Paper Reference(s)

6687

Edexcel GCE

Statistics S5

Advanced/Advanced Subsidiary

Wednesday 23 June 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S5), the paper reference (6687), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. In a school, 30% of students have dark hair, 60% of students have fair hair and 10% of students have red hair. The probabilities of a student with each hair colour having brown eyes are 0.8, 0.6 and 0.1 respectively.

(*a*) Find the probability that a randomly selected student has brown eyes.

(2)

(*b*) Given that a student has brown eyes, find the probability that the student also has red hair.

(2)

Two students are selected at random. Both have brown eyes.

(*c*) Find the probability that at least one of them has red hair.

(3)

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2. In an apartment block, the lift breaks down randomly at a mean rate of 3 times per week. The random variable X represents the time in days between successive lift breakdowns.

(*a*) Write down the probability density function of X.

(2)

(*b*) Calculate the probability that the time interval between successive lift breakdowns is between 2 and 3 days.

(3)

(*c*) Find the probability that, after a breakdown has just occurred, at least 1 week will pass without another breakdown occurring.

(2)

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3. An archer shoots at a target until he hits it. The random variable S is the number of shots needed by the archer to hit the target.

(*a*) State a suitable distribution to model S.

(1)

Given that the mean of S is 8, calculate the probability of the archer

(*b*) hitting the target for the first time on his 5th shot,

(3)

(*c*) taking at least 3 shots to hit the target for the first time.

(3)

(*d*) State any assumptions you have made in using this model.

(2)

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4. The random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{1}{a}, & 0 \le x \le a, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prove that the moment generating function of X is $\mathrm{M}_X(t) = \dfrac{1}{at}(e^{at} - 1)$.

(4)

A second random variable Y has mean $\dfrac{5}{4}$ and moment generating function

$$\mathrm{M}_Y(t) = \frac{1}{4}(1 + Ae^t + Be^{2t}).$$

(b) Find the value of A and the value of B.

(6)

(c) Given that X and Y are independent, write down the moment generating function of $Z = X + Y$.

(1)

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5. The random variable Y is the number of times a biased coin is tossed until 3 heads have occurred. The variance of Y is 60.

(a) Find the probability of obtaining a head.

(5)

(b) Find $\mathrm{P}(Y = 8)$.

(2)

(c) Find $\mathrm{P}(Y \le 10 \mid \text{the first head was gained on the second toss})$.

(5)

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6. Computer chips are produced in batches of size 1000. Samples of size 20 are taken from each batch. The batch is rejected if it contains more than 1 defective computer chip. The proportion of detectives in each batch is denoted by p.

(*a*) Show that the probability P of accepting a batch is given by

$$P = (1 - p)^{19}(1 + 19p).$$

(3)

The table below shows the probabilities of accepting the batch for different values of p.

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
p	0.983	0.940	j	0.810	0.736	0.660	k	0.517

(*b*) Find the value of j and the value of k.

(2)

(*c*) On graph paper draw the Operating Characteristic of the probability of acceptance P against the proportion of detectives p.

(3)

(*d*) Using your graph estimate the probability of rejecting a batch when (i) 1.5%, (ii) 6.5% of the computer chips in the batch are defective.

(3)

(*e*) Use your graph to comment on this sampling scheme.

(2)

7. A shop hires out carpet cleaners by the day. The number of requests X per day to hire a cleaner can be modelled as a Poisson distribution with mean 3.

(*a*) Find, in terms of e, the probability that on a particular day there will be

(i) exactly 2,

(ii) at least 4

requests to hire a cleaner.

(5)

The random variable Y represents the number of carpet cleaners hired on a particular day. The shop has 4 cleaners.

(*b*) Show that the probability generating function of Y, $\mathrm{G}_Y(t)$ is given by

$$\mathrm{G}_Y(t) = \mathrm{e}^{-3}(1 + 3t + 4.5t^2 + 4.5t^3 - 13t^4) + t^4.$$

(3)

(*c*) Use the probability generating function to find the mean and the standard deviation of Y.

(8)

END