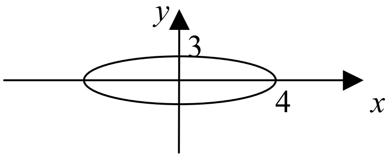
Question Number	Scheme	Marks
1. (*a*)	 Closed shape 3, 4	B1 **(1)**
(*b*)	$b^2 = a^2(1-e^2) \Rightarrow 9 = 16(1-e^2)$	M1
	$e = \frac{\sqrt{7}}{4}$ oe awrt 0.661	A1 **(2)**
(*c*)	Foci are at $(\pm ae, 0)$ use of ae	M1
	$(\sqrt{7}, 0)$ <u>and</u> $(-\sqrt{7}, 0)$ awrt 2.65, 0 is required, ft their e	A1 ft **(2)**
		(5 marks)
2.	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $(= \sqrt{2}$ at $x = ½\sqrt{2})$	B1
	$\frac{d^2y}{dx^2} = -½(1-x^2)^{-3/2}.2x$ $\left(= \frac{x}{(1-x^2)^{3/2}}\right) = 2$ at $x = ½\sqrt{2}$	B1
	Use of $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$	M1
	to obtain $\rho = \frac{\left(1 + \frac{1}{1-x^2}\right)^{3/2}}{x(1-x^2)^{3/2}}$ oe $\left(= \frac{(2-x^2)^{3/2}}{x}\right)$ may be unsimplified or implied correct numerical answer if first M1 clearly gained	A1
	At $x = ½\sqrt{2}$, $\rho = \frac{3}{2}\sqrt{3}$ accept $\frac{9}{2\sqrt{3}}$ or exact equivalents	M1, A1
		(6 marks)

(✪) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
3.	$10\left(\frac{e^x + e^{-x}}{2}\right) + 2\left(\frac{e^x - e^{-x}}{2}\right) = 11$	M1
	$6e^{2x} - 11x^x + 4 = 0$ quadratic in e^x	M1, A1
	$(2e^x - 1)(3e^x - 4) = 0$	M1
	$e^x = ½$ and $^4/_3$	A1
	$x = \ln ½$ and $\ln {}^4/_3$	M1, A1
		(7 marks)
Alt 3.	$10\cosh x + 2\sinh x \equiv R\cosh(x + \alpha)$	
	$R = \sqrt{96}$ <u>and</u> $\tan\alpha = \frac{1}{5}$	M1, A1
	$\cosh(x + \alpha) = \frac{11}{\sqrt{96}}$	
	$x + \alpha = \ln\left[\frac{11}{\sqrt{96}} \pm \sqrt{\left(\frac{121}{96}\right) - 1}\right]$ either	A1
	$= \ln\frac{4}{\sqrt{6}}$ <u>and</u> $\ln\frac{\sqrt{6}}{4}$ both as single ln	A1
	$x = \ln\frac{4}{\sqrt{6}} - ½\ln\frac{3}{2},\ \ln\frac{\sqrt{6}}{4} - ½\ln\frac{3}{2}$	
	$= \ln\left(\frac{4}{\sqrt{6}} - \frac{\sqrt{2}}{\sqrt{3}}\right),\ \ln\left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{\sqrt{3}}\right)$ combine either into single ln. Dependent on first two Ms	M1
	$x = \ln ½$ and $\ln {}^4/_3$	A1, A1
	[One answer by alt. method gains max. M1A1M1M0M1A1A0]	
		(11 marks)

(✪) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
4. (*a*)	$\int x^n \cos x \,dx = x^n \sin x - n\int x^{n-1} \sin x \,dx$	M1, A1
	$= \ldots\ldots \quad -[nx^{n-1}\cos x + \int n(n-1)x^{n-2}\cos x\,dx]$	M1
	Using limits $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$ (✱) cso	M1, A1 **(5)**
(*b*)	$I_0 = \int_0^{\frac{\pi}{2}} \cos x\,dx = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$ at any stage	B1
	$I_6 = \left(\frac{\pi}{2}\right)^6 - 30I_4$	M1
	$= \left(\frac{\pi}{2}\right)^6 - 30\left(\left(\frac{\pi}{2}\right)^6 - 12I_2\right)$	
	$= \left(\frac{\pi}{2}\right)^6 - 30\left(\frac{\pi}{2}\right)^4 + 360\left(\frac{\pi}{2}\right)^2 - 720I_0$	M1
	Hence $I_6 = \left(\frac{\pi}{2}\right)^6 - 30\left(\frac{\pi}{2}\right)^4 + 360\left(\frac{\pi}{2}\right)^2 - 720$ cao	A1 **(4)**
		(9 marks)

