## 6671 Pure Mathematics P1

Mark Scheme

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1. | Forming equation in $x$ or $y$ by attempt to eliminate one variable $\begin{aligned} & (3-y)^{2}+y=15 \text { or } x^{2}+(3-x)=15 \\ & y^{2}-5 y-6=0 \text { or } x^{2}-x-12=0 \text { (Correct } 3 \text { term version) } \end{aligned}$ <br> Attempt at solution i.e. solving 3 term quadratic: $(y-6)(y+1)=0, \quad y=\ldots$ $\text { or }(x-4)(x+3)=0, \quad x=\ldots$ <br> or correct use of formula or correct use of completing the square $x=4 \text { and } x=-3 \text { or } y=-1 \text { and } y=6$ <br> Finding the values of the other coordinates ( M attempt one, A both) | M1 <br> A1 <br> M1 <br> A1 <br> M1 A1 ft <br> (6) |
| 2. | Using $\sin ^{2} \theta+\cos ^{2} \theta=1$ to give a quadratic in $\cos \theta$. <br> Attempt to solve $\cos ^{2} \theta+\cos \theta=0$ $\begin{aligned} & (\cos \theta=0) \Rightarrow \theta=\frac{\pi}{2}, \frac{3 \pi}{2} \\ & (\cos \theta=-1) \Rightarrow \theta=\pi \end{aligned}$ <br> ( Candidate who writes down 3 answers only with no working scores a maximum of 3 ) | M1 <br> M1 <br> B1, B1 <br> B1 <br> (5) |
| 3. (a) <br> (b) <br> (c) | Attempt $\mathrm{f}(2) ;=16-4+4-16=0 \Rightarrow(x-2)$ is a factor $\underline{c=8}$ <br> A complete method to find $b$ <br> Either compare coefficients of $x$ or $x^{2}: \quad-2 b+8=2$, or $-4+b=-1$ <br> Or substitute value of $x$ (may be implied) : e.g. $(x=1) \Rightarrow-13=(-1)(10+b)$ $\underline{b=3}$ <br> Checking $b^{2}-8 c ; \quad-55 \Rightarrow$ no real roots to the quadratic <br> $\Rightarrow \underline{x=2}$ is the only solution | M1; A1 <br> (2) <br> B1 <br> M1 <br> A1 <br> (3) <br> M1; A1 <br> A1 |


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| 4 (a) | Correct strategy for differentiation e.g. $y=4 x^{2}+(5 x-1) x^{-1}$ multiplied out with correct differentiation method, or product or quotient rules applied correctly to $\frac{5 x-1}{x}$. $\frac{\mathrm{d} y}{\mathrm{~d} x}=8 x,+\frac{1}{x^{2}} \quad \text { B1 for } 8 x \text { seen anywhere. }$ | M1 B1, A1 |
|  | Putting $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ <br> So $8 x^{3}+1=0 \quad \Rightarrow x=-\frac{1}{2}$. <br> M1 requires multiplication by denominator and use of a root in the solution | M1 <br> M1 A1 <br> (3) |
| (c) | Complete method: <br> Either $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=8-\frac{2}{x^{3}}$, with $x$ value substituted, or gradient either side checked | M1 |
|  | Completely correct argument, either > 0 with no error seen,( 24 )or -ve to +ve gradient, then minimum stated | A1 (2) |
| 5(a) | $p=15, q=-3$ <br> Special case if B0 B0, allow M1 for method, e.g. $8=\frac{1+p}{2}$ | B1, B1 |
| (b) | Gradient of line $A D C=-\frac{5}{7}$, gradient of perpendicular line $=-\frac{1}{\text { gradient } A D C}\left(\frac{7}{5}\right)$ | B1, M1 |
|  | Equation of $l$ : $y-2=\left(\frac{7}{5}\right)(x-8)$ | M1 A1ft A1cao |
|  |  | (5) |
| (c) | Substituting $y=7$ and finding value for $x$, | M1 |
|  | $x=\frac{81}{7}$ or $11 \frac{4}{7}$ | A1 (2) |


