Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Wednesday 12 January 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is attached to a fixed point A. The particle is moving, with the string taut, in a horizontal circle with centre O vertically below A. The particle is moving with constant angular speed 2.7 rad s^{-1}. Find

(a) the tension in the string,

(4)

(b) the angle, to the nearest degree, that AP makes with the downward vertical.

(3)

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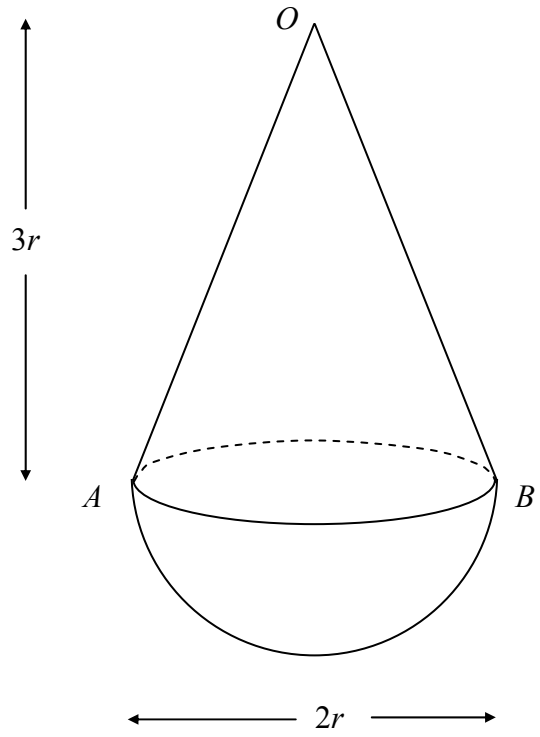
2. **Figure 1**



A child's toy consists of a uniform solid hemisphere, of mass M and base radius r, joined to a uniform solid right circular cone of mass m, where $2m < M$. The cone has vertex O, base radius r and height $3r$. Its plane face, with diameter AB, coincides with the plane face of the hemisphere, as shown in Figure 1.

(a) Show that the distance of the centre of mass of the toy from AB is

$$\frac{3(M-2m)}{8(M+m)}r.$$

(5)

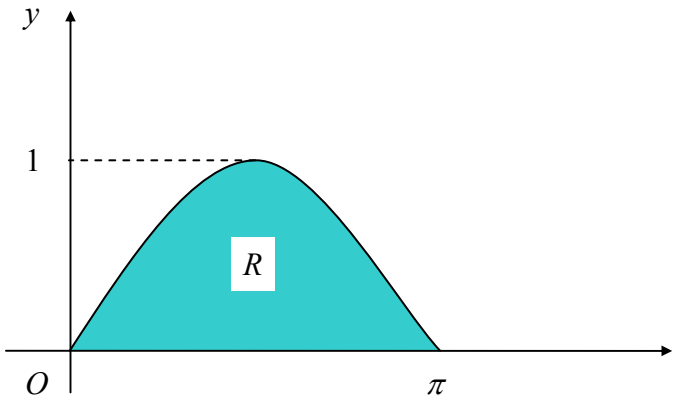
The toy is placed with OA on a horizontal surface. The toy is released from rest and does not remain in equilibrium.

(b) Show that $M > 26m$.

(4)

3.

Figure 2



A uniform lamina occupies the region R bounded by the x-axis and the curve

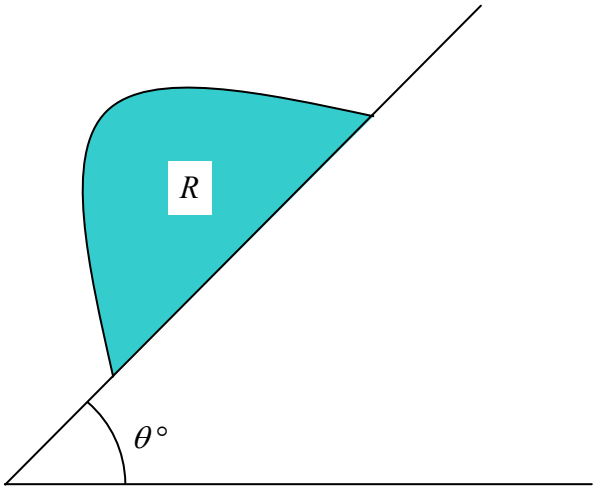
$$y = \sin x, \qquad 0 \leq x \leq \pi,$$

as shown in Figure 2.

(a) Show, by integration, that the y-coordinate of the centre of mass of the lamina is $\frac{\pi}{8}$.

(6)

Figure 3

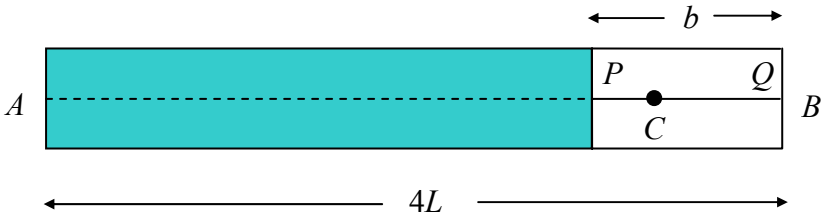


A uniform prism S has cross-section R. The prism is placed with its rectangular face on a table which is inclined at an angle $\theta°$ to the horizontal. The cross-section R lies in a vertical plane as shown in Figure 3. The table is sufficiently rough to prevent S sliding. Given that S does not topple,

(b) find the largest possible value of θ.

