

Answers for C3 January 2006

1) Graphical questions (I don't have a working scanner, so I can't scan a graph into the computer).

$$\begin{aligned} 2) & (2x^2 + 3x)/[(2x + 3)(x - 2)] - 6/(x^2 - x - 2) \\ &= (2x^2 + 3x)/[(2x + 3)(x - 2)] - 6/[(x - 2)(x + 1)] \\ &= [(2x^2 + 3x)(x + 1) - 6(2x + 3)]/[(2x + 3)(x - 2)(x + 1)] \\ &= (2x^3 + 5x^2 + 3x - 12x - 18)/[(2x + 3)(x - 2)(x + 1)] \\ &= (2x^3 + 5x^2 - 9x - 18)/[(2x + 3)(x - 2)(x + 1)] \\ &= [(x - 2)(2x^2 + 9x + 9)]/[(2x + 3)(x - 2)(x + 1)] \\ &= (2x^2 + 9x + 9)/[(2x + 3)(x + 1)] \\ &= [(2x + 3)(x + 3)]/[(2x + 3)(x + 1)] \\ &= \mathbf{(x + 3)/(x + 1)} \end{aligned}$$

3) y co-ordinate of P is $\ln(3/3) = \ln 1 = 0$

$$y = \ln(x/3) = \ln x - \ln 3$$

$$dy/dx = 1/x$$

At P, $dy/dx = 1/3$.

Therefore, gradient of normal at P is -3.

Equation of normal at P is $y = -3(x - 3)$

$$\mathbf{y = -3x + 9}$$

4ai) x^2e^{3x+2}

$$d/dx = \mathbf{2x \cdot e^{3x+2} + 3x^2e^{3x}}$$

4a ii) $\cos(2x^3)/3x$

$$d/dx = [-18x^3\sin(2x^3) - 3\cos(2x^3)]/9x^2$$

$$= \mathbf{-2x\sin(2x^3) - \cos(2x^3)/3x^2}$$

4b) $x = 4\sin(2y + 6)$

$$dx/dy = 8\cos(2y + 6)$$

$$dy/dx = 1/[8\cos(2y + 6)]$$

$$dy/dx = 1/\{8\sqrt{[1 - \sin^2(2y + 6)]}\}$$

$$\frac{dy}{dx} = 1/[8\sqrt{(1 - x^2/16)}]$$

$$5a) x = \sqrt{(2/x + 0.5)}$$

$$x^2 = 2/x + 0.5$$

$$x^3 = 2 + 0.5x$$

$$2x^3 - x - 4 = 0$$

$$5b) x_1 = 1.41 \text{ (2 d.p.)}$$

$$x_2 = 1.39 \text{ (2 d.p.)}$$

$$x_3 = 1.39 \text{ (2 d.p.)}$$

$$5c) f(1.3915) = -0.00285432825$$

$$f(1.3925) = 0.00777165625$$

The sign change indicates alpha = 1.392 (3 d.p.)

6a) (I'm using 'a' for alpha)

$$R\cos(x + a) = R\cos x \cos a - R\sin x \sin a$$

$$R\cos a = 12, R\sin a = 4$$

$$R^2\cos^2 a + R^2\sin^2 a = 160$$

$$R = \sqrt{160} = 4\sqrt{10}$$

$$(4\sqrt{10})\sin a = 4$$

$$\sin a = 1/\sqrt{10}$$

$$a = 18.4349488\dots$$

$$6b) (4\sqrt{10})\cos(x + 18.4349488\dots)^\circ = 7$$

$$\cos(x + 18.4349488\dots)^\circ = 7/4\sqrt{10}$$

$$(x + 18.4349488\dots)^\circ = 56.3995\dots^\circ, 303.6004\dots^\circ$$

$$x^\circ = 37.9645\dots^\circ, 285.165\dots^\circ$$

$$x^\circ = 38.0^\circ, 285.2^\circ \text{ (1 d.p.)}$$

$$6ci) -4\sqrt{10}$$

$$6cii) \cos(x + 18.4349488\dots)^\circ = -1$$

$$(x + 18.4349488\dots)^\circ = 180^\circ$$

$$x^\circ = 161.565\dots$$

$$x^\circ = 161.57^\circ \text{ (2 d.p.)}$$

$$7\text{ai) } \cos 2x / (\cos x + \sin x) \equiv (\cos^2 x - \sin^2 x) / (\cos x + \sin x) \equiv \cos x - \sin x$$

$$7\text{aii) } (\cos 2x - \sin 2x) / 2 \equiv (2\cos^2 x - 2\sin x \cos x - 1) / 2 \equiv \cos^2 x - \sin x \cos x - 1/2$$

$$7\text{b) } \cos \theta [\cos 2\theta / (\cos \theta + \sin \theta)] = 1/2$$

$$\cos \theta (\cos \theta - \sin \theta) = 1/2$$

$$2\cos \theta (\cos \theta - \sin \theta) - 1 = 0$$

$$2\cos^2 \theta - 2\sin \theta \cos \theta - 1 = 0$$

$$\sin 2\theta = \cos 2\theta$$

$$7\text{c) } \sin 2\theta = \cos 2\theta$$

$$\sin 2\theta / \cos 2\theta = 1$$

$$\tan 2\theta = 1$$

$$2\theta = \pi/4, 5\pi/4, 9\pi/4, 13\pi/4$$

$$\theta = \pi/8, 5\pi/8, 9\pi/8, 13\pi/8$$

$$8\text{a) } g f(x) = g(2x + \ln 2)$$

$$= e^{2(2x + \ln 2)}$$

$$= e^{4x + \ln 4}$$

$$= e^{4x} \cdot e^{\ln 4}$$

$$= 4e^{4x}$$

8b) Graphical question (I don't have a working scanner, so I can't scan a graph into the computer).

$$8\text{c) } g f(x) > 0$$

$$8\text{d) } d/dx(4e^{4x}) = 16e^{4x}$$

$$16e^{4x} = 3$$

$$e^{4x} = 3/16$$

$$4x = \ln(3/16)$$

$$x = 0.25\ln(3/16)$$

$$x = -0.418494\dots$$

$$x = -0.418 \text{ (3 s.f.)}$$