## Edexcel GCE

## Mechanics M3

## Advanced Level

## Monday 12 June 2006 - Afternoon

## Time: 1 hour 30 minutes

Materials required for examination<br>Items included with question papers<br>Answer Book (AB16)<br>Nil<br>Mathematical Formulae (Lilac)<br>Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of $g$ is required, take $g=9.8 \mathrm{~m} \mathrm{~s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 7 questions.
The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A uniform solid is formed by rotating the region enclosed between the curve with equation $y=\sqrt{ }$, the $x$-axis and the line $x=4$, through one complete revolution about the $x$-axis. Find the distance of the centre of mass of the solid from the origin $O$.
2. A bowl consists of a uniform solid metal hemisphere, of radius $a$ and centre $O$, from which is removed the solid hemisphere of radius $\frac{1}{2} a$ with the same centre $O$.
(a) Show that the distance of the centre of mass of the bowl from $O$ is $\frac{45}{112} a$.

The bowl is fixed with its plane face uppermost and horizontal. It is now filled with liquid. The mass of the bowl is $M$ and the mass of the liquid is $k M$, where $k$ is a constant. Given that the distance of the centre of mass of the bowl and liquid together from $O$ is $\frac{17}{48} a$,
(b) find the value of $k$.
3. A particle $P$ of mass 0.2 kg oscillates with simple harmonic motion between the points $A$ and $B$, coming to rest at both points. The distance $A B$ is 0.2 m , and $P$ completes 5 oscillations every second.
(a) Find, to 3 significant figures, the maximum resultant force exerted on $P$.

When the particle is at $A$, it is struck a blow in the direction $B A$. The particle now oscillates with simple harmonic motion with the same frequency as previously but twice the amplitude.
(b) Find, to 3 significant figures, the speed of the particle immediately after it has been struck.
4. Figure 1


A hollow cone, of base radius $3 a$ and height $4 a$, is fixed with its axis vertical and vertex $V$ downwards, as shown in Figure 1. A particle moves in a horizontal circle with centre $C$, on the smooth inner surface of the cone with constant angular speed $\sqrt{\frac{8 g}{9 a}}$.

Find the height of $C$ above $V$.
5. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N . A particle $P$ of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points $A$ and $B$, where $A B$ is horizontal and $A B=1.5 \mathrm{~m}$.

## Figure 2



The particle is held at the mid-point of $A B$. The particle is released from rest, as shown in Figure 2.
(a) Find the speed of $P$ when it has fallen a distance of 1 m .

Given instead that $P$ hangs in equilibrium vertically below the mid-point of $A B$, with $\angle A P B=2 \alpha$,
(b) show that $\tan \alpha+5 \sin \alpha=5$.
6. A particle moving in a straight line starts from rest at the point $O$ at time $t=0$. At time $t$ seconds, the velocity $v \mathrm{~m} \mathrm{~s}^{-1}$ of the particle is given by

$$
\begin{array}{lr}
v=3 t(t-4), & 0 \leq t \leq 5, \\
v=75 t^{-1}, & 5 \leq t \leq 10 .
\end{array}
$$

(a) Sketch a velocity-time graph for the particle for $0 \leq t \leq 10$.
(b) Find the set of values of $t$ for which the acceleration of the particle is positive.
(c) Show that the total distance travelled by the particle in the interval $0 \leq t \leq 5$ is 39 m .
(d) Find, to 3 significant figures, the value of $t$ at which the particle returns to $O$.
7. One end of a light inextensible string of length $l$ is attached to a particle $P$ of mass $m$. The other end is attached to a fixed point $A$. The particle is hanging freely at rest with the string vertical when it is projected horizontally with speed $\sqrt{\frac{5 g l}{2}}$.
(a) Find the speed of $P$ when the string is horizontal.

When the string is horizontal it comes into contact with a small smooth fixed peg which is at the point $B$, where $A B$ is horizontal, and $A B<l$. Given that the particle then describes a complete semicircle with centre $B$,
(b) find the least possible value of the length $A B$.

