

Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Thursday 1 July 2004 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Answer Book (AB16)

Graph Paper (ASG2)

Mathematical Formulae (Lilac)

Items included with question papers

Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

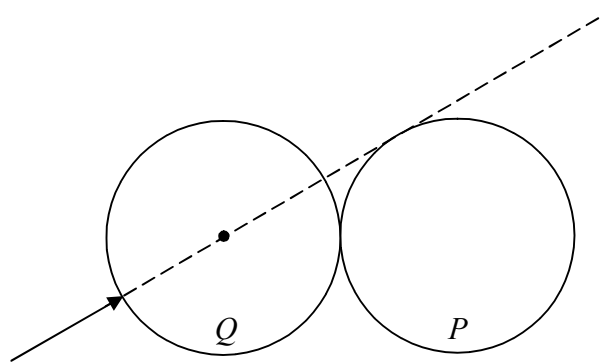
1. [In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively.]

An aeroplane makes a journey from a point P to a point Q which is due east of P . The wind velocity is $w(\cos \theta \mathbf{i} + \sin \theta \mathbf{j})$, where w is a positive constant. The velocity of the aeroplane relative to the wind is $v(\cos \phi \mathbf{i} - \sin \phi \mathbf{j})$, where v is a constant and $v > w$. Given that θ and ϕ are both acute angles,

(a) show that $v \sin \phi = w \sin \theta$, (2)

(b) find, in terms of v , w and θ , the speed of the aeroplane relative to the ground. (4)

2. Figure 1



A smooth uniform sphere P is at rest on a smooth horizontal plane, when it is struck by an identical sphere Q moving on the plane. Immediately before the impact, the line of motion of the centre of Q is tangential to the sphere P , as shown in Fig. 1. The direction of motion of Q is turned through 30° by the impact.

Find the coefficient of restitution between the spheres. (11)

3. At noon, two boats A and B are 6 km apart with A due east of B . Boat B is moving due north at a constant speed of 13 km h^{-1} . Boat A is moving with constant speed 12 km h^{-1} and sets a course so as to pass as close as possible to boat B . Find

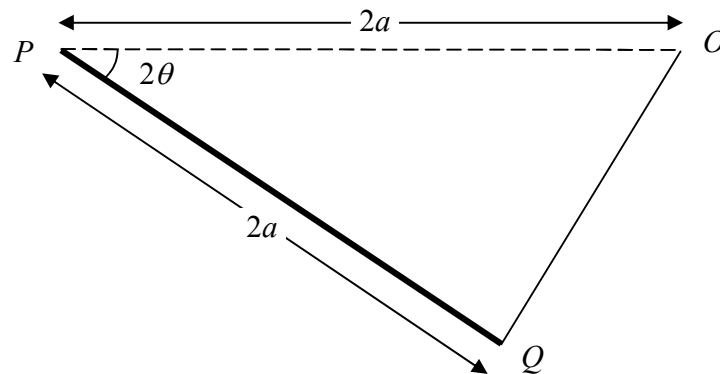
(a) the direction of motion of A , giving your answer as a bearing, (4)

(b) the time when the boats are closest, (5)

(c) the shortest distance between the boats. (2)

4.

Figure 2



A uniform rod PQ , of length $2a$ and mass m , is free to rotate in a vertical plane about a fixed smooth horizontal axis through the end P . The end Q is attached to one end of a light elastic string, of natural length a and modulus of elasticity $\frac{mg}{2\sqrt{3}}$. The other end of the string is attached to a fixed point O , where OP is horizontal and $OP = 2a$, as shown in Fig. 2. $\angle OPQ$ is denoted by 2θ .

(a) Show that, when the string is taut, the potential energy of the system is

$$-\frac{mga}{\sqrt{3}}(2 \cos 2\theta + \sqrt{3} \sin 2\theta + 2 \sin \theta) + \text{constant.} \quad (7)$$

(b) Verify that there is a position of equilibrium at $\theta = \frac{\pi}{6}$. (4)

(c) Determine whether this is a position of stable equilibrium. (4)

5. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mak^2$, where k is a positive constant. The other end of the string is attached to a fixed point A . At time $t = 0$, P is released from rest from a point which is a distance $2a$ vertically below A . When P is moving with speed v , the air resistance has magnitude $2mkv$. At time t , the extension of the string is x .

(a) Show that, while the string is taut,

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = g. \quad (4)$$

You are given that the general solution of this differential equation is

$$x = e^{-kt}(C \sin kt + D \cos kt) + \frac{g}{2k^2}, \quad \text{where } C \text{ and } D \text{ are constants.}$$

- (b) Find the value of C and the value of D . (5)

Assuming that the string remains taut,

- (c) find the value of t when P first comes to rest, (3)

- (d) show that $2k^2a < g(1 + e^\pi)$. (4)

6. A particle P of mass m is attached to one end of a light inextensible string and hangs at rest at time $t = 0$. The other end of the string is then raised vertically by an engine which is working at a constant rate kmg , where $k > 0$. At time t , the distance of P above its initial position is x , and P is moving upwards with speed v .

- (a) Show that $v^2 \frac{dv}{dx} = (k - v)g$. (4)

- (b) Show that $gx = k^2 \ln\left(\frac{k}{k - v}\right) - kv - \frac{1}{2}v^2$. (7)

- (c) Hence, or otherwise, find t in terms of k , v and g . (5)

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