January 2007
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1. (a)	A random variable;	B1
	function of known observations (from a population). data OK	B1 (2)
(b) (i)	Yes	B1 (1)
(ii)	No	B1 (1)
		Total 4
2. (a)	$P(J \geq 10) = 1 - P(J \leq 9)$ or $=1-P(J<10)$	M1
	$= 1 - 0.9919$ implies method	
	$= 0.0081$ awrt 0.0081	A1 (2)
(b)	$P(K \leq 1) = P(K = 0) + P(K = 1)$ both, implied below even with '25' missing	M1
	$= (0.73)^{25} + 25(0.73)^{24}(0.27)$ clear attempt at '25' required	M1
	$= 0.00392$ awrt 0.0039 implies M	A1 (3)
		Total 5

Question Number	Scheme		Marks
3. (a)	Let W represent the number of white plants.		
	$W \sim B(12,0.45)$	use of	B1
	$P(W=5) = P(W \le 5) - P(W \le 4)$	${}^{12}C_5 0.45^5 0.55^7$ or equivalent award B1M1	M1
	$= 0.5269 - 0.3044$	values from correct table implies B	
	$= 0.2225$	awrt 0.222(5)	A1 **(3)**
(b)	$P(W \ge 7) = 1 - P(W \le 6)$	or $=1-P(W<7)$	M1
	$= 1 - 0.7393$	implies method	
	$= 0.2607$	awrt 0.261	A1 **(2)**
(c)	P(3 contain more white than coloured)$=\frac{10!}{3!7!}(0.2607)^3(1-0.2607)^7$	use of B,n=10	M1A1∫
	$= 0.256654\ldots$	awrt 0.257	A1 **(3)**
(d)	mean = np = 22.5 ; var = npq = 12.375		B1B1
	$P(W > 25) \approx P\left(Z > \frac{25.5-22.5}{\sqrt{12.375}}\right)$	± standardise with σ and μ; ±0.5 c.c.	M1;M1
	$\approx P(Z > 0.8528..)$	awrt 0.85	A1
	$\approx 1 - 0.8023$	'one minus'	M1
	≈ 0.1977	awrt 0.197 or 0.198	A1
			(7)
			Total 15

Question Number	Scheme	Marks
4. (a)	λ > 10 or large μ ok	**B1** **(1)**
(b)	The Poisson is discrete and the normal is continuous.	**B1** **(1)**
(c)	Let Y represent the number of yachts hired in winter	
	$\text{P}(Y<3) = \text{P}(Y \leqslant 2)$ $\text{P}(Y \leqslant 2)$ & Po(5)	**M1**
	= 0.1247 awrt 0.125	**A1** **(2)**
(d)	Let X represent the number of yachts hired in summer X~Po(25).	
	N(25,25) all correct, can be implied by standardisation below	**B1**
	$\text{P}(X > 30) \approx \text{P}\left(Z > \frac{30.5-25}{5}\right)$ ± standardise with 25 & 5; ±0.5 c.c.	**M1;M1**
	≈ P(Z > 1.1) 1.1	**A1**
	≈ 1 – 0.8643 'one minus'	**M1**
	≈ 0.1357 awrt 0.136	**A1** **(6)**
(e)	no. of weeks = 0.1357 x 16 ANS (d)x16	**M1**
	= 2.17 or 2 or 3 ans>16 M0A0	**A1∫** **(2)**
		Total 12

Question Number	Scheme		Marks
5.			
(a)	$\text{f}(x) = \begin{cases} \frac{1}{\beta - \alpha}, & \alpha < x < \beta, \\ 0, & \text{otherwise.} \end{cases}$	function including inequality, 0 otherwise	**B1,B1**
			(2)
(b)	$\frac{\alpha + \beta}{2} = 2, \quad \frac{3 - \alpha}{\beta - \alpha} = \frac{5}{8}$	or equivalent	**B1,B1**
	$\alpha + \beta = 4$ $3\alpha + 5\beta = 24$		
	$3(4 - \beta) + 5\beta = 24$ $2\beta = 12$ $\beta = 6$	attempt to solve 2 eqns	**M1**
	$\alpha = -2$	both	**A1**
			(4)
(c)	$\text{E}(X) = \frac{150 + 0}{2} = 75 \text{ cm}$	75	**B1**
			(1)
(d)	Standard deviation = $\sqrt{\frac{1}{12}(150 - 0)^2}$		**M1**
	$= 43.30127\ldots\text{cm}$	$25\sqrt{3}$ or awrt 43.3	**A1**
			(2)
(e)	$\text{P}(X < 30) + \text{P}(X > 120) = \frac{30}{150} + \frac{30}{150}$	1st or at least one fraction, + or double	**M1,M1**
	$= \frac{60}{150}$ or $\frac{2}{5}$ or 0.4 or equivalent fraction		**A1**
			(3)
			Total 12

Question Number	Scheme	Marks
6. (a)	$H_0 : p = 0.20$, $H_1: p < 0.20$	**B1,B1**
	Let X represent the number of people buying family size bar. $X \sim B(30, 0.20)$	
	$P(X \leqslant 2) = 0.0442$ or $P(X \leqslant 2) = 0.0442$ awrt 0.044 $P(X \leqslant 3) = 0.1227$ CR $X \leqslant 2$	**M1A1**
	$0.0442 < 5\%$, so significant. Significant	**M1**
	There is evidence that the no. of family size bars sold is lower than usual.	**A1** **(6)**
(b)	$H_0 : p = 0.02$, $H_1: p \neq 0.02$ $\lambda = 4$ etc ok both	**B1**
	Let Y represent the number of gigantic bars sold.	
	$Y \sim B(200, 0.02) \Rightarrow Y \sim Po(4)$ can be implied below	**M1**
	$P(Y = 0) = \mathbf{0.0183}$ and $P(Y \leqslant 8) = \mathbf{0.9786} \Rightarrow P(Y \geqslant 9) = \mathbf{0.0214}$ first, either	**B1,B1**
	Critical region $Y = 0 \cup Y \geqslant 9$ $Y \leq 0$ ok	**B1,B1**
	N.B. Accept exact Bin: 0.0176 and 0.0202	
(c)	Significance level = 0.0183 + 0.0214 = 0.0397 awrt 0.04	**B1** **(1)**
		Total 13

Question Number	Scheme	Marks
7. (a)	$1 - F(0.3) = 1 - (2 \times 0.3^2 - 0.3^3)$ $= 0.847$ 'one minus' required	M1 A1 (2)
(b)	$F(0.60) = 0.5040$ $F(0.59) = 0.4908$ both required awrt 0.5, 0.49 0.5 lies between therefore median value lies between 0.59 and 0.60.	M1A1 B1 (3)
(c)	$f(x) = \begin{cases} -3x^2 + 4x, & 0 \le x \le 1, \\ 0\ , & \text{otherwise.} \end{cases}$ attempt to differentiate, all correct	M1A1 (2)
(d)	$\int_0^1 x f(x) dx = \int_0^1 -3x^3 + 4x^2 dx$ attempt to integrate $x f(x)$ $= \left[\frac{-3x^4}{4} + \frac{4x^3}{3} \right]_0^1$ sub in limits $= \frac{7}{12}$ or $0.58\dot{3}$ or 0.583 or equivalent fraction	M1 M1 A1 (3)
(e)	$\frac{df(x)}{dx} = -6x + 4 = 0$ attempt to differentiate f(x) and equate to 0 $x = \frac{2}{3}$ or $0.\dot{6}$ or 0.667	M1 A1 (2)
(f)	mean < median < mode, therefore negative skew. Any pair, cao	B1,B1 (2)
		Total 14