## Edexcel GCE

## Decision Mathematics D2

# Advanced/Advanced Subsidiary 

Friday 28 May 2004 - Afternoon

## Time: 1 hour 30 minutes

Materials required for examination Items included with question papers<br>Graph paper (ASG2)<br>D2 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has six questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers in the D2 answer booklet for this paper.

1. In game theory explain what is meant by
(a) zero-sum game,
(b) saddle point.
2. In a quiz there are four individual rounds, Art, Literature, Music and Science. A team consists of four people, Donna, Hannah, Kerwin and Thomas. Each of four rounds must be answered by a different team member.

The table shows the number of points that each team member is likely to get on each individual round.

|  | Art | Literature | Music | Science |
| :--- | :---: | :---: | :---: | :---: |
| Donna | 31 | 24 | 32 | 35 |
| Hannah | 16 | 10 | 19 | 22 |
| Kerwin | 19 | 14 | 20 | 21 |
| Thomas | 18 | 15 | 21 | 23 |

Use the Hungarian algorithm, reducing rows first, to obtain an allocation which maximises the total points likely to be scored in the four rounds. You must make your method clear and show the table after each stage.
3. The table shows the least distances, in km, between five towns, $A, B, C, D$ and $E$.

|  | $A$ | $B$ | $C$ | $D$ | $E$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | - | 153 | 98 | 124 | 115 |
| $B$ | 153 | - | 74 | 131 | 149 |
| $C$ | 98 | 74 | - | 82 | 103 |
| $D$ | 124 | 131 | 82 | - | 134 |
| $E$ | 115 | 149 | 103 | 134 | - |

Nassim wishes to find an interval which contains the solution to the travelling salesman problem for this network.
(a) Making your method clear, find an initial upper bound starting at $A$ and using
(i) the minimum spanning tree method,
(ii) the nearest neighbour algorithm.
(b) By deleting $E$, find a lower bound.
(c) Using your answers to parts (a) and (b), state the smallest interval that Nassim could correctly write down.
4. Emma and Freddie play a zero-sum game. This game is represented by the following pay-off matrix for Emma.

$$
\left(\begin{array}{rrr}
-4 & -1 & 3 \\
2 & 1 & -2
\end{array}\right)
$$

(a) Show that there is no stable solution.
(b) Find the best strategy for Emma and the value of the game to her.
(c) Write down the value of the game to Freddie and his pay-off matrix.
5. (a) Describe a practical problem that could be solved using the transportation algorithm.

A problem is to be solved using the transportation problem. The costs are shown in the table. The supply is from $A, B$ and $C$ and the demand is at $d$ and $e$.

|  | $d$ | $e$ | Supply |
| :---: | :---: | :---: | :---: |
| $A$ | 5 | 3 | 45 |
| $B$ | 4 | 6 | 35 |
| $C$ | 2 | 4 | 40 |
| Demand | 50 | 60 |  |

(b) Explain why it is necessary to add a third demand $f$.
(c) Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.
(d) Calculate shadow costs and improvement indices for this pattern.
(e) Use the stepping-stone method once to obtain an improved solution and its cost.
6. Joan sells ice cream. She needs to decide which three shows to visit over a three-week period in the summer. She starts the three-week period at home and finishes at home. She will spend one week at each of the three shows she chooses travelling directly from one show to the next.

Table 1 gives the week in which each show is held. Table 2 gives the expected profits from visiting each show. Table 3 gives the cost of travel between shows.

Table 1

| Week | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| Shows | A, B, C | D, E | F, G, H |

Table 2

| Show | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ | $H$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expected Profit (£) | 900 | 800 | 1000 | 1500 | 1300 | 500 | 700 | 600 |

Table 3

| Travel costs (£) | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Home | 70 | 80 | 150 |  |  | 80 | 90 | 70 |
| A |  |  |  | 180 | 150 |  |  |  |
| B |  |  |  | 140 | 120 |  |  |  |
| C |  |  |  | 200 | 210 |  |  |  |
| D |  |  |  |  |  | 200 | 160 | 120 |
| E |  |  |  |  |  | 170 | 100 | 110 |

It is decided to use dynamic programming to find a schedule that maximises the total expected profit, taking into account the travel costs.
(a) Define suitable stage, state and action variables.
(b) Determine the schedule that maximises the total profit. Show your working in the table in the answer book.
(c) Advise Joan on the shows that she should visit and state her total expected profit.

## END

