Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Thursday 15 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

2.
$$\mathrm{f}(x) = \frac{3x-1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(*a*) find the values of A and B.

(3)

(*b*) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)

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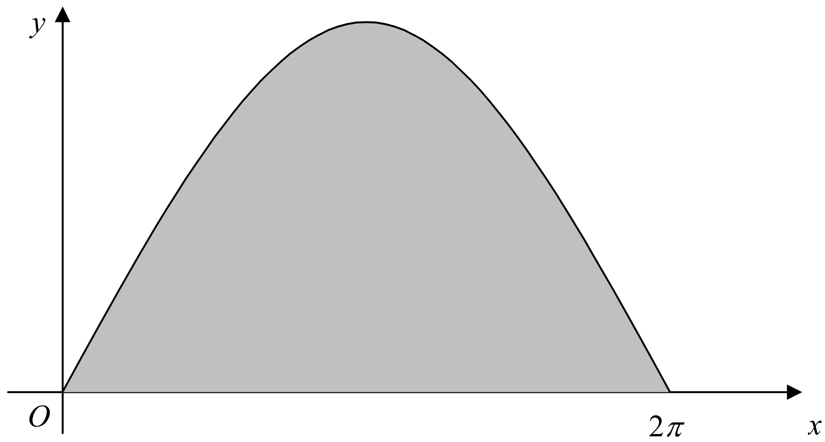
3. **Figure 3**



The curve with equation $y = 3 \sin \frac{x}{2}$, $0 \leq x \leq 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(*a*) Find, by integration, the area of the shaded region.

(3)

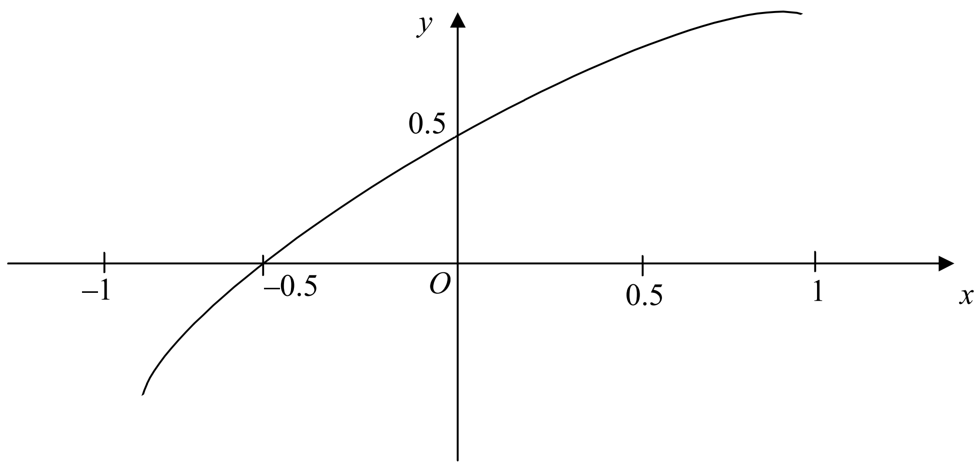
This region is rotated through 2π radians about the x-axis.

(*b*) Find the volume of the solid generated.

(6)

4.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(*a*) Find an equation of the tangent to the curve at the point where $t = \dfrac{\pi}{6}$.

(6)

(*b*) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

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5. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(*a*) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(*b*) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(*c*) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

6. **Figure 3**

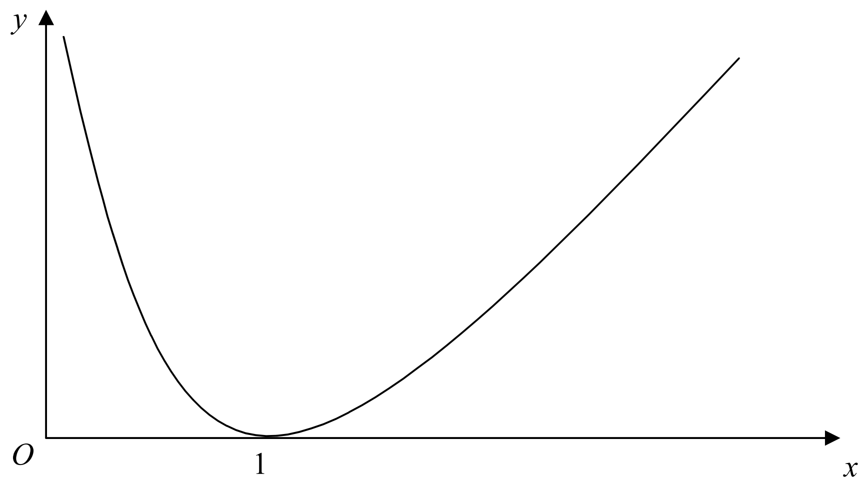


Figure 3 shows a sketch of the curve with equation $y = (x - 1)\ln x,\ x > 0$.

(*a*) Copy and complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		ln 2		2 ln 3

(1)

Given that $I = \int_1^3 (x-1)\ln x \;\, dx$,

(*b*) use the trapezium rule

(i) with values at y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values at y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

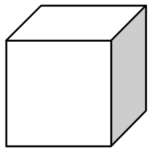
(*c*) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(*d*) Show, by integration, that the exact value of $\int_1^3 (x-1)\ln x \;\, dx$ is $\frac{3}{2}\ln 3$.

(6)

7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(*a*) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(*b*) $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(*c*) solve the differential equation in part (*b*), and find the value of t when $V = 16\sqrt{2}$.

(7)

TOTAL FOR PAPER: 75 MARKS

END