Question Number	Scheme	Marks
1	$x^2 - 2x + 17 = (x-1)^2 + 16$	B1
	$I = \int_1^4 \frac{1}{\sqrt{(x-1)^2+16}}\,dx = [\ \text{arsinh}\ \frac{(x-1)}{4}\ ]$ or equiv. = arsinh ¾	M1 A1√
	[M1 does not require limits; A1 f.t. on completing square, providing arsinh]	
	Into ln form [$\ln\left[\frac{3}{4} + \sqrt{\frac{9}{16}+1}\right]$; = ln2	M1A1
		[5]
	[If straight to ln form : B1, $\ln\left[(x-1) + \sqrt{(x-1)^2+16}\right]$ M1	
	Using limits correctly M1A1√ , ln2 A1]	
2	(a) Using $b^2 = a^2(e^2 - 1)$; [$4 = 16(e^2 - 1)$] $e = \frac{\sqrt{5}}{2}$ or equiv. (1.12)	M1A1 **(2)**
	(b) Distance between foci = 2 a e [2 x 4 x $\frac{\sqrt{5}}{2}$] ; = $4\sqrt{5}$	M1A1√ **(2)**
	[A1√ dependent on both Ms]	
	(c) Ellipse, centred on origin	B1
	Hyperbola, both branches	B1
	Totally correct, touching, with correct intercepts	B1
		(3)
		[7]

3	$\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \sin t,$ (both)	B1
	$s = \int \sqrt{\left((1+\cos t)^2 + (\sin t)^2\right)}\, dt \quad ; \quad = \int \sqrt{2 + 2\cos t}\, dt$	M1A1
	Use of "half-angle formula" $[\int \sqrt{4\cos^2 t}\, dt]$; s = $\left[4 \sin \frac{t}{2}\right]_{(0)}^{\left(\frac{\pi}{2}\right)}$	M1A1√
	Using limits correctly and surd form; $= 2\sqrt{2}$ (allow $\frac{4}{\sqrt{2}}$)	M1A1 **[7]**
4	Using $\cosh x = \frac{e^x + e^{-x}}{2}$ and attempt to progress	M1
	Correct intermediate step as far as $4\left(\frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8}\right) - \left[3\left(\frac{e^x + e^{-x}}{2}\right)\right]$	A1
	$= \frac{e^{3x} + e^{-3x}}{2} = \cosh 3x$	A1 **(3)**
	(b) Using part (a) to reduce to $\cosh^2 x = [\,2\,]$	M1
	Correct method to form $\ln x$ or find e^X or e^{2x}	M1
	$x = \ln\left(\sqrt{2} + 1\right), \quad \ln\left(\sqrt{2} - 1\right)$ or equivalent	A1 A1√
	or $\frac{1}{2}\ln\left(3 + 2\sqrt{2}\right), \quad \frac{1}{2}\ln\left(3 - 2\sqrt{2}\right)$, (after finding e^{2x} = ……)	**(4)** **[7]**

Question Number	Scheme	Marks
5	$\frac{dy}{dx} = \frac{1}{\sec x}.\sec x \tan x = \tan x$; $\psi = x$	B1B1
	$s = \int \sqrt{(1 + \left(\frac{dy}{dx}\right)_c^2} dx = \int \sqrt{(1 + \tan x_c^2} dx$	M1
	$= \int \sec x \, dx$; $= \ln(\sec x + \tan x)$ $(+ c)$ or equivalent	A1A1
	Method to find "c": $s = 0,\ x = \frac{\pi}{3} \Rightarrow 0 = \ln(2 + \sqrt{3}) + c$	M1
	$c = -\ln(2 + \sqrt{3})$ or $-\ln\tan\frac{5\pi}{12}$ or -1.32	A1
	$\Rightarrow s = \ln(\sec\psi + \tan\psi) - \ln(2 + \sqrt{3})$ or equivalent	A1√ **[8]**
6	(a) $\frac{dy}{dx} = 3\cosh^2 x \sinh x$	B1
	$\frac{d^2y}{dx^2} = 3\cosh^3 x + 6\cosh x \sinh^2 x$ or equivalent	M1A1
	Application of formula for radius of curvature [$\rho = \frac{(1 + 9\cosh^4 x \sinh^2 x)^{\frac{3}{2}}}{3\cosh^3 x + 6\cosh x \sinh^2 x}$]	M1
	Use of $\sinh^2 x = \cosh^2 x - 1$	M1
	$\rho = \frac{(1 + 9c^4(c^2 - 1))^{\frac{3}{2}}}{3c\{c^2 + 2(c^2 - 1)\}} = \frac{(9c^6 - 9c^4 + 1)^{\frac{3}{2}}}{3c\{3c^2 - 2)\}}$ AG	A1* (6)
	(b) $\cosh x = \frac{5}{4}$	B1
	Using found value of $\cosh x$ in formula for ρ ; $\rho = 4.8$	M1A1 (3) **[9]**