(3)

4. **Figure 4**



In a game at a fair, a small target C moves horizontally with simple harmonic motion between the points A and B, where $AB = 4L$. The target moves inside a box and takes 3 s to travel from A to B. A player has to shoot at C, but C is only visible to the player when it passes a window PQ, where $PQ = b$. The window is initially placed with Q at the point as shown in Figure 4. The target C takes 0.75 s to pass from Q to P.

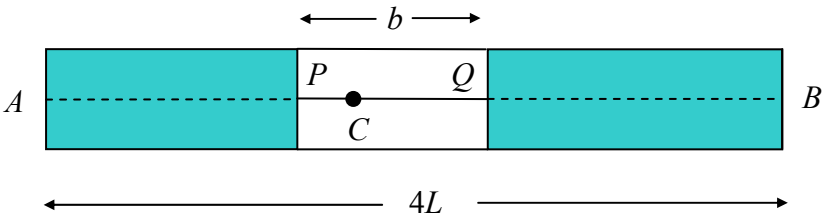
(a) Show that $b = (2 - \sqrt{2})L$.

(5)

(b) Find the speed of C as it passes P.

(2)

Figure 5



For advanced players, the window PQ is moved to the centre of AB so that $AP = QB$, as shown in Figure 5.

(c) Find the time, in seconds to 2 decimal places, taken for C to pass from Q to P in this new position.

(3)

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5. At time $t = 0$, a particle P is at the origin O, moving with speed 18 m s^{-1} along the x-axis, in the positive x-direction. At time t seconds ($t > 0$) the acceleration of P has magnitude $\frac{3}{\sqrt{(t+4)}}$ m s^{-2} and is directed towards O.

(a) Show that, at time t seconds, the velocity of P is $[30 - 6\sqrt{(t+4)}]$ m s^{-1}.

(5)

(b) Find the distance of P from O when P comes to instantaneous rest.

(7)

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6. A light spring of natural length L has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the spring. The particle is moving vertically. As it passes through the point B below A, where $AB = L$, its speed is $\sqrt{(2gL)}$. The particle comes to instantaneous rest at a point C, $4L$ below A.

(a) Show that the modulus of elasticity of the spring is $\frac{8mg}{9}$.

(4)

At the point D the tension in the spring is mg.

(b) Show that P performs simple harmonic motion with centre D.

(5)

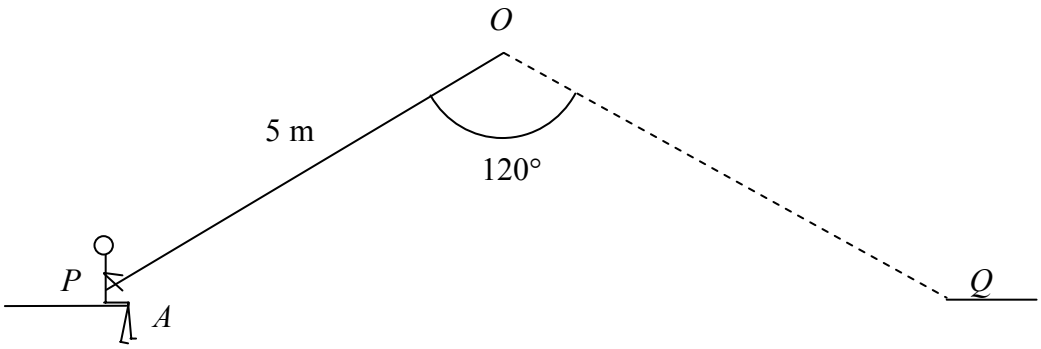
(c) Find, in terms of L and g,

(i) the period of the simple harmonic motion,

(ii) the maximum speed of P.

(5)

7. **Figure 6**



A trapeze artiste of mass 60 kg is attached to the end A of a light inextensible rope OA of length 5 m. The artiste must swing in an arc of a vertical circle, centre O, from a platform P to another platform Q, where PQ is horizontal. The other end of the rope is attached to the fixed point O which lies in the vertical plane containing PQ, with $\angle POQ = 120°$ and $OP = OQ = 5$ m, as shown in Figure 6.

As part of her act, the artiste projects herself from P with speed $\sqrt{15}$ m s^{-1} in a direction perpendicular to the rope OA and in the plane POQ. She moves in a circular arc towards Q. At the lowest point of her path she catches a ball of mass m kg which is travelling towards her with speed 3 m s^{-1} and parallel to QP. After catching the ball, she comes to rest at the point Q.

By modelling the artiste and the ball as particles and ignoring her air resistance, find

(a) the speed of the artiste immediately before she catches the ball,

(4)

(b) the value of m,

(7)

(c) the tension in the rope immediately after she catches the ball.

(3)

TOTAL FOR PAPER: 75 MARKS

END