Paper Reference(s)

6690

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Wednesday 26 June 2002 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Page 8 is blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. **Figure 1**

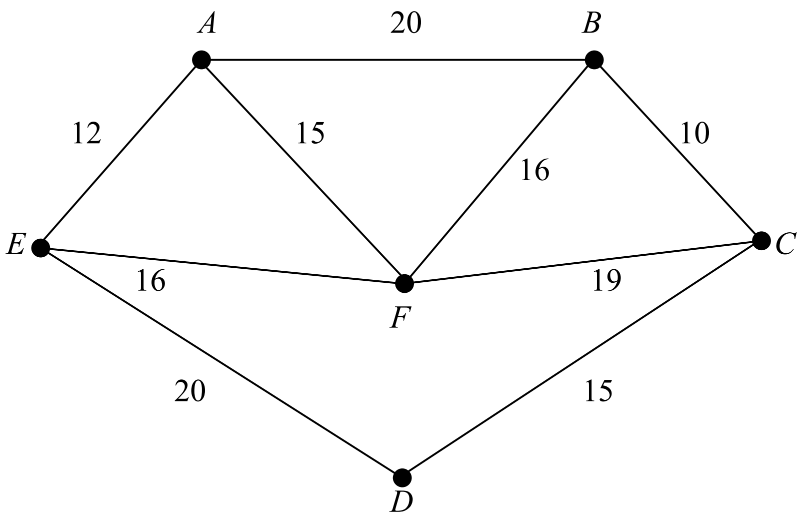


Figure 1 shows a network of roads connecting six villages *A*, *B*, *C*, *D*, *E* and *F*. The lengths of the roads are given in km.

(*a*) Complete the table in the answer booklet, in which the entries are the shortest distances between pairs of villages. You should do this by inspection.

(2)

The table can now be taken to represent a complete network.

(*b*) Use the nearest-neighbour algorithm, starting at *A*, on your completed table in part (*a*). Obtain an upper bound to the length of a tour in this complete network, which starts and finishes at *A* and visits every village exactly once.

(3)

(*c*) Interpret your answer in part (*b*) in terms of the original network of roads connecting the six villages.

(1)

(*d*) By choosing a different vertex as your starting point, use the nearest-neighbour algorithm to obtain a shorter tour than that found in part (*b*). State the tour and its length.

(2)

2. A two-person zero-sum game is represented by the following pay-off matrix for player A.

B

		I	II	III	IV
	I	–4	–5	–2	4
	II	–1	1	–1	2
A	III	0	5	–2	–4
	IV	–1	3	–1	1

(*a*) Determine the play-safe strategy for each player.

(4)

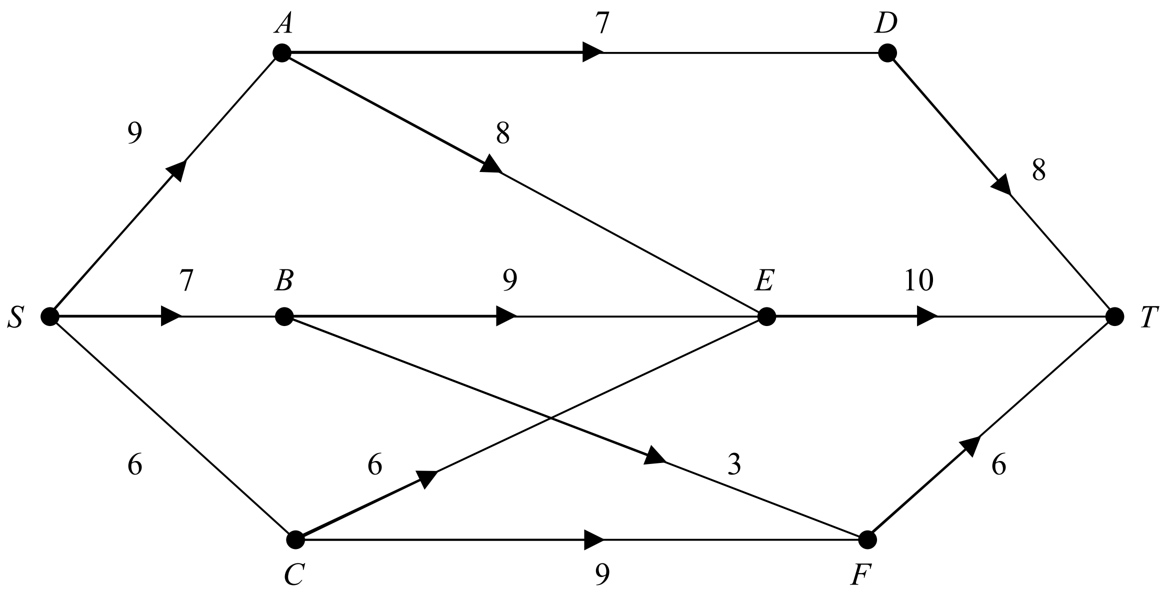
(*b*) Verify that there is a stable solution and determine the saddle points.