7	(a) $I_n = -\frac{2}{3}\left[x^n (4-x)^{\frac{3}{2}}\right]_0^4 + \frac{2}{3}n\int_0^4 x^{n-1}(4-x)^{\frac{3}{2}}\,dx$	M1A1
	$= \frac{2}{3}n\int_0^4 x^{n-1}(4-x)^{\frac{3}{2}}\,dx$	A1√
	$= \frac{2}{3}n\int_0^4 4x^{n-1}(4-x)^{\frac{1}{2}}\,dx - \frac{2}{3}n\int_0^4 x^n (4-x)^{\frac{1}{2}}\,dx$	M1A1
	$\Rightarrow I_n = \frac{8}{3}n I_{n-1} - \frac{2}{3}n I_n$	
	[$(2n+3)I_n = 8nI_{n-1}$] $\Rightarrow I_n = \frac{8n}{2n+3} I_{n-1}$ AG	A1* (6)
	(b) Relating I_2 to I_0 using result from (a)	M1
	$I_2 = \frac{16}{7}.\frac{8}{5} I_0 = \frac{2048}{105} \left(19\frac{53}{105}\right)$	A1A1 (3)
		[9]

8	(a) ar tanh $\frac{1}{\sqrt{2}} = \frac{1}{2}\ln\left(\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right)$ or equivalent	M1
	$= \frac{1}{2}\ln\left[\frac{\sqrt{2}+1}{\sqrt{2}-1} \text{ x } \frac{\sqrt{2}+1}{\sqrt{2}+1}\right]$ or equivalent	M1
	$= \frac{1}{2}\ln\left(\sqrt{2}+1\right)^2 = \ln\left(\sqrt{2}+1\right)$ AG	A1* (3)
	Alternative Approach	
	If using $y =$ ar tanh (sin x) $\Rightarrow$ tanh $y = \left(\frac{1}{\sqrt{2}}\right)$ [or $\cosh^2 y = 2$] and then use exponentials:	
	Progression as far as $e^y = \ldots\ldots$ *or* $e^{2y} = \ldots\ldots$ — M1 Converting to ln form — M1 Answer as given — A1*	
	Note: $\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ can earn M1M1 but for A1* there must be a convincing further step.	
	(b) $\frac{dy}{dx} = \frac{1}{1-\sin^2 x}.\ \cos x$; $= \frac{\cos x}{\cos^2 x} = \sec x$	M1A1 (2)
	Note: If $\tanh y = \sin x$ is differentiated M1 requires $\frac{dy}{dx} = f(x)$	
	(c) Attempt at by parts and use of result in (b)	M1
	$= -\cos x \text{ ar tanh}(\sin x) + \int \cos x \sec x \, d\, dx$	A1
	$= -\cos x \text{ ar tanh}(\sin x) + x$	M1
	Using limits correctly : $= -\frac{1}{\sqrt{2}}\ln\left(1+\sqrt{2}\right) + \frac{\pi}{4}$ or exact equivalent	M1A1 (5) **[10]**

9	(a) Correct method for finding $\frac{dy}{dx}$ $\left[\frac{1}{P}\right]$	M1
	Gradient of normal $= -p$	A1
	Equation of normal: $y - 2ap = (-p)(x - ap^2)$	M1
	$y + px = 2ap + ap^3$ AG	A1* (4)
	(b) Using both equations and eliminating x or y	M1
	$(p-q)x = 2a(p-q) + a(p^3 - q^3)$ may be unsimplified	A1
	$x = 2a + a(p^2 + pq + q^2)$	A1
	Finding the other coordinate	M1
	$y = -apq(p+q)$	A1 (5)
	(c) Using $pq = 3$ in both x and y (in any form)	
	[$x = a(p^2 + q^2 + 5)$, $y = -3a(p+q)$]	M1
	Complete method for relating x and y, independent of p and q	M1
	A correct equation, in any form	A1
	[e.g. : $y^2 = 9a^2(p^2 + q^2 + 2pq) = 9a^2\left\{\left(\frac{x-5a}{a}\right) + 6\right\}$]	
	$y^2 = 9a(x+a)$	A1√ (4) **[13]**