Paper Reference(s)

6674/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Level

Monday 19 June 2006 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P4/Further Pure Mathematics FP1), the paper reference (6674), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N20461A

1. The complex numbers z and w satisfy the simultaneous equations

$$2z + \text{i}w = -1,$$

$$z - w = 3 + 3\text{i}.$$

(a) Use algebra to find z, giving your answers in the form $a + \text{i}b$, where a and b are real.

(4)

(b) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

2.

Figure 1

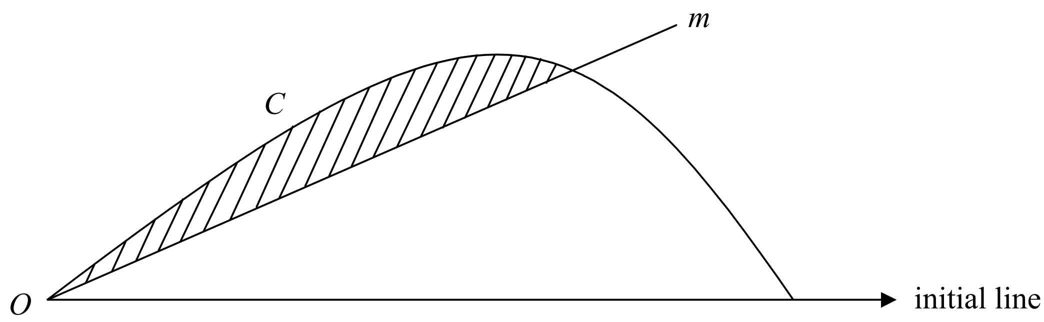

Figure 1 shows a curve C with polar equation $r = 4a \cos 2\theta$, $0 \leq \theta \leq \frac{\pi}{4}$, and a line m with polar equation $\theta = \frac{\pi}{8}$. The shaded region, shown in Figure 1, is bounded by C and m. Use calculus to show that the area of the shaded region is $\frac{1}{2}a^2(\pi - 2)$.

(7)

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3. Given that $3x \sin 2x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4y = k \cos 2x,$$

where k is a constant,

(a) calculate the value of k,

(4)

(b) find the particular solution of the differential equation for which at $x = 0$, $y = 2$, and for which at $x = \frac{\pi}{4}$, $y = \frac{\pi}{2}$.

(4)

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4. Given that $3 - 2i$ is a solution of the equation

$$x^4 - 6x^3 + 19x^2 - 36x + 78 = 0,$$

(a) solve the equation completely,

(7)

(b) show on a single Argand diagram the four points that represent the roots of the equation.

(2)

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5. Given that for all real values of r,

$$(2r + 1)^3 - (2r - 1)^3 = Ar^2 + B,$$

where A and B are constants,

(a) find the value of A and the value of B.

(2)

(b) Hence, or otherwise, prove that $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n + 1)(2n + 1)$.

(5)

(c) Calculate $\sum_{r=1}^{40} (3r - 1)^2$.

(3)

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6. $$\mathrm{f}(x) = 0.25x - 2 + 4 \sin \sqrt{x}.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α between $x = 0.24$ and $x = 0.28$.

(2)

(b) Starting with the interval [0.24, 0.28], use interval bisection three times to find an interval of width 0.005 which contains α.

(3)

The equation $\mathrm{f}(x) = 0$ also has a root β between $x = 10.75$ and $x = 11.25$.

(c) Taking 11 as a first approximation to β, use the Newton-Raphson process on $\mathrm{f}(x)$ once to obtain a second approximation to β. Give your answer to 2 decimal places.

(6)

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7. (a) Use algebra to find the exact solutions of the equation

$$|2x^2 + x - 6| = 6 - 3x.$$

(6)

(b) On the same diagram, sketch the curve with equation $y = |2x^2 + x - 6|$ and the line with equation $y = 6 - 3x$.

(3)

(c) Find the set of values of x for which

$$|2x^2 + x - 6| > 6 - 3x.$$

(3)

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8. During an industrial process, the mass of salt, S kg, dissolved in a liquid t minutes after the process begins is modelled by the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{2S}{120 - t} = \frac{1}{4}, \quad 0 \leq t < 120.$$

Given that $S = 6$ when $t = 0$,

(a) find S in terms of t,

(8)

(b) calculate the maximum mass of salt that the model predicts will be dissolved in the liquid at any one time during the process.

(4)

TOTAL FOR PAPER: 75 MARKS

END