Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 21 January 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

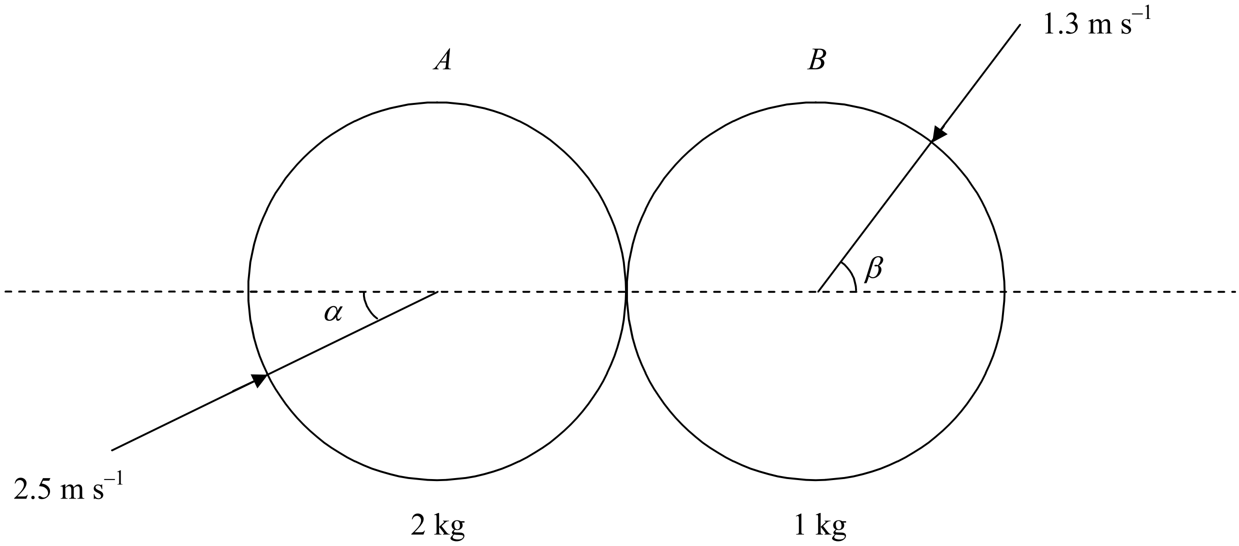
N13900A

1. A particle P of mass 3 kg moves in a straight line on a smooth horizontal plane. When the speed of P is v m s^{-1}, the resultant force acting on P is a resistance to motion of magnitude $2v$ N. Find the distance moved by P while slowing down from 5 m s^{-1} to 2 m s^{-1}.

(5)

2.

Figure 1



Two smooth uniform spheres A and B of equal radius have masses 2 kg and 1 kg respectively. They are moving on a smooth horizontal plane when they collide. Immediately before the collision the speed of A is 2.5 m s^{-1} and the speed of B is 1.3 m s^{-1}. When they collide the line joining their centres makes an angle α with the direction of motion of A and an angle β with the direction of motion of B, where $\tan \alpha = \frac{4}{3}$ and $\tan \beta = \frac{12}{5}$ as shown in Fig. 1.

(*a*) Find the components of the velocities of A and B perpendicular and parallel to the line of centres immediately before the collision.

(4)

The coefficient of restitution between A and B is $\frac{1}{2}$.

(*b*) Find, to one decimal place, the speed of each sphere after the collision.

(9)

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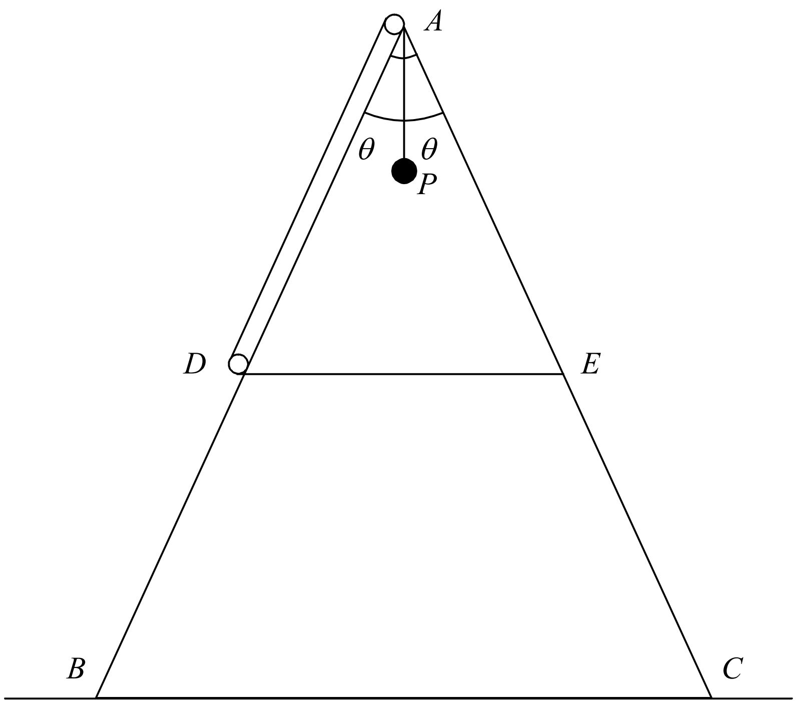
3. **Figure 2**



Two uniform rods AB and AC, each of mass $2m$ and length $2L$, are freely jointed at A. The mid-points of the rods are D and E respectively. A light inextensible string of length s is fixed to E and passes round small, smooth light pulleys at D and A. A particle P of mass m is attached to the other end of the string and hangs vertically. The points A, B and C lie in the same vertical plane with B and C on a smooth horizontal surface. The angles PAB and PAC are each equal to θ ($\theta > 0$), as shown in Fig. 2.

