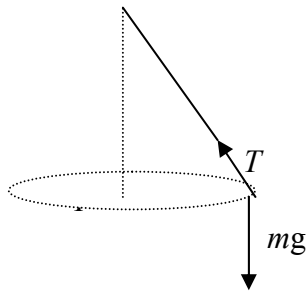


1.



(a) $(\Downarrow) \quad T \cos 60^\circ = mg \Rightarrow T = 2mg \quad *$ B1 (1)

(b) $(\leftrightarrow) \quad T \sin 60^\circ = mr\omega^2$ M1A1
 [Omission of m is M0]

Attempt at $r = L \sin 60^\circ$ M1

$(T \sin 60^\circ = m L \sin 60^\circ \omega^2)$

$$\omega = \sqrt{\frac{2g}{L}}$$
 A1 (4)

(c) Applying Hooke's Law: $2mg = \frac{\lambda}{(\frac{3}{5}L)} (L - \frac{2}{5}L); \quad \lambda = 3mg$ M1;A1 (2)

[L in denominator is M0]

[7]

2.

(a) Integration of $-4e^{-2t}$ w.r.t. t to give $v = 2e^{-2t} + c$ B1

Using initial conditions to find c (-1) or $v - 1 = [f(t)]_0^t$ M1

$v = 2e^{-2t} - 1 \text{ ms}^{-1}$ A1 (3)

(b) **Integrating** v w.r.t. t ; $x = -e^{-2t} - t + c$ M1;A1√

Using $t=0, x=0$ **and** finding value for c ($c=1$) M1

Finding t when $v=0$; $t = \frac{1}{2} \ln 2$ or equiv., 0.347 M1;A1√

[both f.t. marks dependent on v of form $ae^{-2t} \pm b$]

$x = \frac{1}{2} (1 - \ln 2) \text{ m}$ or 0.153 m (awrt) A1 (6)

[9]

[For A1, exact form must be two termed answer]

3. (a) $F = \frac{k}{x^2}$ [k may be seen as Gm_1m_2 , for example] M1
 Equating F to mg at $x = R$, [$mg = \frac{k}{R^2}$] M1
 Convincing completion [$k = mgR^2$] to give $F = \frac{mgR^2}{x^2}$ * A1 (3)

[Note: r may be used instead of x throughout, then $r \rightarrow x$ at end.]

- (b) Equation of motion: $(m)a = (-) \frac{(m)gR^2}{x^2}$; $(m)v \frac{dv}{dx} = - \frac{(m)gR^2}{x^2}$ M1;M1

Integrating: $\frac{1}{2} v^2 = \frac{gR^2}{x}$ (+ c) or equivalent M1A1

[S.C: Allow A1√ if A0 earlier due to “+” only]

Use of $v^2 = \frac{3gR}{2}$, $x = R$ to find c [$c = -\frac{1}{4}gR$] or use in def. int. M1

[Using $x = 0$ is M0] [$v^2 = \frac{2gR^2}{x} - \frac{gR}{2}$]

Substituting $x = 3R$ and finding V ; $V = \sqrt{\frac{gR}{6}}$ M1;A1 (7)

[Using $x = 2R$ is M0]

Alternative in (b)

[10]

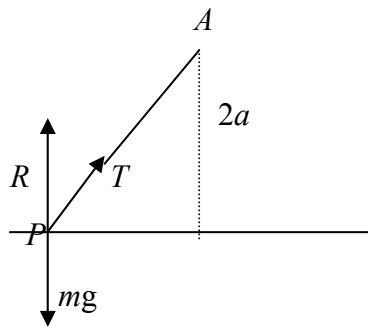
Work/energy $(-)\int_R^a \frac{mgR^2}{x^2} dx$; $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ M1;M1

Integrating: [$\frac{mgR^2}{x} - \frac{mgR^2}{R}$] $= \frac{1}{2}mv^2 - \frac{1}{2}m \frac{3gR}{2}$ M1A1M1

Final 2 marks as scheme M1A1

[Conservation of energy scores 0]

4.



(a) Length of string = $\frac{10}{3} a$

B1

$$\text{EPE} = \frac{\frac{1}{2} mg}{2a} (L - a)^2$$

M1

$$= \frac{49}{36} mga$$

A1 (3)

(b) Energy equation: $\frac{1}{2} mv^2 + \frac{\frac{1}{2} mg}{2a} a^2 = (\frac{49}{36} mga)_C$

M1A1☆

$$v = \frac{2}{3} \sqrt{5ga} \text{ or equivalent}$$

A1 (3)

(c) When string at angle θ to horizontal, length of string = $\frac{2a}{\sin \theta}$

$$\Rightarrow \text{Vert. Comp. of } T, T_v = T \sin \theta = \frac{mg}{2a} \left(\frac{2a}{\sin \theta} - a \right) \sin \theta$$

M1A1

$$= \frac{mg}{2} (2 - \sin \theta)$$

(⇓) $R + T_v = mg$ and find $R = \dots$

M1

$$R = mg - \frac{mg}{2} (2 - \sin \theta) = \frac{mg}{2} \sin \theta$$

A1

$$\Rightarrow R > 0 \text{ (as } \sin \theta > 0), \text{ so stays on table}$$

A1 (5)

