<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>
<table>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>A</td><td>0</td><td>20</td><td>30</td><td>32</td><td>12</td><td>15</td></tr>
<tr><td>B</td><td>20</td><td>0</td><td>10</td><td>25</td><td>32</td><td>16</td></tr>
<tr><td>C</td><td>30</td><td>10</td><td>0</td><td>15</td><td>35</td><td>19</td></tr>
<tr><td>D</td><td>32</td><td>25</td><td>15</td><td>0</td><td>20</td><td>34</td></tr>
<tr><td>E</td><td>12</td><td>32</td><td>35</td><td>20</td><td>0</td><td>16</td></tr>
<tr><td>F</td><td>15</td><td>16</td><td>19</td><td>34</td><td>16</td><td>0</td></tr>
</table>
</td><td>M1 A1 (2)</td></tr>
<tr><td>(b)</td><td>AE (12), EF (16), FB (16), BC (10), CD (15), DA (32), i.e. AEFBCDA
Upper bound = 101</td><td>M1 A1
A1 (3)</td></tr>
<tr><td>(c)</td><td>In the original network AD is not a direct path.
The tour becomes AEFBCDEA</td><td>B1 (1)</td></tr>
<tr><td>(d)</td><td>For example,
B C D E A F B
C D E A F B C
D C B F A E D length 98
E A F B C D E
F A E D C B F</td><td>M1 A1 (2)
(8 marks)</td></tr>
<tr><td>2. (a)</td><td>Row minima: –5, –1, –4, –1 max is –1
Column minima: 0, 5, –1, 4 min is –1
Play safe is A plays II or IV and B plays III</td><td>M1
A1
A1 A1 (4)</td></tr>
<tr><td>(b)</td><td>Since (–1) – (–1) = 0 there is a stable solution
Saddle point (II, III) and (IV, III)</td><td>B1
M1 A1 ft (3)</td></tr>
<tr><td>(c)</td><td>Value of game to B is –(–1) = 1</td><td>B1 (1)
(8 marks)</td></tr>
</table>

ft = follow-through mark

3. (*a*)

Stage	Initial state	Action	Destination	Value
1	*D*	*DT*	*T*	8 *
	E	*ET*	*T*	10 *
	F	*FT*	*T*	6 *
2	*A*	*AD*	*D*	max (7, 8) = 8 *
		AE	*E*	max (8, 10) = 10
	B	*BE*	*E*	max (9, 10) = 10
		BF	*F*	max (3, 6) = 6 *
	C	*CE*	*E*	max (6, 10) = 10
		CF	*F*	max (9, 6) = 9 *
3	*S*	*SA*	*A*	max (9, 8) = 9
		SB	*B*	max (7, 6) = 7 *
		SC	*C*	max (6, 9) = 9

M1 A1

M1 A1 ft

A1 ft

A1 ft

M1 A1 ft **(8)**

(*b*) Minimax route is *SBFT* — M1

Maximum amount of fuel used is 7 units — A1 **(2)**

(10 marks)

4. (*a*) Row 1 dominates row 3
Column 1 dominates column 3 — M1 A1

Thus row 3 and column 3 may be deleted — A1 **(3)**

(*b*) Let *A* play row 3 with probability p and hence row 3 with probability $(1 - p)$

If *B* plays 1, *A*'s expected gain is $3p + 6(1 - p) = 6 - 3p$
If *B* plays 2, *A*'s expected gain is $5p + 3(1 - p) = 2p + 3$ — M1 A1

Optimal when $6 - 3p = 2p + 3$

$5p = 3$

$p = \frac{3}{5}$ — A1

Hence *A* should play row 1 with probability $\frac{3}{5}$
and row 2 with probability $\frac{2}{5}$ — A1 ft **(4)**

Similarly, let *B* play column 1 with probability q

$3q + 5(1 - q) = 6q + 3(1 - q)$ — M1 A1

$5q = 2$

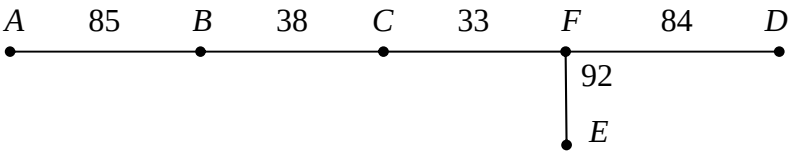
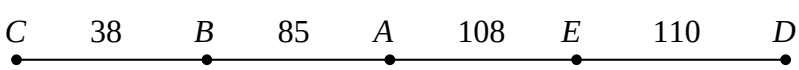
$q = \frac{2}{5}$ — A1 ft