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| 6 (a) | $P=r \theta+2 r, \quad A=\frac{1}{2} r^{2} \theta$ | B1, B1 | (2) |
| (b) | Substituting value for $r$ and equating $P$ to $A$. [2 $\left.2 \sqrt{2}(2+\theta)=\frac{1}{2}(2 \sqrt{2})^{2} \theta\right]$ | M1 |  |
|  | Correct process to find $\theta \quad[\theta(\sqrt{2}-1)=2]$ | M1 |  |
|  | $\theta=\frac{2}{\sqrt{2}-1} \quad * \text { often see } \quad \theta=\frac{4 \sqrt{2}}{4-2 \sqrt{2}}$ | A1 c.s.o. | (3) |
| (c) | Multiply numerator and denominator by ( $\sqrt{2}+1)$ | M1 |  |
|  | $2,+2 \sqrt{2}$ | A1, A1 | (3) |
| 7 (a) | Applying correct formula [325 = $120+5(\mathrm{n}-1)$ ] | M1 |  |
|  | Solving to give $n=42 \quad * \quad$ (or verifying in correct equation) | A1 |  |
| (b) | Using formula for sum of AP: $S=\frac{42}{2}\{240+5(42-1)\}$ or use $\frac{n}{2}\{a+l\}$ | M1 A1 |  |
|  | $=9345$ | A1 |  |
| (c) | Recognising GP with $r=0.98$ | M1 |  |
|  | $\begin{aligned}\text { Value ( in £ }) & =7200(0.98)^{24} \\ & =4434(\text { only this value })\end{aligned}$ | M1 |  |
|  |  | A1 |  |
|  |  |  | (3) |


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| 8 (a) | Substitute $x=0, y=\sqrt{3}$ to give $\sqrt{3}=k \frac{\sqrt{3}}{2} \Rightarrow k=2 \quad$ (or verify result) must see $\frac{\sqrt{3}}{2}$ | B1 (1) |
| (b) | $p=120, \quad q=300 \quad \text { (f.t. on } p+180 \text { ) }$ | B1, B1ft |
| (c) | $\arcsin (-0.8)=-53.1$ or $\arcsin (0.8)=53.1$ | B1 |
|  | $(x+60)=180-\arcsin (-0.8)$ or equivalent $180+\arcsin 0.8$ | M1 |
|  | First value of $x=233.1-60$, i.e. $\quad x=173.1$ | A1 |
|  | OR $(x+60)=360+\arcsin (-0.8)$ or equivalent $360-\arcsin 0.8, \quad$ i.e. $x=246.9$ | M1, A1 |
|  |  | (5) |
| 9 (a) | $(x-3)^{2},+9$ isw . $a=3$ and $b=9$ may just be written down with no method shown. | B1, M1 A1 <br> (3) |
| (b) | $P$ is (3, 9) | B1 ft, B1ft |
| (c) | $A=(0,18)$ | B1 |
|  | $\frac{\mathrm{d} y}{\mathrm{~d} x}=2 x-6, \text { at } A \quad m=-6$ <br> Equation of tangent is $y-18=-6 x$ (in any form) | M1 A1 <br> A1ft |
|  |  | (4) |
| (d) | Showing that line meets $x$ axis directly below $P$, i.e. at $x=3$. | A1cso <br> (1) |
| (e) | $A=\int x^{2}-6 x+18 \mathrm{~d} x=\left[\frac{1}{3} x^{3}-3 x^{2}+18 x\right]$ | M1 A1 |
|  | Substituting $x=3$ to find area $A$ under curve $A[=36]$ | M1 |
|  | Area of $R=A$ - area of triangle $=A-\frac{1}{2} \times 18 \times 3,=9$ | M1 A1 |
|  | Alternative: $\int x^{2}-6 x+18-(18-6 x) \mathrm{d} x$ M1 | (5) |
|  | $=\frac{1}{3} x^{3}$ <br> M1 A1 ft |  |
|  | Use $x=3$ to give answer 9 M1 A1 |  |

