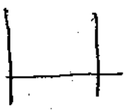
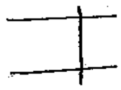
January 2006
6690 Decision D2
Mark Scheme

Question Number	Scheme	Marks
1)	To maximise, subtract all entries from $n \geq 278$ e.g. $\begin{bmatrix} 11 & 6 & 2 & 17 \\ 14 & 7 & 0 & 15 \\ 11 & 5 & 3 & 15 \\ 17 & 9 & 4 & 21 \end{bmatrix}$	M1 A1 (2)
	Reduce rows $\begin{bmatrix} 9 & 4 & 0 & 15 \\ 14 & 7 & 0 & 15 \\ 8 & 2 & 0 & 12 \\ 13 & 5 & 0 & 17 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 6 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 \end{bmatrix}$	M1 A1√ A1√ (3)
	 min element = 1 $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 5 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$	M1 A1√ A1√ (3)
	 min element = 1 $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 4 & 1 & 0 & 3 \end{bmatrix}$ then  min element 1 or  min element = 2 $\begin{bmatrix} 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$ optimal	M1 A1√ A1√ (3)
	$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$ optimal So A – H, B – P, C – S, D – I or H, S, I, P (both £1077)	M1 A1 (2) 13

Question Number	Scheme	Marks
2) e.g.		

stage	state	Action	Dest	value	Marks
1 (Sept)	2	2	0	200 + 200 = 400 *	M1 A1
	1	3	0	200 + 100 = 300 *	
	0	4	0	200 = 200 *	
2 (Aug)	2	5	2	200 + 200 + 500 + 400 = 1300	
		4	1	200 + 200 + 300 = 700	M1
		3	0	200 + 200 + 200 = 600 *	
	1	5	1	200 + 100 + 500 + 300 = 1100	A1√
		4	0	200 + 100 + 200 = 500 *	
	0	5	0	200 + 500 + 200 = 900 *	(4)
3 (July)	2	5	0	200 + 200 + 500 + 900 = 1800 *	M1 A1√
4 (June)	2	3	2	200 + 200 + 1800 = 2200 *	
	1	4	2	200 + 100 + 1800 = 2100 *	A1√
	0	5	2	200 + 500 + 1800 = 2500 *	
5 (may)	0	5	2	200 + 500 + 2200 = 2900	M1
		4	1	200 + 2100 = 2300 *	A1√
		3	0	200 + 2500 = 2700	(5)

month	may	June	July	August	September
production schedule	4	4	5	5	4

M1 A1√

cost £2300

A1√ (3)

[12]

3)	Let x_{ij} be the <u>number</u> of units transported from i to j, in 1000 litres where $i \in \{F, G, H\}$ and $j \in \{S, T, U\}$	B2, 1, 0 (2)
	minimise $C = 23x_{fs} + 31x_{ft} + 46x_{fu} + 35x_{gs} + 38x_{gt} + 51x_{gu} + 41x_{hs} + 50x_{ht} + 63x_{hu}$	B1 B1 (2)
	unblanced	
	Subject to $x_{fs} + x_{ft} + x_{fu} \leq 540$	
	$x_{gs} + x_{gt} + x_{gu} \leq 789$	M1
	$x_{hs} + x_{ht} + x_{hu} \leq 673$	A1
	$x_{fs} + x_{gs} + x_{hs} \leq 257$	
	$x_{ft} + x_{gt} + x_{ht} \leq 348$ (accept = here)	A1
	$x_{fu} + x_{gu} + x_{hu} \leq 412$	(3)
	$x_{ij} \geq 0$	B1 (1)
		8
	Accept introduction of a dummy demand methods.	

