

The questions
 28/01/05

Question Number	Scheme	Marks
1.	<p>(a) $P(R=5) = P(R \leq 5) - P(R \leq 4) = 0.7216 - 0.5755$ $= 0.2061$ (OR: ${}^{15}C_5 (0.3)^5 (0.7)^{10} = 0.206130\dots$)</p> <p>(b) $P(S=5) = 0.2414 - 0.1321 = 0.1093$ (OR: $\frac{7.5^5 e^{-7.5}}{5!} = 0.10937459\dots$)</p> <p>(c) $P(T=5) = 0$</p>	<p>Can be implied M1 ANSWER 0.2061 A1 (2)</p> <p>Accept B1 (1) 0.1093 or 0.1094 ANSWER ANSWER</p> <p>cao B1 (1)</p>
2.	<p>(a) (i) A <u>collection</u> of <u>individuals</u> or <u>items</u></p> <p>(ii) A <u>list</u> of <u>all sampling units</u> in the population</p> <p>(b) Not always possible to keep this list up to date</p> <p>(c) (i) eg:- Pupils in year 12 - small easily listed sample ^{population} <u>Population known & easily accessed</u></p> <p>(ii) Students in a University - large not easily listed ^{population} <u>Population known but too time consuming/expensive to interview all of them.</u></p>	<p>B1</p> <p>B1 (2)</p> <p>B1 (1)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1 (4)</p>
	<p>(c) SR (i) Definition of census <u>by example</u></p> <p>(ii) - - <u>sample</u> - -</p>	<p>B1</p> <p>B1</p>

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3.	<p>(a) <u>Continuous uniform/Rectangular</u></p> $f(x) = \begin{cases} \frac{1}{l}, & 0 \leq x \leq l \\ 0 & \text{otherwise} \end{cases}$ <p>(b) $P(X < \frac{1}{3}l) = \frac{1}{l} \times \frac{l}{3} = \frac{1}{3}$ Their $\frac{1}{3} \times \frac{l}{3}$</p> <p>(c) $E(X) = \frac{1}{2}l$</p> <p>(d) $P(\text{Both} < \frac{1}{3}l) = (\frac{1}{3})^2 = \frac{1}{9}$ (b)²</p>	<p>B1</p> <p>B1</p> <p>B1 (3)</p> <p>M1A1 (2)</p> <p>B1 (1)</p> <p>M1</p> <p>A1/2</p>
4.	<p>(a) Probability of success/failure is constant <u>Trials are independent</u></p> <p>(b) Let p represent proportion of students who can distinguish between brands</p> <p>$H_0: p = 0.1$; $H_1: p > 0.1$ (both) B1</p> <p>$\alpha = 0.01$; CR: $z > 2.3263$ 2.3263 B1</p> <p>$np = 25$; $npq = 22.5$ both B1</p> <p>$z = \frac{39.5 - 25}{\sqrt{22.5}} = 3.0568\dots$ can be implied</p> <p><u>Reject H_0: claim cannot be accepted</u> Standardisation with ± 0.5 & their \sqrt{npq} AWT 3.06 A1</p> <p>Based on clear evidence from z test A1 (6)</p> <p>(c) eg:- np, nq both > 5 - true so acceptable</p> <p>p close to 0.5 - not true, assumption not met B1 (2)</p> <p>success/failure not clear cut necessarily</p> <p>independence - one student influences another B1</p>	<p>B1</p> <p>B1 (2)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 (6)</p> <p>B1 (2)</p> <p>B1</p>

(b) Alter $z = 3.06 \Rightarrow p = 0.9989 > 0.99$ } B1 equir to 2.3263
 or $p = 0.0011 < 0.01$ }

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5.	<p>Let X represent the number of defective articles $\therefore X \sim B(10, 0.032)$</p> <p>(a) $P(X=2) = \binom{10}{2} (0.032)^2 (1-0.032)^8$ $= 0.0355234 \dots$</p> <p>(b) Large n, small $p \Rightarrow$ Poisson approximation with $\lambda = 10 \times 0.032 = 3.2$</p> <p>$P(X < 4) = P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$ $= \frac{e^{-3.2}}{1} \left\{ 1 + 3.2 + \frac{(3.2)^2}{2} + \frac{(3.2)^3}{6} \right\}$ $= 0.602519 \dots$</p> <p>(c) np & npq both $> 5 \Rightarrow$ Normal approximation with $np = 32$ and $npq = 30.976$</p> <p>$P(X > 42) \approx P(Y > 42.5)$ where $Y \sim N(32, 30.976)$ $= P\left(Z > \frac{42.5 - 32}{\sqrt{30.976}}\right)$ $= P(Z > 1.8865 \dots)$ $= 0.0294$</p>	<p>Use of $\binom{n}{r} p^r q^{n-r}$ M1 All correct A1 AWRT 0.0355 A1 (3)</p> <p>Seen or implied B1</p> <p>$P(X \leq 3)$ stated or implied M1 All correct A1 AWRT 0.603 A1 (4)</p> <p>N approx M1 both A1</p> <p>Standardised this np, \sqrt{npq} M1 All correct A1 AWRT 1.89 A1 0.0294 - 0.0297 A1 (6)</p>

