



1. Express as a single fraction in its simplest form

$$\frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x-5)^2}. \quad (4)$$

2. (i) Given that  $\sin x = \frac{3}{5}$ , use an appropriate double angle formula to find the exact value of  $\sec 2x$ .

(4)

- (ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \quad \left(x \neq \frac{n\pi}{2}, n \in \mathbb{Z}\right).$$

(4)

3. For the binomial expansion, in descending powers of  $x$ , of

$$\left(x^3 - \frac{1}{2x}\right)^{12},$$

- (a) find the first 4 terms, simplifying each term.

(5)

- (b) Find, in its simplest form, the term independent of  $x$  in this expansion.

(3)

4.

Figure 1

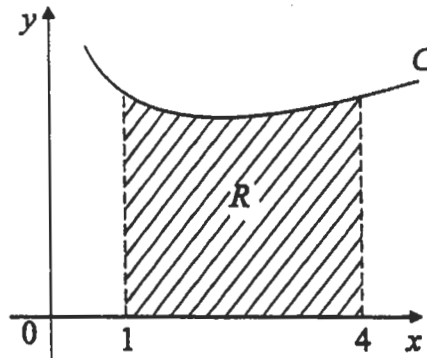


Figure 1 shows parts of the curve  $C$  with equation

$$y = \frac{x+2}{\sqrt{x}}$$

The shaded region  $R$  is bounded by  $C$ , the  $x$ -axis and the lines  $x=1$  and  $x=4$ .

This region is rotated through  $360^\circ$  about the  $x$ -axis to form a solid  $S$ .

(a) Find, by integration, the exact volume of  $S$ .

(7)

The solid  $S$  is used to model a wooden support with a circular base and a circular top.

(b) Show that the base and the top have the same radius.

(1)

Given that the actual radius of the base is 6 cm,

(c) show that the volume of the wooden support is approximately  $630 \text{ cm}^3$ .

(2)

5.

$$f(x) = x^3 + x^2 - 4x - 1.$$

The equation  $f(x) = 0$  has only one positive root,  $\alpha$ .

(a) Show that  $f(x) = 0$  can be rearranged as

$$x = \sqrt{\left(\frac{4x+1}{x+1}\right)}, \quad x \neq -1. \quad (2)$$

The iteration formula  $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$  is used to find an approximation to  $\alpha$ .

(b) Taking  $x_1 = 1$ , find, to 2 decimal places, the values of  $x_2$ ,  $x_3$  and  $x_4$ . (3)

(c) By choosing values of  $x$  in a suitable interval, prove that  $\alpha = 1.70$ , correct to 2 decimal places. (3)

(d) Write down a value of  $x_1$  for which the iteration formula  $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$  does not produce a valid value for  $x_2$ .

Justify your answer.

(2)

6. Given that  $\log_5 x = a$  and  $\log_5 y = b$ , find, in terms of  $a$  and  $b$ ,

(a)  $\log_5\left(\frac{x^2}{y}\right)$ , (2)

(b)  $\log_5(25x\sqrt{y})$ . (3)

It is given that  $\log_5\left(\frac{x^2}{y}\right) = 1$  and that  $\log_5(25x\sqrt{y}) = 1$ .

(c) Form simultaneous equations in  $a$  and  $b$ . (1)

(d) Show that  $a = -0.25$  and find the value of  $b$ . (2)

Using the values of  $a$  and  $b$ , or otherwise,

(e) calculate, to 3 decimal places, the value of  $x$  and the value of  $y$ . (3)

7.

$$f(x) = x + \frac{e^x}{5}, \quad x \in \mathbb{R}.$$

(a) Find  $f'(x)$ .

(2)

The curve  $C$ , with equation  $y = f(x)$ , crosses the  $y$ -axis at the point  $A$ .

(b) Find an equation for the tangent to  $C$  at  $A$ .

(3)

(c) Complete the table, giving the values of  $\sqrt{\left(x + \frac{e^x}{5}\right)}$  to 2 decimal places.

$x$	0	0.5	1	1.5	2
$\sqrt{\left(x + \frac{e^x}{5}\right)}$	0.45	0.91			

(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x + \frac{e^x}{5}\right)} dx.$$

(4)

8. The function  $f$  is given by

$$f: x \mapsto \ln(3x - 6), \quad x \in \mathbb{R}, \quad x > 2.$$

(a) Find  $f^{-1}(x)$ .

(3)

(b) Write down the domain of  $f^{-1}$  and the range of  $f^{-1}$ .

(2)

(c) Find, to 3 significant figures, the value of  $x$  for which  $f(x) = 3$ .

(2)

The function  $g$  is given by

$$g: x \mapsto \ln|3x - 6|, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(d) In the space at the top of page 15, sketch the graph of  $y = g(x)$ .

(3)

(e) Find the exact coordinates of all the points at which the graph of  $y = g(x)$  meets the coordinate axes.

(3)