FINAL Publication

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6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $P(X > 19.023) = 0.025$ or $P(X < 19.023) = 0.975$ both	B1
	$P(X > 2.088) = 0.990$ or $P(X < 2.088) = 0.010$	
	$\therefore P(2.088 < X < 19.023) = 0.990 - 0.025$ or $0.975 - 0.010$	M1
	$= 0.965$	A1 (3)
	(b) Upper Critical value of $F_{12,5} = 4.68$	B1
	Lower Critical value of $F_{12,5} = \frac{1}{F_{5,12}}$	M1
	$= \frac{1}{3.11} = 0.3215...$ AWRT 0.322	A1 (3)
2.	(a) $H_0: \sigma_1^2 = \sigma_2^2$; $H_1: \sigma_1^2 \neq \sigma_2^2$ both	B1
	$\frac{s_1^2}{s_2^2} = \frac{14^2}{8^2} = 3.0625$ or $\frac{8^2}{14^2} = 0.32653....$ AWRT 3.06 or 0.327	M1 A1
	C.V $F_{12,7} = 3.57$ CV: $F_{7,12} = \frac{1}{3.57} = 0.28011$	B1
	Since 3.0625 is not in the Critical Region their is insufficient evidence to reject H_0. There is insufficient evidence of a difference in the variances of the lengths of the fence posts.	A1√ (5)
	(b) The distribution of the population of lengths of fence posts is normally distributed.	B1 (1)

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3.	Let x represent weight of flour	
	$\sum x = 8055 \therefore \bar{x} = 1006.875$ AWRT 1006.9	B1
	$\sum x^2 = 8110611 \therefore s^2 = \frac{1}{7}\left\{8110611 - \frac{8055^2}{8}\right\} = 33.26785...$ AWRT 33.7	M1
	Allow from Calculator $\therefore s = 5.767828...$ or AWRT 5.77	A1
	$H_0: \mu = 1010$; $H_1: \mu < 1010$ both	B1
	CV: $\lvert t \rvert = 1.895$	B1
	$t = \left\lvert \frac{1006.875 - 1010}{5.767828.../\sqrt{8}} \right\rvert = \pm 1.5324$ Using $\frac{\bar{x}-\mu}{s/\sqrt{n}}$	M1
	AWRT -1.53	A1
	Since -1.53 is not in the critical region ($t < -1.895$) there is insufficient evidence to reject H_0 and consistent thus the mean weight of flour delivered by the machine is 1010g.	A1√ (8)

4.	(a) The data were not collected in pairs.	B1 (1)
	(b) Use data from twin lambs.	B1 (1)
	(c) Age, weight, gender — Any Two sensible factors.	B1; B1 (2)
	(d) $d = B - A$ $d: 2, 1.2, 1, 1.8, -1, 2.2, 2, -1.2, 1.1, 2.8$	M1
	$\sum d = 11.9$; $\sum d^2 = 30.01$ $\therefore \bar{d} = 1.19$; $s^2 = 1.761$ $(s = 1.327)$	A1 ; A1
	$H_0: \delta = 0$; $H_1: \delta \neq 0$ Allow μ_D for δ — both	B1
	$t = \dfrac{1.19 - 0}{\sqrt{\frac{1.761}{10}}} = 2.83574\ldots$	M1
	AWRT 2.84	A1
	$\nu = 9$; CV: $t = 2.262$	B1
	Since 2.8357... is in the critical region $(t > 2.262)$ there is evidence to reject H_0. The (mean) weight gained by the lambs is different for each diet.	A1$\sqrt{}$ (8)
	(e) Diet B – it has the higher mean	B1 (1)
	(d) Using non-paired t-test. $H_0: \mu_A = \mu_B$; $H_1: \mu_A \neq \mu_B$	B1
	$t = \dfrac{\mu_A - \mu_B}{\sqrt{s_p^2\left(\frac{1}{10} + \frac{1}{10}\right)}} = -2.30$ — AWRT -2.30	B1
	CV: $\lvert t \rvert = 2.101$	B1
	Conclusion: Mean weight gained is different	B1 (4)
	NB $\mu_A = 5.6$ $\mu_B = 6.79$ $S_p^2 = 1.342722\ldots$	

