Paper Reference(s)
6689/01

## Edexcel GCE

## Decision Mathematics D1

# Advanced/Advanced Subsidiary 

# Thursday 16 June 2005 - Afternoon 

## Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
| :--- | :--- |
| Nil Answer book |  |

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets:
e.g. (2)

This paper has eight questions. The total mark for this question paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers in the D1 answer booklet for this paper.

1. 

| Ali | 74 |
| :---: | :---: |
| Bobby | 28 |
| Eun-Jung | 63 |
| Katie | 54 |
| Marciana | 54 |
| Peter | 49 |
| Rory | 37 |
| Sophie | 68 |

The table shows the marks obtained by students in a test. The students are listed in alphabetical order. Carry out a quick sort to produce a list of students in descending order of marks. You should show the result of each pass and identify your pivots clearly.
(Total 5 marks)
2.

Figure 1

(a) Starting from $A$; write down a Hamiltonian cycle for the graph in Figure 1.
(b) Use the planarity algorithm to show that the graph in Figure 1 is planar.

Arcs $A F$ and $E F$ are now added to the graph.
(c) Explain why the new graph is not planar.
3.

Figure 2


Figure 2 models a network of roads which need to be inspected to assess if they need to be resurfaced. The number on each arc represents the length, in km, of that road.

Each road must be traversed at least once and the length of the inspection route must be minimised.
(a) Starting and finishing at $A$, solve this route inspection problem. You should make your method and working clear. State the length of the shortest route.
(The weight of the network is 77 km .)

Given that it is now permitted to start and finish the inspection at two distinct vertices,
(b) state which two vertices you should choose to minimise the length of the route. Give a reason for your answer.
4. The precedence table shows the activities involved in a project.

| Activity | Immediately preceding activities |
| :---: | :---: |
| $A$ | - |
| $B$ | - |
| $C$ | - |
| $D$ | $A$ |
| $E$ | $A$ |
| $F$ | $B$ |
| $G$ | $B$ |
| $H$ | $E, D$ |
| $I$ | $G, H$ |
| $J$ | $G, J$ |
| $K$ | $G$ |
| $L$ | $L$ |
| $M$ | $L$ |
| $N$ |  |

(a) Draw the activity network for this project, using activity on arc and using two dummies.
(b) Explain why each of the two dummies is necessary.
5.

Figure 3


Figure 4


A film critic, Verity, must see five films A, B, C, D and E over two days.
The films are being shown at five special critics' preview times:
1 (Monday 4 pm ),
2 (Monday 7 pm ),
3 (Tuesday 1 pm ),
4 (Tuesday 4 pm ),
5 (Tuesday 7 pm ).
The bipartite graph in Figure 3 shows the times at which each film is showing.
Initially Verity intends to see
Film A on Monday at 4 pm , Film B on Tuesday at 4 pm , Film C on Tuesday at 1 pm , Film D on Monday at 7 pm .

This initial matching is shown in Figure 4.
Using the maximum matching algorithm and the given initial matching,
(a) find two distinct alternating paths and complete the matchings they give.

Verity's son is very keen to see film D, but he can only go with his mother to the showing on Monday at 7 pm .
(b) Explain why it will not be possible for Verity to take her son to this showing and still see all five films herself.
6.

Figure 5


Figure 5 shows a network of roads. The number on each arc represents the length of that road in km .
(a) Use Dijkstra's algorithm to find the shortest route from $A$ to $J$. State your shortest route and its length.
(b) Explain how you determined the shortest route from your labelled diagram.

The road from $C$ to $F$ will be closed next week for repairs.
(c) Find the shortest route from $A$ to $J$ that does not include $C F$ and state its length.
7. Polly has a bird food stall at the local market. Each week she makes and sells three types of packs $A, B$ and $C$.

Pack $A$ contains 4 kg of bird seed, 2 suet blocks and 1 kg of peanuts.
Pack $B$ contains 5 kg of bird seed, 1 suet block and 2 kg of peanuts.
Pack $C$ contains 10 kg of bird seed, 4 suet blocks and 3 kg of peanuts.
Each week Polly has 140 kg of bird seed, 60 suet blocks and 60 kg of peanuts available for the packs.

The profit made on each pack of $A, B$ and $C$ sold is $£ 3.50, £ 3.50$ and $£ 6.50$ respectively. Polly sells every pack on her stall and wishes to maximise her profit, $P$ pence.

Let $x, y$ and $z$ be the numbers of packs $A, B$ and $C$ sold each week.
An initial Simplex tableau for the above situation is

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 4 | 5 | 10 | 1 | 0 | 0 | 140 |
| $s$ | 2 | 1 | 4 | 0 | 1 | 0 | 60 |
| $t$ | 1 | 2 | 3 | 0 | 0 | 1 | 60 |
| $P$ | -350 | -350 | -650 | 0 | 0 | 0 | 0 |

(a) Explain the meaning of the variables $r, s$ and $t$ in the context of this question.
(b) Perform one complete iteration of the Simplex algorithm to form a new tableau $T$. Take the most negative number in the profit row to indicate the pivotal column.
(c) State the value of every variable as given by tableau $T$.
(d) Write down the profit equation given by tableau $T$.
(e) Use your profit equation to explain why tableau $T$ is not optimal.

Taking the most negative number in the profit row to indicate the pivotal column,
(f) identify clearly the location of the next pivotal element.
8.

Figure 6


Figure 6 shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow through the network. Take this as the initial flow.
(a) On Diagram 1 and Diagram 2 in the answer book, add a supersource $S$ and a supersink $T$. On Diagram 1 show the minimum capacities of the arcs you have added.

Diagram 2 in the answer book shows the first stage of the labelling procedure for the given initial flow.
(b) Complete the initial labelling procedure in Diagram 2.
(c) Find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow, and state the maximal flow.
(d) Show a maximal flow pattern on Diagram 3.
(e) Prove that your flow is maximal.
(f) Describe briefly a situation for which this network could be a suitable model.
(Total 16 marks)

## END