(3)

(*c*) State the value of the game to *B*.

(1)

3. **Figure 2**



The network in Fig. 2 shows possible routes that an aircraft can take from S to T. The numbers on the directed arcs give the amount of fuel used on that part of the route, in appropriate units. The airline wishes to choose the route for which the maximum amount of fuel used on any part of the route is as small as possible. This is the rninimax route.

(*a*) Complete the table in the answer booklet.

(8)

(*b*) Hence obtain the minimax route from S to T and state the maximum amount of fuel used on any part of this route.

(2)

4. Andrew (A) and Barbara (B) play a zero-sum game. This game is represented by the following pay-off matrix for Andrew.

$$\begin{array}{cc} & B \\ A & \begin{pmatrix} 3 & 5 & 4 \\ 1 & 4 & 2 \\ 6 & 3 & 7 \end{pmatrix} \end{array}$$

(a) Explain why this matrix may be reduced to

$$\begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}.$$

(b) Hence find the best strategy for each player and the value of the game.

(8)

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5. An engineering company has 4 machines available and 4 jobs to be completed. Each machine is to be assigned to one job. The time, in hours, required by each machine to complete each job is shown in the table below.

	Job 1	Job 2	Job 3	Job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Use the Hungarian algorithm, *reducing rows first*, to obtain the allocation of machines to jobs which minimises the total time required. State this minimum time.

(11)

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6. The table below shows the distances, in km, between six towns A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	85	110	175	108	100
B	85	–	38	175	160	93
C	110	38	–	148	156	73
D	175	175	148	–	110	84
E	108	160	156	110	–	92
F	100	93	73	84	92	–

(a) Starting from A, use Prim's algorithm to find a minimum connector and draw the minimum spanning tree. You must make your method clear by stating the order in which the arcs are selected.

(4)

(b) (i) Using your answer to part (a) obtain an initial upper hound for the solution of the travelling salesman problem.

(ii) Use a short cut to reduce the upper bound to a value less than 680.

(4)

(c) Starting by deleting F, find a lower bound for the solution of the travelling salesman problem.

(4)

7. A steel manufacturer has 3 factories F_1, F_2 and F_3 which can produce 35, 25 and 15 kilotonnes of steel per year, respectively. Three businesses B_1, B_2 and B_3 have annual requirements of 20, 25 and 30 kilotonnes respectively. The table below shows the cost C_{ij} in appropriate units, of transporting one kilotonne of steel from factory F_i to business B_j.

		Business		
		B_1	B_2	B_3
Factory	F_1	10	4	11
	F_2	12	5	8
	F_3	9	6	7

The manufacturer wishes to transport the steel to the businesses at minimum total cost.

(*a*) Write down the transportation pattern obtained by using the North-West corner rule.

(2)

(*b*) Calculate all of the improvement indices I_{ij}, and hence show that this pattern is not optimal.

(5)

(*c*) Use the stepping-stone method to obtain an improved solution.

(3)

(*d*) Show that the transportation pattern obtained in part (*c*) is optimal and find its cost.

(5)

END