Paper Reference(s)

6672

Edexcel GCE

Pure Mathematics P2

Advanced/Advanced Subsidiary

Tuesday 4 November 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P2), the paper reference (6672), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. (*a*) Express as a fraction in its simplest form

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}.$$

(3)

(*b*) Hence solve

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}=1.$$

(3)

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2. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £y, where

$$y = 1.04^x.$$

(*a*) Sketch the graph of $y = 1.04^x$, $x \geq 0$.

(2)

(*b*) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years.

(2)

(*c*) Use logarithms to find the number of years it takes to double the total value of any initial investment.

(3)

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3. (*a*) Write down the first 4 terms of the binomial expansion, in ascending powers of x, of $(1 + ax)^n$, $n > 2$.

(2)

Given that, in this expansion, the coefficient of x is 8 and the coefficient of x^2 is 30,

(*b*) calculate the value of n and the value of a,

(4)

(*c*) find the coefficient of x^3.

(2)

4.

Figure 1

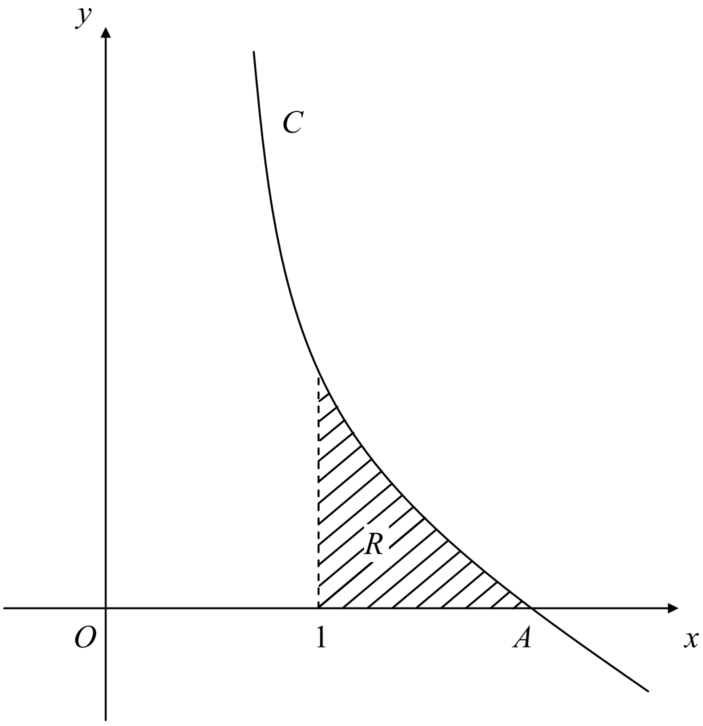

Figure 1 shows the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{8}{x} - x^2, \quad x > 0.$$

Given that C crosses the x-axis at the point A,

(a) find the coordinates of A.

(3)

The finite region R, bounded by C, the x-axis and the line $x = 1$, is rotated through 2π radians about the x-axis.

(b) Use integration to find, in terms of π, the volume of the solid generated.

(7)

5. (*a*) Prove that

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv \cos 2\theta .$$

(4)

(*b*) Hence, or otherwise, prove

$$\tan^2\frac{\pi}{8} = 3 - 2\sqrt{2}.$$

(5)

6.

Figure 2

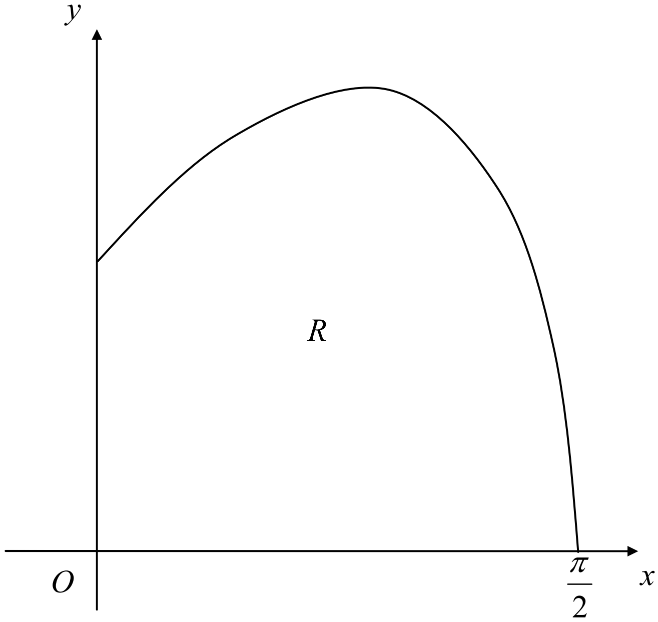

Figure 2 shows part of the curve with equation

$$y = e^x \cos x, \; 0 \leq x \leq \frac{\pi}{2}.$$

The finite region R is bounded by the curve and the coordinate axes.

(a) Calculate, to 2 decimal places, the y-coordinates of the points on the curve where $x = 0, \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$.

(3)

(b) Using the trapezium rule and all the values calculated in part (a), find an approximation for the area of R.

(4)

(c) State, with a reason, whether your approximation underestimates or overestimates the area of R.

(2)

7. The functions f and g are defined by

$$\mathrm{f}: x \mapsto |x - a| + a, \quad x \in \mathbb{R},$$

$$\mathrm{g}: x \mapsto 4x + a, \qquad x \in \mathbb{R}.$$

where a is a positive constant.

(*a*) On the same diagram, sketch the graphs of f and g, showing clearly the coordinates of any points at which your graphs meet the axes.

(5)

(*b*) Use algebra to find, in terms of a, the coordinates of the point at which the graphs of f and g intersect.

(3)

(*c*) Find an expression for $\mathrm{fg}(x)$.

(2)

(*d*) Solve, for x in terms of a, the equation

$$\mathrm{fg}(x) = 3a.$$

(3)

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8. The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3 \ln x + \frac{1}{x}, \quad x > 0.$$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.

(4)

(b) Show that the y-coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found.

(2)

The point Q on C has x-coordinate 1.

(c) Find an equation for the normal to C at Q.

(4)

The normal to C at Q meets C again at the point R.

(d) Show that the x-coordinate of R

(i) satisfies the equation $6 \ln x + x + \frac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14.

(4)

END