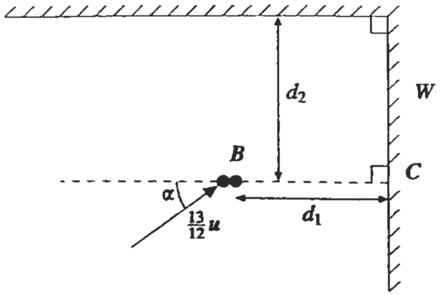
6.

Figure 2



A small ball Q of mass $2m$ is at rest at the point B on a smooth horizontal plane. A second small ball P of mass m is moving on the plane with speed $\frac{13}{12}u$ and collides with Q. Both the balls are smooth, uniform and of the same radius. The point C is on a smooth vertical wall W which is at a distance d_1 from B, and BC is perpendicular to W. A second smooth vertical wall is perpendicular to W and at a distance d_2 from B. Immediately before the collision occurs, the direction of motion of P makes an angle α with BC, as shown in Fig. 2, where $\tan \alpha = \frac{5}{12}$. The line of centres of P and Q is parallel to BC. After the collision Q moves towards C with speed $\frac{3}{5}u$.

(a) Show that, after the collision, the velocity components of P parallel and perpendicular to CB are $\frac{1}{5}u$ and $\frac{5}{12}u$ respectively. **(4)**

(b) Find the coefficient of restitution between P and Q. **(2)**

(c) Show that when Q reaches C, P is at a distance $\frac{4}{3}d_1$ from W. **(3)**

For each collision between a ball and a wall the coefficient of restitution is $\frac{1}{2}$.

Given that the balls collide with each other again,

(d) show that the time between the two collisions of the balls is $\frac{15d_1}{u}$, **(4)**

(e) find the ratio $d_1 : d_2$. **(5)**

END

M4 January 2003

1. A boy enters a large horizontal field and sees a friend 100 m due north. The friend is walking in an easterly direction at a constant speed of 0.75 m s^{-1}. The boy can walk at a maximum speed of 1 m s^{-1}.

Find the shortest time for the boy to intercept his friend and the bearing on which he must travel to achieve this. **(6)**

2. Boat A is sailing due east at a constant speed of 10 km h^{-1}. To an observer on A, the wind appears to be blowing from due south. A second boat B is sailing due north at a constant speed of 14 km h^{-1}. To an observer on B, the wind appears to be blowing from the south west. The velocity of the wind relative to the earth is constant and is the same for both boats.

Find the velocity of the wind relative to the earth, stating its magnitude and direction. **(7)**

3. A small pebble of mass m is placed in a viscous liquid and sinks vertically from rest through the liquid. When the speed of the pebble is v the magnitude of the resistance due to the liquid is modelled as mkv^2, where k is a positive constant.

Find the speed of the pebble after it has fallen a distance D through the liquid. **(11)**

4.

Figure 1

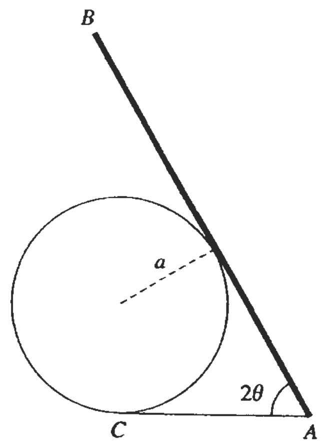


Figure 1 shows a uniform rod AB, of mass m and length $4a$, resting on a smooth fixed sphere of radius a. A light elastic string, of natural length a and modulus of elasticity $\frac{3}{4}mg$, has one end attached to the lowest point C of the sphere and the other end attached to A. The points A, B and C lie in a vertical plane with $\angle BAC = 2\theta$, where $\theta < \frac{\pi}{4}$. Given that AC is always horizontal,

(a) show that the potential energy of the system is

$$\frac{mga}{8}(16\sin 2\theta + 3\cot^2\theta - 6\cot\theta) + \text{constant},$$

(7)

(b) show that there is a value of θ for which the system is in equilibrium such that $0.535 < \theta < 0.545$.

(6)

(c) Determine whether this position of equilibrium is stable or unstable.

(3)

5. A particle P moves in a straight line. At time t seconds, its displacement from a fixed point O on the line is x metres. The motion of P is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 12\cos 2t - 6\sin 2t.$$

When $t = 0$, P is at rest at O.

(a) Find, in terms of t, the displacement of P from O.

(11)

(b) Show that P comes to instantaneous rest when $t = \frac{\pi}{4}$.

(2)

(c) Find, in metres to 3 significant figures, the displacement of P from O when $t = \frac{\pi}{4}$.

(2)

(d) Find the approximate period of the motion for large values of t.

(2)