5.	(a) A Type I error occurs when H_0 is rejected when in fact it is true.	B1 (1)
	(b) The size of a test is the probability of a Type I error	B1 (1)
	(c) $X \sim B(8, 0.25)$ Can be implied	B1
	$\text{Size} = P(X > 6) = 1 - P(X \leq 6 \mid n=8, p=0.25)$	M1
	$= 1 - 0.9996$	
	$= \underline{0.0004}$	A1 (3)
	(d) $\text{Power} = P(X > 6 \mid p = p, n = 8)$	
	$= P(X=7) + P(X=8)$	M1
	$= \frac{8!}{7!1!} p^7(1-p) + p^8$	A1
	$= 8p^7 - 8p^8 + p^8$	
	$* = \underline{8p^7 - 7p^8} *$	A1 (3)
	(e) Power when $p = 0.3 = 8 \times 0.3^7 - 7 \times 0.3^8$	B1 (1)
	$= \underline{0.00129}$ AWRT 0.0013	
	(f) $P(\text{Type II error}) = 1 - \text{Power}(0.3)$	M1
	$= \underline{0.99870\ldots}$	A1 (2)
	(g) Increase the number of trials	B1
	Increase the critical region	B1 (2)

6.	(a) Confidence interval is given by		
	$\bar{x} \pm t_{19} \times s/\sqrt{n}$	2.539	B1
	ie:- $207.1 \pm 2.539 \times \sqrt{\frac{3.2}{20}}$	Using $\bar{x} \pm t \times s/\sqrt{n}$	M1
		All correct	A1
	ie:- 207.1 ± 1.0156		
	ie:- $\underline{(206.08\ldots,\ 208.1156)}$	AWRT (206, 208)	A1 (4)
	(b) $s_p^2 = \dfrac{19 \times 3.2 + 9 \times 10.217\dot{3}}{28}$	$10.217\dot{3}$ AWRT 10.2	B1
			M1
	$= \underline{5.45557\ldots}$	AWRT 5.46	A1
	Confidence interval is given by		
	$\bar{x}_B - \bar{x}_G \pm t_{28} \times \sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$		
		1.701	B1
	ie:- $(207.1 - 204.62) \pm 1.701\sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$		
	ie:- 2.48 ± 1.53875	Using $\bar{x} - \bar{y} \pm t\sqrt{s^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$	M1
	ie:- $\underline{(0.94125,\ 4.0187)}$	All correct	A1
		AWRT 0.941; 4.02	A1;A1 (8)

7.	(a) $E(X) = np$, $E(Y) = mp$ both; can be implied	B1
	$E(\hat{p}_1) = \frac{a\,E(X)}{n} + \frac{b\,E(Y)}{m} = p$; $\Rightarrow \frac{anp}{n} + \frac{bmp}{m} = p$	M1 A1
	$\Rightarrow (a+b) = 1$	A1 (4)
	(b) $E(\hat{p}_2) = \frac{1}{n+m}\{E(X) + E(Y)\}$	M1
	$= \frac{1}{n+m}\{np + mp\}$	A1
	$= \frac{1}{(n+m)} \cdot p(n+m) = p \Rightarrow \hat{p}_2$ is unbiased	A1 (3)
	(c) $Var(X) = np(1-p)$; $Var(Y) = mp(1-p)$ both; can be implied	B1
	$Var(\hat{p}_1) = \frac{a^2 Var(X)}{n^2} + \frac{b^2 Var(Y)}{m^2}$ Use of $Var(aX) = a^2 Var(X)$	M1
	$= \frac{a^2 np(1-p)}{n^2} + \frac{b^2 mp(1-p)}{m^2}$	
	$= p(1-p)\left\{\frac{a^2}{n} + \frac{b^2}{m}\right\}$	A1 (3)
	(d) $Var(\hat{p}_2) = \frac{1}{(n+m)^2}\{np(1-p) + mp(1-p)\}$	M1 A1
	$= \frac{p(1-p)}{n+m}$	A1 (3)
	(e) $Var(\hat{p}_1) = 0.044p(1-p)$; $Var(\hat{p}_2) = 0.033\dot{3}p(1-p)$	B1; B1
	Use $\hat{p}_2$; $Var(\hat{p}_2) < Var(\hat{p}_1)$	B1; B1 (4) dep

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