## Edexcel GCE

## Decision Mathematics D1

## Advanced/Advanced Subsidiary

## Tuesday 2 November 2004 - Afternoon

## Time: 1 hour 30 minutes

Materials required for examination Items included with question papers Nil D1 Answer booklet


#### Abstract

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.


## Instructions to Candidates

Write your answers for this paper in the D1 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initial(s) and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has eight questions.
The total mark for this paper is 75 . Page 12 is blank.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers for this paper in the D1 answer book.

1. 

Figure 1


Figure 1 shows a directed, capacitated network where the number on each arc is its capacity. A possible flow is shown from $S$ to $T$ and the value in brackets on each arc is the flow in that arc.
(a) Find the values of $x, y$ and $z$.
(b) Find, by inspection, the maximal flow from $S$ to $T$ and verify that it is maximal.
2. (a) Define the following terms
(i) planar graph,
(ii) Hamiltonian cycle.
(b) (i) Draw a graph of $\mathrm{K}_{3,2}$ in such a way as to show that it is planar.
(ii) Explain why the planarity algorithm cannot be used when drawing $\mathrm{K}_{3,2}$ as a planar graph.
3. Six newspaper reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1,2 or 4.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  | $\checkmark$ |  |
| B | $\checkmark$ |  |  | $\checkmark$ |  |  |
| C | $\checkmark$ | $\checkmark$ |  | $\checkmark$ |  |  |
| D |  |  |  |  | $\checkmark$ |  |
| E |  |  | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |
| F |  |  |  | $\checkmark$ |  |  |

(a) Show these possible allocations on the bipartite graph on the diagram in the answer book.

A possible matching is

$$
\text { A to } 5, \quad \text { C to } 1, \quad \text { E to } 6, \quad \text { F to } 4
$$

(b) Show this information, in a distinctive way, on the diagram in the answer book.
(c) Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.
(d) Explain why it is not possible to find a complete matching.
4. $\quad 45, \quad 56, \quad 37, \quad 79, \quad 46, \quad 18, \quad 90, \quad 81, \quad 51$
(a) Using the quick sort algorithm, perform one complete iteration towards sorting these numbers into ascending order.
(b) Using the bubble sort algorithm, perform one complete pass towards sorting the original list into descending order.

Another list of numbers, in ascending order, is

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\(7, \quad 23, \quad 31, \quad 37, \quad 41, \quad 44, \quad 50, \quad 62, \quad 71, \quad 73, \quad 94\)
```

(c) Use the binary search algorithm to locate the number 73 in this list.


Figure 2 shows a network of roads connecting villages. The length of each road, in km, is shown. Village $B$ has only a small footbridge over the river which runs through the village. It can be accessed by two roads, from $A$ and $D$.

The driver of a snowplough, based at $F$, is planning a route to enable her to clear all the roads of snow. The route should be of minimum length. Each road can be cleared by driving along it once. The snowplough cannot cross the footbridge.

Showing all your working and using an appropriate algorithm,
(a) find the route the driver should follow, starting and ending at $F$, to clear all the roads of snow. Give the length of this route.

The local authority decides to build a road bridge over the river at $B$. The snowplough will be able to cross the road bridge.
(b) Reapply the algorithm to find the minimum distance the snowplough will have to travel (ignore the length of the new bridge).
6.

Figure 3


Peter wishes to minimise the time spent driving from his home $H$, to a campsite at $G$. Figure 3 shows a number of towns and the time, in minutes, taken to drive between them. The volume of traffic on the roads into $G$ is variable, and so the length of time taken to drive along these roads is expressed in terms of $x$, where $x \geq 0$.
(a) On the diagram in the answer book, use Dijkstra's algorithm to find two routes from $H$ to $G$ (one via $A$ and one via $B$ ) that minimise the travelling time from $H$ to $G$. State the length of each route in terms of $x$.
(b) Find the range of values of $x$ for which Peter should follow the route via $A$.
7.

Figure 4


The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y .

The constraints are modelled in Figure 4 where
$x=$ the number (in thousands) of type X batteries produced each day,
$y=$ the number (in thousands) of type Y batteries produced each day.
The profit on each type $X$ battery is 40 p and on each type $Y$ battery is 20 p. The company wishes to maximise its daily profit.
(a) Write this as a linear programming problem, in terms of $x$ and $y$, stating the objective function and all the constraints.
(b) Find the optimal number of batteries to be made each day. Show your method clearly.
(c) Find the daily profit, in $£$, made by EXYCEL.
8.


The network in Figure 5 shows activities that need to be undertaken in order to complete a project. Each activity is represented by an arc. The number in brackets is the duration of the activity in hours. The early and late event times are shown at each node. The project can be completed in 24 hours.
(a) Find the values of $x, y$ and $z$.
(b) Explain the use of the dummy activity in Figure 5.
(c) List the critical activities.
(d) Explain what effect a delay of one hour to activity $B$ would have on the time taken to complete the whole project.

The company which is to undertake this project has only two full time workers available. The project must be completed in 24 hours and in order to achieve this, the company is prepared to hire additional workers at a cost of $£ 28$ per hour. The company wishes to minimise the money spent on additional workers. Any worker can undertake any task and each task requires only one worker.
(e) Explain why the company will have to hire additional workers in order to complete the project in 24 hours.
(f) Schedule the tasks to workers so that the project is completed in 24 hours and at minimum cost to the company.
(g) State the minimum extra cost to the company.

