and reducing the equation to $\tan 2x$ or $\cot 2x$ equals any constant other than zero.

$2x = -4.0688\ ...,\ -0.9272\ ...,\ 2.2142\ ...,\ 5.3558\ ...$

$x = -2.03, -0.46, 1.11, 2.68$ A1 + A1 + A1 Awarded as in main scheme.

8. (a) Using mappings

$\mathrm{f} : x \xrightarrow{\text{cube}} x^3 \xrightarrow{\times(-2)} -2x^3 \xrightarrow{+1} 1-2x^3$

Reversing

$\mathrm{f}^{-1} : x \xrightarrow{-1} x-1 \xrightarrow{\div(-2)} \dfrac{1-x}{2} \xrightarrow{\text{cube root}} \sqrt[3]{\dfrac{1-x}{2}}$

For the M, the candidate must break the function up into 3, or possibly 4 steps ($\times(-2)$ can be broken into 2 steps) and f^{-1} must be formed by reversing their steps.
The A is then awarded for carrying out the procedures correctly.