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8.

Figure 2

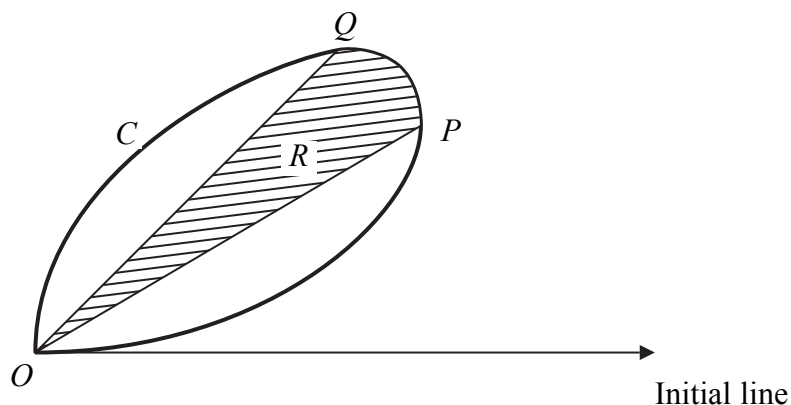


Figure 2 shows a sketch of the curve C with polar equation

$$r = 4 \sin \theta \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The tangent to C at the point P is perpendicular to the initial line.

- (a) Show that P has polar coordinates $\left(\frac{3}{2}, \frac{\pi}{6}\right)$. (6)

The point Q on C has polar coordinates $\left(\sqrt{2}, \frac{\pi}{4}\right)$.

The shaded region R is bounded by OP , OQ and C , as shown in Figure 2.

- (b) Show that the area of R is given by

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\sin^2 2\theta \cos 2\theta + \frac{1}{2} - \frac{1}{2} \cos 4\theta \right) d\theta. \quad (3)$$

- (c) Hence, or otherwise, find the area of R , giving your answer in the form $a + b\pi$, where a and b are rational numbers. (5)



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