# Advanced/Advanced Subsidiary 

Thursday 22 May 2003 - Afternoon
Time: 1 hour 30 minutes

Materials required for examination<br>Graph paper (ASG2)

Items included with question papers
D2 Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has six questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

## Write your answers in the D2 answer booklet for this paper.

1. A two person zero-sum game is represented by the following pay-off matrix for player $A$.

|  | $B$ plays I | $B$ plays II | $B$ plays III |
| :---: | :---: | :---: | :---: |
| $A$ plays I | -3 | 2 | 5 |
| $A$ plays II | 4 | -1 | -4 |

(a) Write down the pay off matrix for player $B$.
(b) Formulate the game as a linear programming problem for player $B$, writing the constraints as equalities and stating your variables clearly.
2. (a) Explain the difference between the classical and practical travelling salesman problems.

## Figure 1



The network in Fig. 1 shows the distances, in kilometres, between eight McBurger restaurants. An inspector from head office wishes to visit each restaurant. His route should start and finish at $A$, visit each restaurant at least once and cover a minimum distance.
(b) Obtain a minimum spanning tree for the network using Kruskal's algorithm. You should draw your tree and state the order in which the arcs were added.
(c) Use your answer to part (b) to determine an initial upper bound for the length of the route.
(2)
(d) Starting from your initial upper bound and using an appropriate method, find an upper bound which is less than 135 km . State your tour.
3. Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at $10 \mathrm{a} . \mathrm{m}$. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below.

|  | Talk I | Talk II | Talk III | Talk IV |
| :--- | :---: | :---: | :---: | :---: |
| Clive | 12 | 34 | 28 | 16 |
| Julie | 13 | 32 | 36 | 12 |
| Nicky | 15 | 32 | 32 | 14 |
| Steve | 11 | 33 | 36 | 10 |

(a) Use the Hungarian algorithm to find the earliest time that the meeting could end. You must make your method clear and show
(i) the state of the table after each stage in the algorithm,
(ii) the final allocation.
(b) Modify the table so it could be used to find the latest time that the meeting could end. (You do not have to find this latest time.)
4. A two person zero-sum game is represented by the following pay-off matrix for player $A$.

|  | $B$ plays I | $B$ plays II | $B$ plays III |
| :---: | :---: | :---: | :---: |
| $A$ plays I | 2 | -1 | 3 |
| $A$ plays II | 1 | 3 | 0 |
| $A$ plays III | 0 | 1 | -3 |

(a) Identify the play safe strategies for each player.
(b) Verify that there is no stable solution to this game.
(c) Explain why the pay-off matrix above may be reduced to

|  | $B$ plays I | $B$ plays II | $B$ plays III |
| :---: | :---: | :---: | :---: |
| $A$ plays I | 2 | -1 | 3 |
| $A$ plays II | 1 | 3 | 0 |

(d) Find the best strategy for player $A$, and the value of the game.
5. The manager of a car hire firm has to arrange to move cars from three garages $A, B$ and $C$ to three airports $D, E$ and $F$ so that customers can collect them. The table below shows the transportation cost of moving one car from each garage to each airport. It also shows the number of cars available in each garage and the number of cars required at each airport. The total number of cars available is equal to the total number required.

|  | Airport $D$ | Airport $E$ | Airport $F$ | Cars available |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Garage $A$ | $£ 20$ | $£ 40$ | $£ 10$ | 6 |  |
| Garage $B$ | $£ 20$ | $£ 30$ | $£ 40$ | 5 |  |
| Garage $C$ | $£ 10$ | $£ 20$ | $£ 30$ | 8 |  |
| Cars required | 6 | 9 | 4 |  |  |

(a) Use the North-West corner rule to obtain a possible pattern of distribution and find its cost.
(b) Calculate shadow costs for this pattern and hence obtain improvement indices for each route.
(c) Use the stepping-stone method to obtain an optimal solution and state its cost.
6. Kris produces custom made racing cycles. She can produce up to four cycles each month, but if she wishes to produce more than three in any one month she has to hire additional help at a cost of $£ 350$ for that month. In any month when cycles are produced, the overhead costs are $£ 200$. A maximum of 3 cycles can be held in stock in any one month, at a cost of $£ 40$ per cycle per month. Cycles must be delivered at the end of the month. The order book for cycles is

| Month | August | September | October | November |
| :---: | :---: | :---: | :---: | :---: |
| Number of cycles required | 3 | 3 | 5 | 2 |

Disregarding the cost of parts and Kris' time,
(a) determine the total cost of storing 2 cycles and producing 4 cycles in a given month, making your calculations clear.

There is no stock at the beginning of August and Kris plans to have no stock after the November delivery.
(b) Use dynamic programming to determine the production schedule which minimises the costs, showing your working in the table provided in the answer booklet.

The fixed cost of parts is $£ 600$ per cycle and of Kris’ time is $£ 500$ per month. She sells the cycles for $£ 2000$ each.
(c) Determine her total profit for the four month period.

## END

