Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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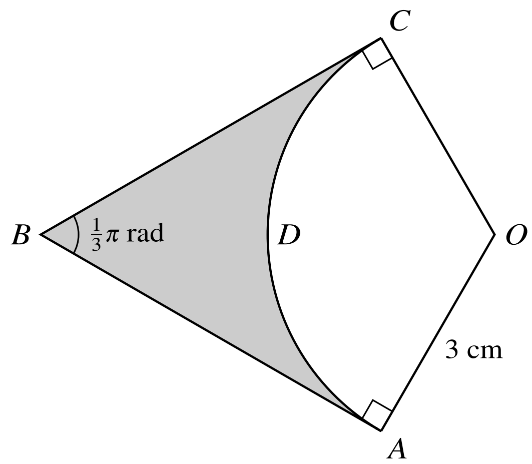
[Turn over

1 In the expansion of $(2 + ax)^6$, the coefficient of x^2 is equal to the coefficient of x^3. Find the value of the non-zero constant a. [4]

2



In the diagram, $OADC$ is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π,

(i) the perimeter of the shaded region, [3]

(ii) the area of the shaded region. [3]

3 **(i)** Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]

(ii) Determine whether $3x^3 - 6x^2 + 5x - 12$ is an increasing function, a decreasing function or neither. [3]

4 Three geometric progressions, P, Q and R, are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression P is $2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \ldots$.

Progression Q is $3,\ 1,\ \frac{1}{3},\ \frac{1}{9},\ \ldots$.

(i) Find the sum to infinity of progression R. [3]

(ii) Given that the first term of R is 4, find the sum of the first three terms of R. [3]

5 **(i)** Show that $\sin^4\theta - \cos^4\theta \equiv 2\sin^2\theta - 1$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta = \frac{1}{2}$ for $0° \leqslant \theta \leqslant 360°$. [4]

6 A is the point $(a,\ 2a - 1)$ and B is the point $(2a + 4,\ 3a + 9)$, where a is a constant.

(i) Find, in terms of a, the gradient of a line perpendicular to AB. [3]

(ii) Given that the distance AB is $\sqrt{(260)}$, find the possible values of a. [4]

7 Three points, O, A and B, are such that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + p\mathbf{k}$ and $\overrightarrow{OB} = -7\mathbf{i} + (1 - p)\mathbf{j} + p\mathbf{k}$, where p is a constant.

(i) Find the values of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [3]

(ii) The magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are a and b respectively. Find the value of p for which $b^2 = 2a^2$. [2]

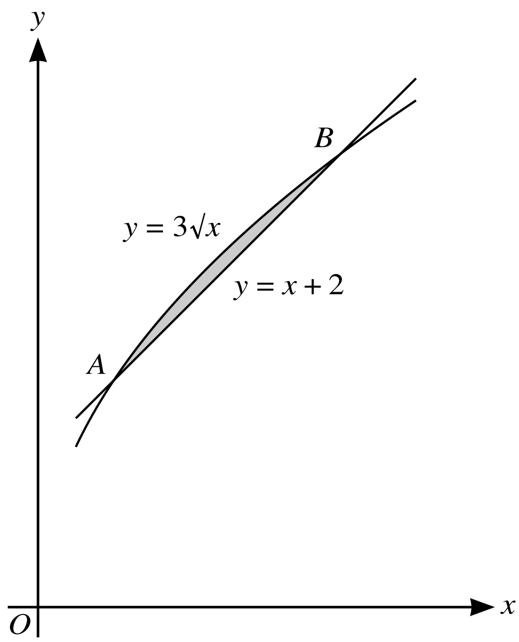
(iii) Find the unit vector in the direction of $\overrightarrow{AB}$ when $p = -8$. [3]

8 A curve $y = \mathrm{f}(x)$ has a stationary point at $(3, 7)$ and is such that $\mathrm{f}''(x) = 36x^{-3}$.

(i) State, with a reason, whether this stationary point is a maximum or a minimum. [1]

(ii) Find $\mathrm{f}'(x)$ and $\mathrm{f}(x)$. [7]

9



The diagram shows parts of the graphs of $y = x + 2$ and $y = 3\sqrt{x}$ intersecting at points A and B.

(i) Write down an equation satisfied by the x-coordinates of A and B. Solve this equation and hence find the coordinates of A and B. [4]

(ii) Find by integration the area of the shaded region. [6]

[Question 10 is printed on the next page.]

10 **(a)** The functions f and g are defined for $x \geqslant 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$
$$g : x \mapsto x^2.$$

Given that $fg(1) = 2$ and $gf(9) = 16$,

(i) calculate the values of a and b, [4]

(ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(b) A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x-coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y-coordinate of P at the instant when P is at the point (3, 4). [5]