So *B* should play column 1 with probability $\frac{2}{5}$
and column 2 with probability $\frac{3}{5}$

Value of game is $4\frac{1}{5}$ to *A* — A1 **(4)**

(11 marks)

ft = follow-through mark

Question Number	Scheme	Marks
5. (*a*)	Reducing rows $\begin{matrix}9&0&3&2\\0&10&4&3\\4&5&0&6\\0&2&4&8\end{matrix}$ reducing → columns $\begin{matrix}9&0&3&0\\0&10&4&1\\4&5&0&4\\0&2&4&6\end{matrix}$	M1 A1 A1 **(3)**
(*b*)	Testing for optimality – 3 lines are enough ⌐ or ⌐¬ Minimum uncovered element is 1 $\begin{matrix}10&0&3&0\\0&9&3&0\\5&5&0&4\\0&1&3&6\end{matrix}$ or $\begin{matrix}10&0&4&0\\0&9&3&0\\4&4&0&3\\0&1&4&5\end{matrix}$ 4 lines now needed	M1 A1 A1 M1 A1 **(5)**
(*c*)	Final matching Machine 1 – Job 2 (5) Machine 2 – Job 4 (5) Machine 3 – Job 3 (3) Machine 4 – Job 1 (2) Minimum time: 15 hours	M1 A1 A1 **(3)** **(11 marks)**
6. (*a*)	Order of arcs: *AB*, *BC*, *CF*, *FD*, *FG* 	M1 A1 A1 A1 **(6)**
(*b*) (i)	$2 \times 372 = 744$	M1 A1 **(2)**
(ii)	e.g. *DA* saves 105 giving 639 or *AE* saves 180 giving 564	M1 A1 **(2)**
(*c*)	Residual MST *AB*, *BC*, *AE*, *ED*  Lower bound = 341 + 73 + 84 = 498	M1 A1 M1 A1 **(4)** **(12 marks)**

Question Number	Scheme	Marks
7. (*a*)	<table><tr><th></th><th>B_1</th><th>B_2</th><th>B_3</th></tr><tr><td>F_1</td><td>20</td><td>15</td><td></td></tr><tr><td>F_2</td><td></td><td>10</td><td>15</td></tr><tr><td>F_3</td><td></td><td></td><td>15</td></tr></table>	M1 A1 **(2)**
(*b*)	$S(F_1) = 0$ $S(F_2) = 1$ $S(F_3) = 0$ $D(B_1) = 10$ $D(B_2) = 4$ $D(B_3) = 7$ $I_{13} = 11 - 0 - 7 = 4$ $I_{21} = 12 - 1 - 10 = 1$ $I_{31} = 9 - 0 - 10 = -1$ $I_{33} = 6 - 0 - 4 = 2$ Since I_{31} is negative, pattern is not optimal	M1 A1 M1 A1 A1 **(5)**
(*c*)	<table><tr><th></th><th>B_1</th><th>B_2</th><th>B_3</th></tr><tr><td>F_1</td><td>$20 - \theta$</td><td>$15 + \theta$</td><td></td></tr><tr><td>F_2</td><td></td><td>$10 - \theta$</td><td>$15 + \theta$</td></tr><tr><td>F_3</td><td>θ</td><td></td><td>$15 - \theta$</td></tr></table> Entering square F_3 B_1 Exiting square F_2 B_2 $\theta = 10$	M1 A1
	<table><tr><th></th><th>B_1</th><th>B_2</th><th>B_3</th></tr><tr><td>F_1</td><td>10</td><td>25</td><td></td></tr><tr><td>F_2</td><td></td><td></td><td>25</td></tr><tr><td>F_3</td><td>10</td><td></td><td>5</td></tr></table>	A1 **(3)**
(*d*)	$S(F_1) = 0$ $S(F_2) = 0$ $S(F_3) = -1$ $D(B_1) = 10$ $D(B_2) = 4$ $D(B_3) = 8$ $I_{13} = 11 - 0 - 8 = 3$ $I_{21} = 12 - 0 - 10 = 2$ $I_{31} = 5 - 0 - 4 = 1$ $I_{33} = 6 - (-1) - 4 = 3$ all positive $\therefore$ optimal Cost = $(10 \times 10) + (25 \times 4) + (25 \times 8) + (10 \times 9) + (5 \times 7) = 525$ units	M1 A1 A1 M1 A1 **(5)**
		(15 marks)