(✱) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
5. (*a*)	$\frac{dy}{d\psi} = \frac{\frac{1}{2}\sec^2\frac{\psi}{2}}{\tan\frac{\psi}{2}}$	M1
	$= \frac{1}{2\sin\frac{\psi}{2}\cos\frac{\psi}{2}}$	M1
	$= \frac{1}{\sin\psi} = \operatorname{cosec}\psi$ (✪)	A1 **(3)**
(*b*)	$\frac{dy}{d\psi} = \sin\psi$; $\frac{dy}{d\psi} = \frac{dy}{ds}.\frac{ds}{d\psi} = \sin\psi\operatorname{cosec}\psi = 1$	
	$y = \psi$ $(+c)$	M1
	Using $y = 0, \psi = \frac{\pi}{2}$ to obtain $y = \psi - \frac{\pi}{2}$ (✪) cso	A1 **(2)**
(*c*)	$\frac{dx}{ds} = \cos\psi$; $\frac{dx}{d\psi} = \frac{dx}{ds}.\frac{ds}{d\psi} = \cos\psi\operatorname{cosec}\psi = \frac{\cos\psi}{\sin\psi}$	M1
	$x = \ln\sin\psi$ $(+c)$	A1
	Using $x = 0$, $\psi = \frac{\pi}{2}$ to obtain $x = \ln\sin\psi$	A1 **(3)**
(*d*)	$x = \ln\sin\left(y + \frac{\pi}{2}\right)$ (or $\cos y = e^x$ or any equivalent)	A1
	$A = {}^4/_5, B = -{}^4/_5$	A1
	The A1 in (d) depends of the M1 in (c)	
		(14 marks)

(✪) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
6. (*a*)	$y = \arctan 3x \quad \Rightarrow \quad \tan y = 3x$	M1
	$\sec^2 y \dfrac{dy}{dx} = 3$	A1
	$\dfrac{dy}{dx} = \dfrac{3}{1+\tan^2 y} = \dfrac{3}{1+9x^2}$ (🍽)	M1, A1 **(4)**
(*b*)	$\int 6x \arctan 3x \, dx = 3x^2 \arctan 3x - \int \dfrac{9x^2}{1+9x^2} dx$	M1, A1
	$= \ldots\ldots\ldots \quad - \int \dfrac{1+9x^2-1}{1+9x^2} dx$	M1
	$= \ldots\ldots \quad - x + \dfrac{1}{3}\arctan 3x$	A1
	$\Big[\quad \Big]_0^{\frac{\sqrt{3}}{3}} = \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{3} + \dfrac{\pi}{9}$	M1
	$= \dfrac{1}{9}(4\pi - 3\sqrt{3})$ (🍽) cso	A1 **(6)**
		(10 marks)

(🍽) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
7. (*a*)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{x^2}$; at $x = 2p$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{p^2}$	M1, A1
	Equation of tangent at P, $y - \frac{2}{p} = -\frac{1}{p^2}(x - 2p)$	M1
	$(y = -\frac{1}{p^2}x + \frac{4}{p}$, $p^2y + x = 4p$ etc)	
	At Q $q^2y + x + 4q$ Two correct equations in any form	A1
	$(p^2 - q^2)y = 4(p - q)$	M1
	$y = \frac{4}{p+q}$ (🍽)	A1
	$x = 4p - \frac{4p^2}{p+q} = \frac{4pq}{p+q}$ (🍽)	M1, A1 **(8)**
(*b*)	$\frac{4pq}{p+q} \times \frac{4}{p+q} = 3$	M1
	$3p^2 - 10pq + 3q^2 = 0$	A1
	$(3p - q)(p - 3q) = 0$	M1
	$q = 3p$, $q = \frac{1}{3}p$	A1, A1 **(5)**
		(13 marks)

