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2. Obtain the general solution of the differential equation

$$x \frac{dy}{dx} + 2y = \cos x, \quad x > 0,$$

giving your answer in the form $y = f(x)$.

(8)

A series of horizontal lines for writing the solution.



3. The complex numbers z_1 and z_2 are given by

$$z_1 = 5 + 3i,$$
$$z_2 = 1 + pi,$$

where p is an integer.

(a) Find $\frac{z_2}{z_1}$ in the form $a + ib$, where a and b are expressed in terms of p . (3)

Given that $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{4}$,

(b) find the value of p . (2)



Question 4 continued

(This area contains horizontal lines for writing an answer.)

(Total 9 marks)

Q4



5.

Figure 1

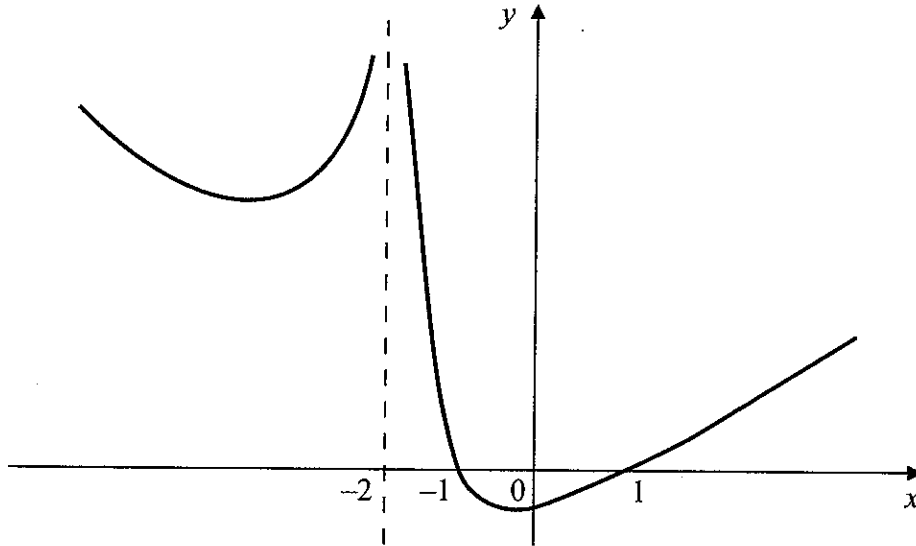


Figure 1 shows a sketch of the curve with equation

$$y = \frac{x^2 - 1}{|x + 2|}, \quad x \neq -2.$$

The curve crosses the x -axis at $x = 1$ and $x = -1$ and the line $x = -2$ is an asymptote of the curve.

(a) Use algebra to solve the equation $\frac{x^2 - 1}{|x + 2|} = 3(1 - x)$.

(6)

(b) Hence, or otherwise, find the set of values of x for which

$$\frac{x^2 - 1}{|x + 2|} < 3(1 - x).$$

(3)



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Question 5 continued

Lined writing area for the answer to Question 5.



Question 6 continued

Lined writing area for the answer to Question 6.

(Total 12 marks)

Q6



7. A scientist is modelling the amount of a chemical in the human bloodstream. The amount x of the chemical, measured in mg l^{-1} , at time t hours satisfies the differential equation

$$2x \frac{d^2x}{dt^2} - 6 \left(\frac{dx}{dt} \right)^2 = x^2 - 3x^4, \quad x > 0.$$

(a) Show that the substitution $y = \frac{1}{x^2}$ transforms this differential equation into

$$\frac{d^2y}{dt^2} + y = 3. \quad \boxed{\text{I}}$$

(5)

(b) Find the general solution of differential equation $\boxed{\text{I}}$.

(4)

Given that at time $t = 0$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$,

(c) find an expression for x in terms of t ,

(4)

(d) write down the maximum value of x as t varies.

(1)



Question 7 continued

Handwriting practice lines for question 7.

(Total 14 marks)

Q7



8.

Figure 2

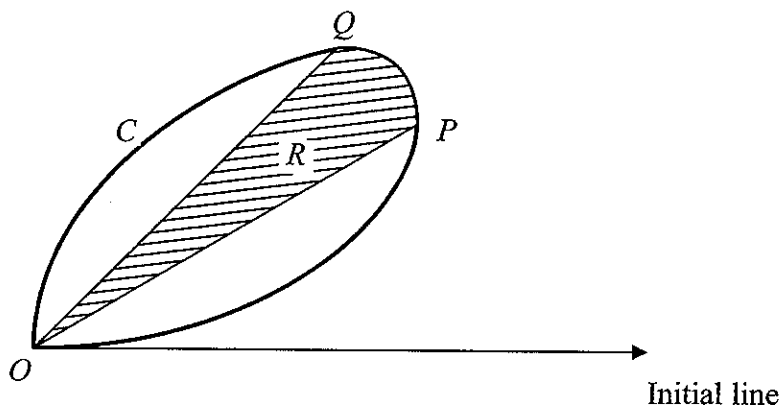


Figure 2 shows a sketch of the curve C with polar equation

$$r = 4 \sin \theta \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The tangent to C at the point P is perpendicular to the initial line.

(a) Show that P has polar coordinates $\left(\frac{3}{2}, \frac{\pi}{6}\right)$.

(6)

The point Q on C has polar coordinates $\left(\sqrt{2}, \frac{\pi}{4}\right)$.

The shaded region R is bounded by OP , OQ and C , as shown in Figure 2.

(b) Show that the area of R is given by

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\sin^2 2\theta \cos 2\theta + \frac{1}{2} - \frac{1}{2} \cos 4\theta \right) d\theta.$$

(3)

(c) Hence, or otherwise, find the area of R , giving your answer in the form $a + b\pi$, where a and b are rational numbers.

(5)



Question 8 continued

Lined area for writing the answer to Question 8.

(Total 14 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

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