Paper Reference(s)

6675

Edexcel GCE

Pure Mathematics P5

Advanced/Advanced Subsidiary

Tuesday 11 June 2002 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P5), the paper reference (6675), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. An ellipse has equation $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$.

(*a*) Sketch the ellipse.

(1)

(*b*) Find the value of the eccentricity e.

(2)

(*c*) State the coordinates of the foci of the ellipse.

(2)

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2. Find the exact value of the radius of curvature of the curve with equation $y = \arcsin x$ at the point where $x = \frac{1}{2}\sqrt{2}$.

(6)

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3. Solve the equation

$$10 \cosh x + 2 \sinh x = 11.$$

Give each answer in the form $\ln a$ where a is a rational number.

(7)

4.
$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx, \quad n \geq 0.$$

(*a*) Prove that $I_n = \left(\dfrac{\pi}{2}\right)^n - n(n-1)I_{n-2}, \; n \geq 2$.

(5)

(*b*) Find an exact expression for I_6.

(4)

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5. Given that the intrinsic equation of a curve C is

$$s = \ln\left(\tan\frac{\psi}{2}\right), \quad 0 < \psi < \pi,$$

(a) show that $\dfrac{ds}{d\psi} = \operatorname{cosec}\psi$.

(3)

Taking $x = 0$ and $y = 0$ at $\psi = \dfrac{\pi}{2}$,

(b) show that for all points on C, $y = \psi - \dfrac{\pi}{2}$,

(2)

(c) find an expression for x in terms of ψ.

(3)

(d) Hence write down a cartesian equation of C.

(1)

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6. (a) Given that $y = \arctan 3x$, and assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{3}{1+9x^2}.$$

(4)

(b) Show that

$$\int_0^{\frac{\sqrt{3}}{3}} 6x \arctan 3x \; dx = \tfrac{1}{9}(4\pi - 3\sqrt{3}).$$

(6)

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7. The point $P\left(2p, \frac{2}{p}\right)$ and the point $Q\left(2q, \frac{2}{q}\right)$, where $p \neq -q$, lie on the rectangular hyperbola with equation $xy = 4$.

The tangents to the curve at the points P and Q meet at the point R.

(a) Show that at the point R,

$$x = \frac{4pq}{p+q} \text{ and } y = \frac{4}{p+q}.$$

(8)

As p and q vary, the locus of R has equation $xy = 3$.

(b) Find the relationship between p and q in the form $q = \mathrm{f}(p)$.

(5)

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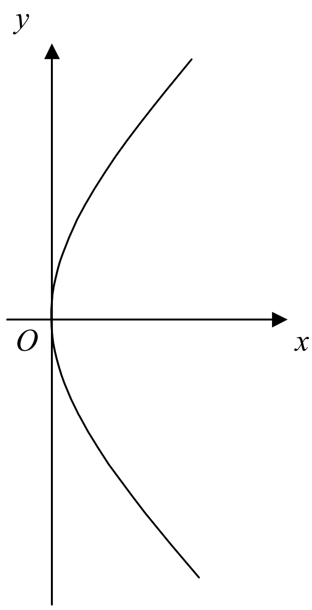
8. **Figure 1**



The curve C shown in Fig. 1 has equation $y^2 = 4x$, $0 \leq x \leq 1$.

The part of the curve in the first quadrant is rotated through 2π radians about the x-axis.

(*a*) Show that the surface area of the solid generated is given by

$$4\pi \int_0^1 \sqrt{(1+x)} \, \mathrm{d}x.$$

(4)

(*b*) Find the exact value of this surface area.

(3)

(*c*) Show also that the length of the curve C, between the points (1, –2) and (1, 2), is given by

$$2\int_0^1 \sqrt{\left(\frac{x+1}{x}\right)} \, \mathrm{d}x.$$

(3)

(*d*) Use the substitution $x = \sinh^2 \theta$ to show that the exact value of this length is

$$2[\sqrt{2} + \ln(1 + \sqrt{2})].$$

(6)

END