Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Thursday 19 June 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. *In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular unit vectors in a horizontal plane and* $\mathbf{k}$ *is a unit vector vertically upwards.*

A small smooth ring of mass 0.1 kg is threaded onto a smooth horizontal wire which is parallel to $(\mathbf{i} + 2\mathbf{j})$. The only forces acting on the ring are its weight, the normal reaction from the wire and a constant force $(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ N. The ring starts from rest at the point A on the wire, whose position vector relative to a fixed origin is $(2\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ m, and passes through the point B with speed 5 m s^{-1}. Find the position vector of B.

(6)

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2. With respect to a fixed origin O, the position vector, $\mathbf{r}$ metres, of a particle P at time t seconds satisfies

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{r} = (\mathbf{i} - \mathbf{j})\mathrm{e}^{-2t}.$$

Given that P is at O when $t = 0$, find

(*a*) $\mathbf{r}$ in terms of t,

(7)

(b) a cartesian equation of the path of P.

(2)

3.

Figure 1

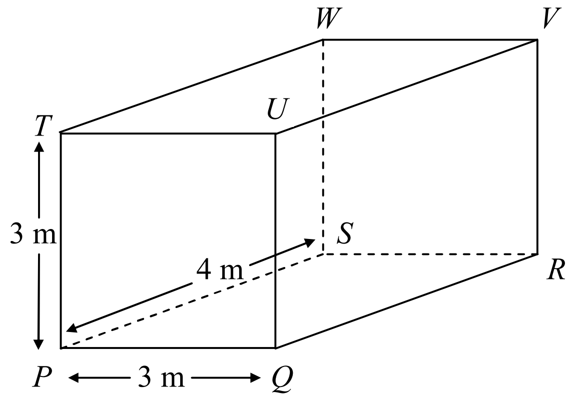

Figure 1 shows a box in the shape of a cuboid *PQRSTUVW* where $\overrightarrow{PQ} = 3\mathbf{i}$ metres, $\overrightarrow{PS} = 4\mathbf{j}$ metres and $\overrightarrow{PT} = 3\mathbf{k}$ metres. A force $(4\mathbf{i} - 2\mathbf{j})$ N acts at Q, a force $(4\mathbf{i} + 2\mathbf{j})$ N acts at R, a force $(-2\mathbf{j} + \mathbf{k})$ N acts at T, and a force $(2\mathbf{j} + \mathbf{k})$ N acts at W. Given that these are the only forces acting on the box, find

(*a*) the resultant force acting on the box,

(2)

(*b*) the resultant vector moment about P of the four forces acting on the box.

(5)

When an additional force $\mathbf{F}$ acts on the box at a point X on the edge PS, the box is in equilibrium.

(*c*) Find $\mathbf{F}$.

(1)

(*d*) Find the length of PX.

(5)

4. A rocket-driven car propels itself forwards in a straight line on a horizontal track by ejecting burnt fuel backwards at a constant rate λ kg s^{-1} and at a constant speed U m s^{-1} relative to the car. At time t seconds, the speed of the car is v m s^{-1} and the total resistance to the motion of the car has magnitude kv N, where k is a positive constant. When $t = 0$ the total mass of the car, including fuel, is M kg. Assuming that at time t seconds some fuel remains in the car,

(a) show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\lambda U - kv}{M - \lambda t},$$

(7)

(b) find the speed of the car at time t seconds, given that it starts from rest when $t = 0$ and that $\lambda = k = 10$.

(6)

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5. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. The rod is hanging in equilibrium with B below A when it is hit by a particle of mass m moving horizontally with speed v in a vertical plane perpendicular to the axis. The particle strikes the rod at B and immediately adheres to it.

(a) Show that the angular speed of the rod immediately after the impact is $\dfrac{3v}{8a}$.

(5)

Given that the rod rotates through 120° before first coming to instantaneous rest,

(b) find v in terms of a and g.

(6)

(c) find, in terms of m and g, the magnitude of the vertical component of the force acting on the rod at A immediately after the impact.

(5)

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6. (*a*) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a, about an axis through its centre O perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(4)

The line AB is a diameter of the disc and P is the mid-point of OA. The disc is free to rotate about a fixed smooth horizontal axis L. The axis lies in the plane of the disc, passes through P and is perpendicular to OA. A particle of mass m is attached to the disc at A and a particle of mass $2m$ is attached to the disc at B.

(*b*) Show that the moment of inertia of the loaded disc about L is $\frac{21}{4}ma^2$.

(6)

At time $t = 0$, PB makes a small angle with the downward vertical through P and the loaded disc is released from rest. By obtaining an equation of motion for the disc and using a suitable approximation,

(*c*) find the time when the loaded disc first comes to instantaneous rest.

(8)

END