1. The function f is given by

$$f: x \mapsto 2 + \frac{3}{x+2}, \quad x \in \mathbb{R}, \quad x \neq -2.$$

(a) Express $2 + \frac{3}{x+2}$ as a single fraction. **(1)**

(b) Find an expression for $f^{-1}(x)$. **(3)**

(c) Write down the domain of f^{-1}. **(1)**

2. A sequence is defined by the recurrence relation

$$u_{n+1} = \sqrt{\left(\frac{u_n}{2} + \frac{a}{u_n}\right)}, \quad n = 1, 2, 3, \ldots,$$

where a is a constant.

(a) Given that $a = 20$ and $u_1 = 3$, find the values of u_2, u_3 and u_4, giving your answers to 2 decimal places. **(3)**

(b) Given instead that $u_1 = u_2 = 3$,

(i) calculate the value of a, **(3)**

(ii) write down the value of u_5. **(1)**

3. Given that $\log_2 x = a$, find, in terms of a, the simplest form of

(a) $\log_2(16x)$, **(2)**

(b) $\log_2\left(\frac{x^4}{2}\right)$. **(3)**

(c) Hence, or otherwise, solve

$$\log_2(16x) - \log_2\left(\frac{x^4}{2}\right) = \frac{1}{2},$$

giving your answer in its simplest surd form. **(4)**

4. The function f is even and has domain $\mathbb{R}$. For $x \geqslant 0$, $f(x) = x^2 - 4ax$, where a is a positive constant.

(a) In the space below, sketch the curve with equation $y = f(x)$, showing the coordinates of all the points at which the curve meets the axes. **(3)**

(b) Find, in terms of a, the value of $f(2a)$ and the value of $f(-2a)$. **(2)**

Given that $a = 3$,

(c) use algebra to find the values of x for which $f(x) = 45$. **(4)**

5. Given that $y = \log_a x$, $x > 0$, where a is a positive constant,

(a) (i) express x in terms of a and y, **(1)**

(ii) deduce that $\ln x = y \ln a$. **(1)**

(b) Show that $\frac{dy}{dx} = \frac{1}{x \ln a}$. **(2)**

The curve C has equation $y = \log_{10} x$, $x > 0$. The point A on C has x-coordinate 10. Using the result in part (b),

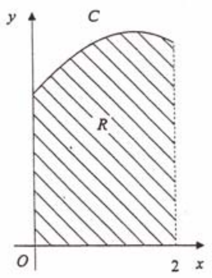
(c) find an equation for the tangent to C at A. **(4)**

The tangent to C at A crosses the x-axis at the point B.

(d) Find the exact x-coordinate of B. **(2)**

6.

Figure 1



The curve C has equation $y = f(x)$, $x \in \mathbb{R}$. Figure 1 shows the part of C for which $0 \leqslant x \leqslant 2$.

Given that

$$\frac{dy}{dx} = e^x - 2x^2,$$

and that C has a single maximum, at $x = k$,

(a) show that $1.48 < k < 1.49$. **(3)**

Given also that the point $(0, 5)$ lies on C,

(b) find $f(x)$. **(4)**

The finite region R is bounded by C, the coordinate axes and the line $x = 2$.

(c) Use integration to find the exact area of R. **(4)**

7. A student tests the accuracy of the trapezium rule by evaluating I, where

$$I = \int_{0.5}^{1.5} \left(\frac{3}{x} + x^4\right) dx.$$

(a) Complete the student's table, giving values to 2 decimal places where appropriate.

x	0.5	0.75	1	1.25	1.5
$\frac{3}{x} + x^4$	6.06	4.32			

(2)

(b) Use the trapezium rule, with all the values from your table, to calculate an estimate for the value of I. **(4)**

(c) Use integration to calculate the exact value of I. **(4)**

(d) Verify that the answer obtained by the trapezium rule is within 3% of the exact value. **(2)**

8. (i) (a) Express $(12\cos\theta - 5\sin\theta)$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. **(4)**

(b) Hence solve the equation

$$12\cos\theta - 5\sin\theta = 4,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place. **(3)**

(ii) Solve

$$8\cot\theta - 3\tan\theta = 2,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place. **(5)**