Paper Reference(s)

6687

Edexcel GCE

Statistics S5

Advanced/Advanced Subsidiary

Tuesday 17 June 2003 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Statistics S5), the paper reference (6687), your surname, other name and signature.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. There are three strains of influenza A, B and C. Of those people infected with influenza, the probability of being infected with each of the strains is 0.4, 0.1 and 0.5 respectively. The probability of a full recovery within one week if infected with strain A is 0.8 and the corresponding probabilities for strains B and C are 0.6 and 0.9 respectively. Susan has influenza.

Given that she makes a full recovery within one week, find the probability that she was infected with strain A.

(4)

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2. The probability of Richard winning a coconut in a game at the fair is 0.12.

Richard plays a number of games.

(*a*) Find

(i) the probability of Richard winning his second coconut on his 8th game,

(2)

(ii) the expected number of games Richard will need to play in order to win 3 coconuts.

(1)

(*b*) State two assumptions that you have made in part (*a*).

(2)

Mary plays the same game, but has a different probability of winning a coconut. She plays until she has won r coconuts. The random variable G represents the total number of games Mary plays.

(*c*) Given that the mean and standard deviation of G are 18 and 6 respectively, determine whether Richard or Mary has the greater probability of winning a coconut in a game.

(5)

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3. An unbiased 6-sided die is thrown repeatedly. The first 6 appears on the *R*th throw.

(*a*) Calculate the probability that

(i) $R = 3$,

(ii) the value of R is at least 4.

(5)

(*b*) Show, from first principles, that the probability generating function for R is given by

$$G_R(t) = \frac{1}{6 - 5t}.$$

(4)

The third 6 appears on the *S*th throw.

(*c*) Write down the probability generating function for the random variable S.

(1)

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4. The random variable P has moment generating function given by

$$M_P(t) = 1 + \frac{t}{2} + \frac{t^2}{4} + \frac{t^3}{8} + \ldots$$

(*a*) Write down the value of $E(P)$.

(1)

(*b*) Calculate $\text{Var}(P)$.

(3)

(*c*) Find $E(P^3)$.

(2)

The random variable $Y = 4P - 1$.

(*d*) Find $M_Y(t)$ in the form $a + bt + ct^2 + \ldots$ where a, b and c are constants.

(5)

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5. The lifetime Y, in hours, of a certain type of light bulb has an exponential distribution with parameter λ ($\lambda > 0$).

(*a*) Write down the probability density function of Y.

(2)

Given that the mean lifetime of these light bulbs is 1500 hours,

(*b*) write down the value of λ,

(1)

(*c*) calculate the probability of a light bulb having a lifetime of less than 200 hours.

(3)

The manufacturer aims to lengthen the lifetime of the light bulbs. It is required that on average no more than 1 in 20 light bulbs fail to work after 200 hours.

(*d*) Find, to the nearest hour, the lowest mean lifetime needed to meet this requirement.

(5)

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6. The probability generating function of the random variable X is given by

$$\mathrm{G}_X(t) = k(1 + t + 3t^2)^2.$$

(*a*) Show that $k = \dfrac{1}{25}$.

(2)

(*b*) Find $\mathrm{P}(X = 2)$.

(2)

(*c*) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(8)

(*d*) Write down the probability generating function of $2X + 1$.

(2)

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7. Two sampling plans, *A* and *B*, are proposed for deciding whether or not to accept a batch of components from a factory.

Plan *A*: A single random sample of 20 components is taken and tested. The batch is accepted if less than 2 components are defective.

Plan *B*: A random sample of 10 components is taken and the batch is accepted if none of the components are defective and rejected if 2 or more are defective. If 1 defective component is found a second random sample of 10 components is taken. The batch is accepted if no more than 1 component is found to be defective in this second sample.

The table shows the probability of acceptance associated with each plan for different proportions of defective components.

	Proportion defective				
Plan	0.02	0.04	0.06	0.1	0.2
A	0.94	0.81	x	0.39	0.07
B	0.98	y	0.84	0.63	0.21

(*a*) Find x and y.
(7)

(*b*) Using the same axes, plot the operating characteristic for each plan.
(4)

(*c*) For Plan *A*, estimate the proportion of defective components in a batch that has a 90% chance of being rejected.
(2)

(*d*) State which plan you would recommend to be used.
(2)

END