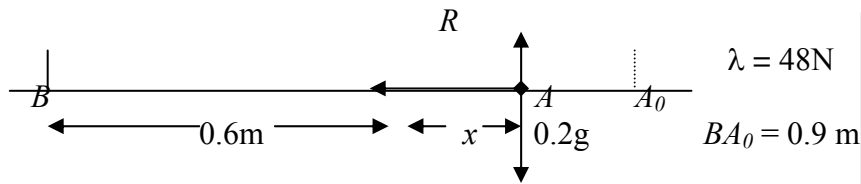
[Alternative final 3 marks: As θ increases so T_v decreases M1

Initial T_v (string at β to hor.) = $\frac{7}{10} mg$ A1

$\Rightarrow T_v \leq \frac{7}{10} mg < mg$, so stays on table A1]

[11]

5. (a)



Applying Hooke's Law correctly : e.g. $T = \frac{48x}{0.6}$

M1

Equation of motion: $(-) T = 0.2 \ddot{x}$

M1

Correct equation of motion: e.g. $-\frac{48x}{0.6} = 0.2 \ddot{x}$

A1

Writing in form $\ddot{x} = -\omega^2 x$, and stating motion is SHM

A1√

Period = $\frac{2\pi}{\omega} = \frac{2\pi}{20} = \frac{\pi}{10}$ * (no incorrect working seen)

A1 (5)

[If measure x from B or A , final 2 marks only available if equation of motion is reduced to $\ddot{X} = -\omega^2 X$]

(b) $\max v = a\omega$ with values substituted; $= 0.3 \times 20 = 6 \text{ ms}^{-1}$

M1A1(2)

(c) Using $x = 0.3 \cos 20t$ or $x = 0.3 \sin 20T$

M1

Using $x = 0.15$ to give either $\cos 20t = \frac{1}{2}$ or $\sin 20T = \frac{1}{2}$

M1

Either $t = \frac{\pi}{60}, \frac{5\pi}{60}$ or $T = \frac{\pi}{120}$

A1

Complete method for time:

$t_2 - t_1$, or $\frac{\pi}{10} - 2t_1$, or $2(\frac{\pi}{40} + T)$

M1

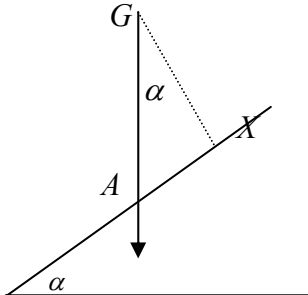
Time = $\frac{\pi}{15}$ s (must be in terms of π)

A1 (5)

[12]

6.	(a)	Cylinder	Hemisphere	S		
		Masses	$(\rho)\pi(2a)^2(\frac{3}{2}a)$ [6 πa^3] [18]	$(\rho)\frac{2}{3}\pi a^3$ [2]	$(\rho)(\frac{16}{3}\pi a^3)$ [16]	M1A1
		Distance of CM from O	$\frac{1}{8}a$	$\frac{3}{8}a$	\bar{x}	B1B1
		Moments equation:	$6\pi a^3(\frac{3}{4}a) - \frac{2}{3}\pi a^3(\frac{3}{8}a) = \frac{16}{3}\pi a^3 \bar{x}$			M1
			$\bar{x} = \frac{51}{64}a$			A1 (6)

(b)



G above “ A ” seen or implied
 or $mg \sin \alpha (GX) = mg \cos \alpha (AX)$

$\tan \alpha = \frac{AX}{XG} = \frac{2a}{\frac{3}{2}a - \bar{x}}$

$[GX = \frac{3}{2}a - \frac{51}{64}a = \frac{45}{64}a, \tan \alpha = \frac{128}{45}] \quad \alpha = 70.6^\circ$

(c) Finding F and R : $R = mg \cos \beta, F = mg \sin \beta$ M1

Using $F = \mu R$ and finding $\tan \beta$ [= 0.8] M1

$\beta = 38.7^\circ$ A1 (3)

[12]

7. (a) Energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \sin \theta$

M1

$$v^2 = \frac{3}{2}ga + 2ga \sin \theta$$

A1 (2)

(b) Radial equation: $T - mg \sin \theta = m \frac{v^2}{a}$

M1A1

$$T = \frac{3mg}{2}(1 + 2\sin \theta) \text{ any form}$$

A1☆ (3)

(c) Setting $T = 0$ and solving trig. equation; $(\sin \theta = -\frac{1}{2}) \Rightarrow \theta = 210^\circ *$

M1;A1(2)

(d) Setting $v = 0$ in (a) and solving for θ

M1

$$\sin \theta = -\frac{3}{4} \text{ so not complete circle}$$

A1 (2)

OR Substituting $\theta = 270^\circ$ in (a); $v^2 < 0$ so not possible to complete

(e) No change in PE \Rightarrow no change in KE (Cof E) so $v = u$

B1 (1)

(f) When string becomes slack, $V^2 = \frac{1}{2}ga$ [$\sin \theta = -\frac{1}{2}$ in (a)]

B1☆

Using fact that horizontal component of velocity is unchanged

M1

$$\sqrt{\frac{ga}{2}} \cos 60^\circ = \sqrt{\frac{3ga}{2}} \cos \phi$$

$$\cos \phi = \sqrt{\frac{1}{12}} \Rightarrow \phi = 73.2^\circ$$

M1A1 (4)

[14]