(🍽) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
8. (*a*)	$y = 2x^{½}$, $\dfrac{dy}{dx} = x^{-½}$	M1, A1
	$\int_{\dots}^{\dots} 2\pi y\left[1 + \left(\dfrac{d}{dx}\right)^2\right] dx = 4\pi \int_{\dots}^{\dots} x^{½}\left[1 + \dfrac{1}{x}\right]^{½} dx$	M1
	$= 4\pi \int_0^1 \sqrt{1+x}\, dx$ (*)	A1 **(4)**
(*b*)	$S = 4\pi \int_{\dots}^{\dots} \sqrt{1+x}\, dx = \left[4\pi \dfrac{2}{3}(1+x)^{3/2}\right]_{(0)}^{(1)}$	M1, A1
	$= \dfrac{8\pi}{3}\left(2^{3/2} - 1\right)$ or any exact equivalent	A1 **(3)**
(*c*)	$\int \left(1 + \left(\dfrac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx = \int \left(1 + \dfrac{1}{x}\right)^{\frac{1}{2}} dx$	M1
	$\int \sqrt{\dfrac{x+1}{x}}\, dx$	A1
	Using symmetry, $s = 2\int_0^1 \sqrt{\dfrac{x+1}{x}}\, dx$ (*)	A1 **(3)**
(*d*)	$x = \sinh^2\theta$, $\dfrac{dx}{d\theta} = 2\sinh\theta\cosh\theta$ oe	B1
	$I = 2\int \sqrt{\dfrac{1+\sinh^2\theta}{\sinh^2\theta}}.2\sinh\theta\cosh\theta\ d\theta$	M1
	$= 4\int \cosh^2\theta\ d\theta$	A1
	$= 2\int (1 + \cosh 2\theta)\ d\theta$	M1
	$= 2\theta + \sinh 2\theta$	A1
	Limits are 0 and arsinh1 $(= \ln(1+\sqrt{2}))$	
	$s = \left[2\theta + 2\sinh\theta\sqrt{1+\sinh^{2\theta}}\right]_0^{\text{arsinh}1}$	
	$= 2\,\text{arsinh}\,1 + 2\sqrt{1+1^2}$	
	$= 2\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$ (*)	M1, A1 **(6)**
		(16 marks)

(*) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark
oe = or equivalent; awrt = answers which round to; cao = correct answer only

Question Number	Scheme	Marks
8. (d) Alt	The last four marks can be gained:	
	$I = 4\int\left(\frac{e^{\theta} + e^{-\theta}}{2}\right)^2 d\theta = \int(e^{2\theta} + 2 + e^{-2\theta})d\theta$	M1
	$= \frac{e^{2\theta}}{2} + 2\theta - \frac{e^{-2\theta}}{2}$	A1
	$s = 2\,\text{arsinh}\,1 + \frac{1}{2}\left[(1+\sqrt{2})^2 - \frac{1}{(1+\sqrt{2})^2}\right]$	
	$= .... \quad + \frac{1}{2}\left[1 + 2 + 2\sqrt{2} - \frac{1}{3+2\sqrt{2}}.\frac{3-2\sqrt{2}}{3-2\sqrt{2}}\right]$	
	$= 2\ln(1+\sqrt{2}) + ½(3 + 2\sqrt{2} - 3 + 2\sqrt{2}) = 2[\sqrt{2} + \ln(1+\sqrt{2})]$ (✪)	M1, A1
8. (d) Alt	The last two marks may be gained by substituting back to the variable x	
	$s = [2\theta + \sinh 2\theta]_{...}^{...} = [2\theta + 2\sinh\theta\cosh\theta]_{...}^{...}$	
	$= \left[2\text{arsinh}\sqrt{x} + 2\sqrt{x}\sqrt{1+x}\right]_0^1$	
	$= 2\text{arsinh}1 + 2\sqrt{2} = 2\ln(1+\sqrt{2}) = 2\sqrt{2}$	
	$= 2\left[\sqrt{2} + \ln(1+\sqrt{2}\right]$ (✪)	M1, A1