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6.	<p>Let X represent number of accidents/month $\therefore X \sim Po(3)$</p> <p>(a) $P(X > 4) = 1 - P(X \leq 4) = 1 - 0.8513 = 0.1487$</p> <p>(b) Let Y represent number of accidents in 3 months $\therefore Y \sim Po(3 \times 3 = 9)$ $P(Y > 4) = 1 - 0.0550 = 0.9450$</p> <p>(c) $H_0: \lambda = 3; H_1: \lambda < 3$ $\alpha = 0.05$ $P(X \leq 1 \lambda = 3) = 0.1991; > 0.05$ \therefore Insufficient evidence to support the claim that the mean number of accidents has been reduced. (NB: CR: $X \leq 0; X = 1$ not in CR; same conclusion \Rightarrow B1, M1, A1)</p> <p>(d) $H_0: \lambda = 24 \times 3 = 72; H_1: \lambda < 72$ $\alpha = 0.05 \Rightarrow$ CR: $Z < -1.6449$</p> <p>Using Normal approximation with $\mu = \sigma^2 = 72$</p> $Z = \frac{55.5 - 72}{\sqrt{72}} = -1.94454\dots$ <p>Since $-1.944\dots$ is in the CR, H_0 is rejected. There is evidence that the restriction has reduced the number of accidents.</p>	<p>B1</p> <p>M1; A1 (3)</p> <p>Can be implied B1</p> <p>B1 (2)</p> <p>both B1</p> <p>B1; M1</p> <p>A1 (4)</p> <p>Can be implied $\lambda = 72$ B1 both H_0 & H_1 B1 -1.6449 B1 Can be implied B1</p> <p>Stand \approx with M1 $\pm 0.5, \mu \pm \sigma$ AWRT -1.9445 A1</p> <p>Context & clear evidence A1 (7)</p>
	<p>Alter (d) $p = 0.0262 < 0.05$ AWRT 0.026 B1 equiv to -1.6449</p>	

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7.	<p>(a) $k \int_1^4 (-x^2 + 5x - 4) dx = 1$</p> <p>$\therefore k \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right]_1^4 = 1$</p> <p>* $\Rightarrow k = \frac{2}{9}$ *</p> <p>(b) $E(X) = \int_1^4 \frac{2}{9} (-x^3 + 5x^2 - 4x) dx$</p> <p>$= \frac{2}{9} \left[-\frac{x^4}{4} + \frac{5x^3}{3} - \frac{4x^2}{2} \right]_1^4$</p> <p>$= \frac{5}{2}$</p> <p>(c) $\frac{d}{dx} f(x) = \frac{2}{9} (-2x + 5) = 0; \Rightarrow \text{Mode} = \frac{5}{2}$</p> <p><i>Se: $\frac{5}{2}$ only; no working B1</i></p> <p>(d) $F(x) = \int_1^{x_0} \frac{2}{9} (-x^2 + 5x - 4) dx$</p> <p>$= \left[\frac{2}{9} \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right) \right]_1^{x_0}$</p> <p>$= \frac{2}{9} \left\{ -\frac{x_0^3}{3} + \frac{5x_0^2}{2} - 4x_0 + \frac{11}{6} \right\}$</p> <p>$\therefore F(x) = \begin{cases} 0 & x < 1 \\ \frac{2}{9} \left\{ -\frac{x^3}{3} + \frac{5x^2}{2} - 4x + \frac{11}{6} \right\} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$</p> <p>(e) $P(X \leq 2.5) = F(2.5) = 0.5$</p> <p>(f) Median = 2.5; Distribution is symmetrical</p>	<p>Use of $\int f(x) dx = 1$ M1</p> <p>All correct integⁿ with limits A1</p> <p>c.s.o. A1 (3)</p> <p>Use of $\int x f(x) dx$ M1</p> <p>Correct integⁿ with limits A1</p> <p>cao A1 (3)</p> <p>Diffⁿ of $f(x)$ at $x=0$ M1; A1 (2)</p> <p>Use of $\int f(x) dx$ M1</p> <p>Integⁿ with limits 1 & symbol A1</p> <p>anf A1</p> <p>$x < 1$ B1</p> <p>$1 \leq x \leq 4$ B1</p> <p>$x > 4$ B1 (5)</p> <p>F(2.5) or integral etc M1 A1 (2)</p> <p>B1; B1 (2) cao cao</p>