(a) Find the length of AP in terms of s, L and θ.

(2)

(b) Show that the potential energy V of the system is given by

$$V = 2mgL\,(3\cos\theta + \sin\theta) + \text{constant}.$$

(4)

(c) Hence find the value of θ for which the system is in equilibrium.

(4)

(d) Determine whether this position of equilibrium is stable or unstable.

(4)

4. A particle P of mass m is attached to the mid-point of a light elastic string, of natural length $2L$ and modulus of elasticity $2mk^2L$, where k is a positive constant. The ends of the string are attached to points A and B on a smooth horizontal surface, where $AB = 3L$. The particle is released from rest at the point C, where $AC = 2L$ and ACB is a straight line. During the subsequent motion P experiences air resistance of magnitude $2mkv$, where v is the speed of P. At time t, $AP = 1.5L + x$.

(*a*) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 4k^2x = 0.$

(6)

(*b*) Find an expression, in terms of t, k and L, for the distance AP at time t.

(8)

5.

Figure 3

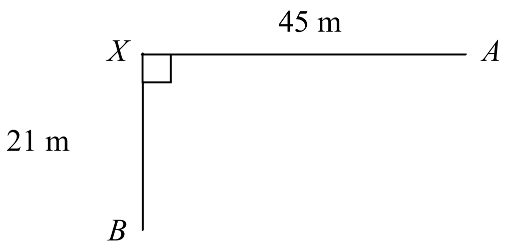


Figure 3 represents the scene of a road accident. A car of mass 600 kg collided at the point X with a stationary van of mass 800 kg. After the collision the van came to rest at the point A having travelled a horizontal distance of 45 m, and the car came to rest at the point B having travelled a horizontal distance of 21 m. The angle AXB is 90°.

The accident investigators are trying to establish the speed of the car before the collision and they model both vehicles as small spheres.

(*a*) Find the coefficient of restitution between the car and the van.

(5)

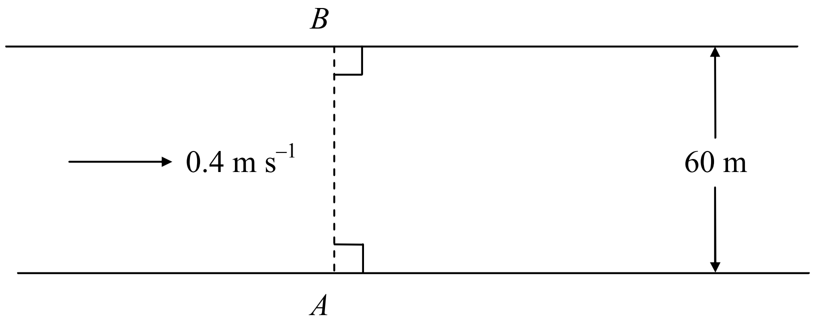
The investigators assume that after the collision, and until the vehicles came to rest, the van was subject to a constant horizontal force of 500 N acting along AX and the car to a constant horizontal force of 300 N along BX.

(*b*) Find the speed of the car immediately before the collision.

(9)

6.

Figure 4



Mary swims in still water at 0.85 m s^{-1}. She swims across a straight river which is 60 m wide and flowing at 0.4 m s^{-1}. She sets off from a point A on the near bank and lands at a point B, which is directly opposite A on the far bank, as shown in Fig. 4.

Find

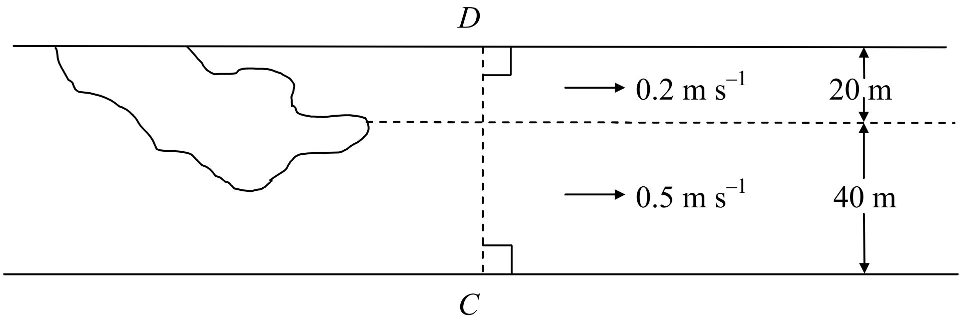
(*a*) the angle between the near bank and the direction in which Mary swims,

(3)

(*b*) the time she takes to cross the river.

(3)

Figure 5



A little further downstream a large tree has fallen from the far bank into the river. The river is modelled as flowing at 0.5 m s^{-1} for a width of 40 m from the near bank, and 0.2 m s^{-1} for the 20 m beyond this. Nassim swims at 0.85 m s^{-1} in still water. He swims across the river from a point C on the near bank. The point D on the far bank is directly opposite C, as shown in Fig. 5. Nassim swims at the same angle to the near bank as Mary.

(*c*) Find the maximum distance, downstream from CD, of Nassim during the crossing.

(5)

(*d*) Show that he will land at the point D.

(4)

END