Paper Reference(s)

6680

Edexcel GCE

Mechanics M4
(New Syllabus)
Advanced/Advanced Subsidiary
Friday 25 January 2002 − Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions. Pages 6, 7 and 8 are blank.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A river of width 40 m flows with uniform and constant speed between straight banks. A swimmer crosses as quickly as possible and takes 30 s to reach the other side. She is carried 25 m downstream.

Find

(*a*) the speed of the river,

(2)

(*b*) the speed of the swimmer relative to the water.

(2)

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2. A ball of mass m is thrown vertically upwards from the ground with an initial speed u. When the speed of the ball is v, the magnitude of the air resistance is mkv, where k is a positive constant.

By modelling the ball as a particle, find, in terms of u, k and g, the time taken for the ball to reach its greatest height.

(8)

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3. A smooth uniform sphere P of mass m is falling vertically and strikes a fixed smooth inclined plane with speed u. The plane is inclined at an angle θ, $\theta < 45°$, to the horizontal. The coefficient of restitution between P and the inclined plane is e. Immediately after P strikes the plane, P moves horizontally.

(*a*) Show that $e = \tan^2 \theta$.

(6)

(*b*) Show that the magnitude of the impulse exerted by P on the plane is $mu \sec \theta$.

(4)

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4. A pilot flying an aircraft at a constant speed of 2000 kmh^{-1} detects an enemy aircraft 100 km away on a bearing of 045°. The enemy aircraft is flying at a constant velocity of 1500 kmh^{-1} due west. Find

(i) the course, as a bearing to the nearest degree, that the pilot should set up in order to intercept the enemy aircraft,

(ii) the time, to the nearest s, that the pilot will take to reach the enemy aircraft.

(11)

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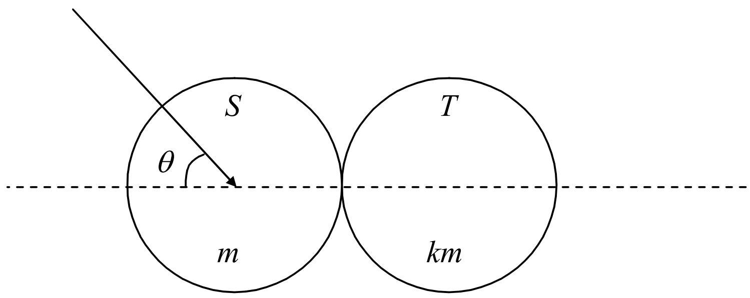
5. **Figure 1**



A smooth uniform sphere S of mass m is moving on a smooth horizontal table. The sphere S collides with another smooth uniform sphere T, of the same radius as S but of mass km, $k > 1$, which is at rest on the table. The coefficient of restitution between the spheres is e. Immediately before the spheres collide the direction of motion of S makes an angle θ with the line joing their centres, as shown in Fig. 1.

Immediately after the collision the directions of motion of S and T are perpendicular.

(*a*) Show that $e = \dfrac{1}{k}$.

(6)

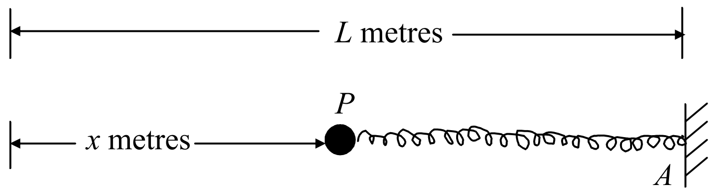
Given that $k = 2$ and that the kinetic energy lost in the collision is one quarter of the initial kinetic energy,

(*b*) find the value of θ.

(6)

TURN OVER FOR QUESTION 6

6. **Figure 2**



In a simple model of a shock absorber, a particle P of mass m kg is attached to one end of a light elastic horizontal spring. The other end of the spring is fixed at A and the motion of P takes place along a fixed horizontal line through A. The spring has natural length L metres and modulus of elasticity $2mL$ newtons. The whole system is immersed in a fluid which exerts a resistance on P of magnitude $3mv$ newtons, where v m s^{-1} is the speed of P at time t seconds. The compression of the spring at time t seconds is x metres, as shown in Fig. 2.

(*a*) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 0.$$

(4)

Given that when $t = 0$, $x = 2$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -4$,

(*b*) find x in terms of t.

(8)

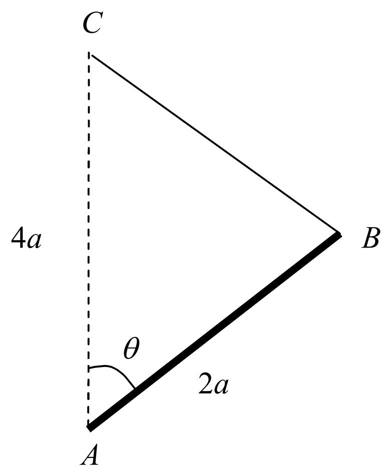
(*c*) Sketch the graph of x against t.

(2)

(*d*) State, with a reason, whether the model is realistic.

(1)

7. **Figure 3**



A uniform rod AB, of mass m and length $2a$, can rotate freely in a vertical plane about a fixed smooth horizontal axis through A. The fixed point C is vertically above A and $AC = 4a$. A light elastic string, of natural length $2a$ and modulus of elasticity $\frac{1}{2}mg$, joins B to C. The rod AB makes an angle θ with the upward vertical at A, as shown in Fig. 3.

(a) Show that the potential energy of the system is

$$-mga[\cos\theta + \sqrt{(5 - 4\cos\theta)}] + \text{constant}.$$

(9)

(b) Hence determine the values of θ for which the system is in equilibrium.

(6)

END