Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 12 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (*a*) Simplify $\frac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(*b*) Hence, or otherwise, express $\frac{3x^2 - x - 2}{x^2 - 1} - \frac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

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2. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(*b*) $(5 + x^2)^{\frac{3}{2}}$.

(3)

3.

Figure 1

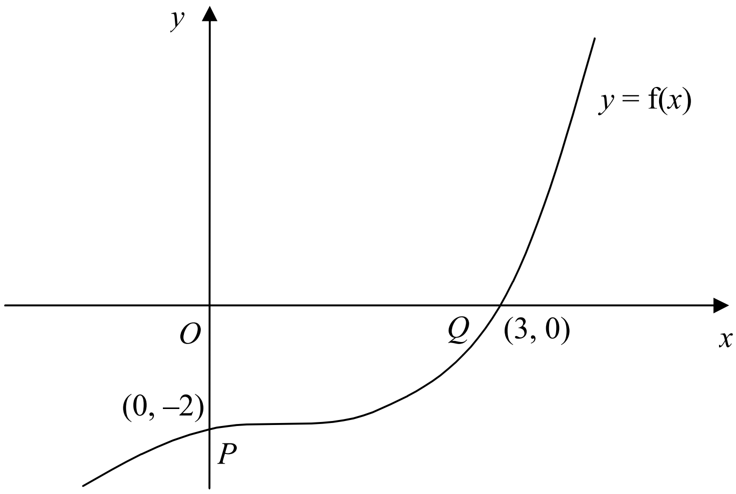

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$,

(3)

(*c*) $y = \frac{1}{2}\mathrm{f}(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\mathrm{e}^{-0.05t} + 25, \quad t \geq 0.$$

(*a*) Find the temperature of the ball as it enters the liquid.

(1)

(*b*) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(*c*) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(*d*) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

5.

Figure 2

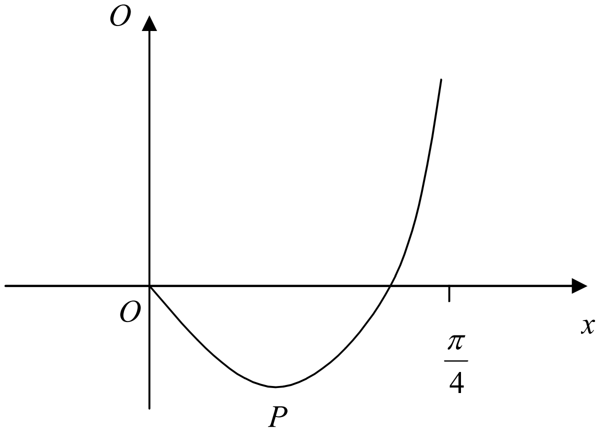

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \tfrac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimals places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

6. (*a*) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that the $\text{cosec}^2\,\theta - \cot^2\,\theta \equiv 1$.

(2)

(*b*) Hence, or otherwise, prove that

$$\text{cosec}^4\,\theta - \cot^4\,\theta \equiv \text{cosec}^2\,\theta + \cot^2\,\theta.$$

(2)

(*c*) Solve, for $90° < \theta < 180°$,

$$\text{cosec}^4\,\theta - \cot^4\,\theta = 2 - \cot\,\theta.$$

(6)

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7. For the constant k, where $k > 1$, the functions f and g are defined by

$$\text{f}: x \mapsto \ln(x+k), \quad x > -k,$$

$$\text{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(*a*) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(*b*) Write down the range of f.

(1)

(*c*) Find $\text{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \text{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(*d*) Find the value of k.

(4)

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8. (*a*) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(*b*) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$.

(4)

TOTAL FOR PAPER: 75 MARKS

END