Paper Reference(s)

6673

Edexcel GCE

Pure Mathematics P3
Advanced/Advanced Subsidiary

Tuesday 3 June 2003 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may only use one of the basic scientific calculators approved by the Qualifications and Curriculum Authority.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P3), the paper reference (6673), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. The curve C is described by the parametric equations

$$x = 3\cos t, \quad y = \cos 2t, \quad 0 \leq t \leq \pi.$$

(a) Find a cartesian equation of the curve C.

(2)

(b) Draw a sketch of the curve C.

(2)

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2. $$\mathrm{f}(x) = px^3 + 6x^2 + 12x + q.$$

Given that the remainder when $\mathrm{f}(x)$ is divided by $(x - 1)$ is equal to the remainder when $\mathrm{f}(x)$ is divided by $(2x + 1)$,

(a) find the value of p.

(4)

Given also that $q = 3$, and p has the value found in part (a),

(b) find the value of the remainder.

(1)

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3. A circle C has equation

$$x^2 + y^2 - 6x + 8y - 75 = 0.$$

(a) Write down the coordinates of the centre of C, and calculate the radius of C.

(3)

A second circle has centre at the point (15, 12) and radius 10.

(b) Sketch both circles on a single diagram and find the coordinates of the point where they touch.

(4)

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4. A curve has equation $7x^2 + 48xy - 7y^2 + 75 = 0$.

A and B are two distinct points on the curve. At each of these points the gradient of the curve is equal to $\frac{2}{11}$.

(a) Use implicit differentiation to show that $x + 2y = 0$ at the points A and B.

(5)

(b) Find the coordinates of the points A and B.

(4)

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5. **Figure 1**

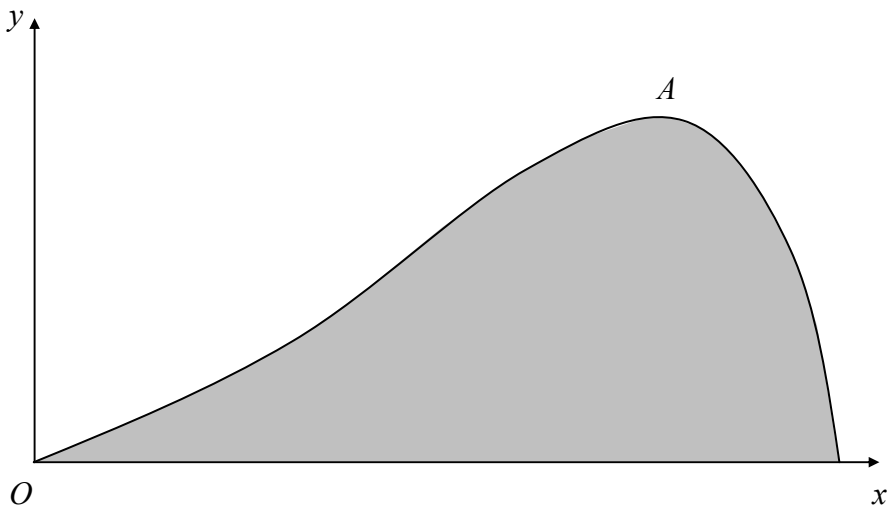

Figure 1 shows a graph of $y = x\sqrt{\sin x}$, $0 < x < \pi$. The maximum point on the curve is A.

(a) Show that the x-coordinate of the point A satisfies the equation $2 \tan x + x = 0$.

(4)

The finite region enclosed by the curve and the x-axis is shaded as shown in Fig. 1.

A solid body S is generated by rotating this region through 2π radians about the x-axis.

(b) Find the exact value of the volume of S.

(7)

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6. Relative to a fixed origin O, the point A has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, the point B has position vector $5\mathbf{i} + \mathbf{j} + \mathbf{k}$, and the point C has position vector $7\mathbf{i} - \mathbf{j}$.

(a) Find the cosine of angle ABC.

(4)

(b) Find the exact value of the area of triangle ABC.

(3)

The point D has position vector $7\mathbf{i} + 3\mathbf{k}$.

(c) Show that AC is perpendicular to CD.

(2)

(d) Find the ratio $AD:DB$.

(2)

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7. Fluid flows out of a cylindrical tank with constant cross section. At time t minutes, $t \geq 0$, the volume of fluid remaining in the tank is V m^3. The rate at which the fluid flows, in m^3 min^{-1}, is proportional to the square root of V.

(a) Show that the depth h metres of fluid in the tank satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}, \qquad \text{where } k \text{ is a positive constant.}$$

(3)

(b) Show that the general solution of the differential equation may be written as

$$h = (A - Bt)^2, \qquad \text{where } A \text{ and } B \text{ are constants.}$$

(4)

Given that at time $t = 0$ the depth of fluid in the tank is 1 m, and that 5 minutes later the depth of fluid has reduced to 0.5 m,

(c) find the time, T minutes, which it takes for the tank to empty.

(3)

(d) Find the depth of water in the tank at time $0.5T$ minutes.

(2)

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8. $$f(x) = \frac{25}{(3+2x)^2(1-x)}, \quad |x| < 1.$$

(*a*) Express f(x) as a sum of partial fractions.

(4)

(*b*) Hence find $\int f(x)\, dx$.

(5)

(*c*) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^2. Give each coefficient as a simplified fraction.

(7)

END