4) (a)	Adds zeros for cost in third column Adds 14 as the demand value	B1 B1 (2)
(b)	The total supply is greater than the total demand	B2, 1, 0 (2)
(c)	The solution would otherwise be degenerate	B1 (1)
(d)		

		J (10)	K (15)	L (0)
0	A		8	1
0	B			13
−6	C	9	3	

$I_{AJ} = 12 - 0 - 10 = 2$

$I_{BJ} = 8 - 0 - 10 = -2$ *

$I_{BK} = 17 - 0 - 15 = 2$

$I_{CL} = 0 + 6 - 0 = 6$

M1 A1 A1 A1 (4)

	J	K	L
A		$8-\theta$	$1+\theta$
B	θ		$13-\theta$
C	$9-\theta$	$3+\theta$	

$\theta = 8$

Entering square BJ

Exiting square AK

M1 A1✓ (2)

		J (8)	K (13)	L (0)
0	A			9
0	B	8		5
−4	C	1	11	

$I_{AJ} = 12 - 0 - 8 = 4$

$I_{AK} = 15 - 0 - 13 = 2$

$I_{BK} = 17 - 0 - 13 = 4$

$I_{CL} = 0 + 4 - 0 = 4$

No negatives, so optimal

M1 A1✓ A1✓ A1✓ A1 (5)

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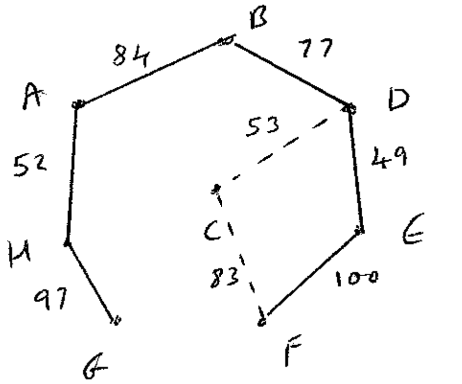
5) (a)	Row minimums $\{-2, -1, -4, -2\}$ row maximin $= -1$	M1
	column maximums $\{1, 3, 3, 3\}$ column minimax $= 1$	A1
	Since $1 \neq -1$ not stable	A1 (3)
(b)	Row 2 dominates Row 3	B1
	Column 1 dominates column 4	B1 (2)
(c)	Let A play row R_1 with probability p_1, R_2 with probability p_2 and "R_3" with probability p_3.	B1
	$\begin{pmatrix} -2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{pmatrix} \xrightarrow[+3]{\text{eg}} \begin{pmatrix} 1 & 4 & 6 \\ 2 & 6 & 5 \\ 4 & 1 & 2 \end{pmatrix}$	M1 (2)
	e.g. maximise $P = V$	M1 A1
	subject to $V - p_1 - 2p_2 - 4p_3 \leq 0$	A 4/3/2/1/0
	$V - 4p_1 - 6p_2 - p_3 \leq 0$	
	$V - 6p_1 - 5p_2 - 2p_3 \leq 0$	
	$p_1 + p_2 + p_3 \leq 1$	(6)
	$V, p_1, p_2, p_3 \geqslant 0$	
	OR	
	eg Let $x_i = \frac{p_i}{V}$ $\therefore \frac{1}{V} = x_1 + x_2 + x_3$.	M1
	minimise $P = x_1 + x_2 + x_3$	A1
	subject to $x_1 + 2x_2 + 4x_3 \geqslant 1$	
	$4x_1 + 6x_2 + x_3 \geqslant 1$	A 4/3/2/1/0
	$6x_1 + 5x_1 + 2x_3 \geqslant 1$	(6)
	$x_1, x_2, x_3 \geqslant 0$.	
	+ other equivalent methods.	13

6) (a)



R.M.S.T.

eg. AH, AB, BD, DE
HG, EF using prim — m1 A1

length of RMST = 459

∴ lower bound = 459 + 53 + 83 = 595 km (deleting C) — A1

∴ Best lower bound is 595 km, by deleting C — m1 A1√ (5)

(b) Add 167 to AF and FA
137 to CH and HC — B4,3,2,1,0
136 to DF and FD
145 to DG and GD — (4)

(c) C D E F H A B G C — m1 A1
53 49 100 115 52 84 222 92

upper bound, starting at C = 767 km — A1

∴ Best upper bound is 707 starting at F — B1∫(4)

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